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ABSTRACT

This two-volume teacher's guide is to be used with a first-semester, third-grade mathematics program. The program is in large part the outgrowth of involvement with children and was influenced by information gathered in an extended pilot project conducted nationwide during 1975-79. The manner of presentation and the pedagogical ideas and tools are based on the works of Georges and Frederique Papy. They are recognized as having introduced colored arrow drawings ("papygrams") and models of our numeration system (the Papy "minicomputer"). This contains five sections: (1) General Introduction: (2) World of Whole Numbers: (3) Languages of Strings and Arrows: (4) Geometry and Measurement: and (5) Workbooks. Each section has its own table of contents. A series of lesson plans include capsule lesson summaries, lists of materials needed for both teacher and student, and lesson descriptions which suggest possible classroom dialogue and indicate the use of worksheets and manipulative materials. (Author/MK)

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Comprehensive School Mathematics Program
CEMREL, Inc.
Final Experimental Version
3-41401

CSMP Mathematics for the Upper Primary Grades
Part III

Teacher's Guide

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
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General Introduction
The World of Numbers

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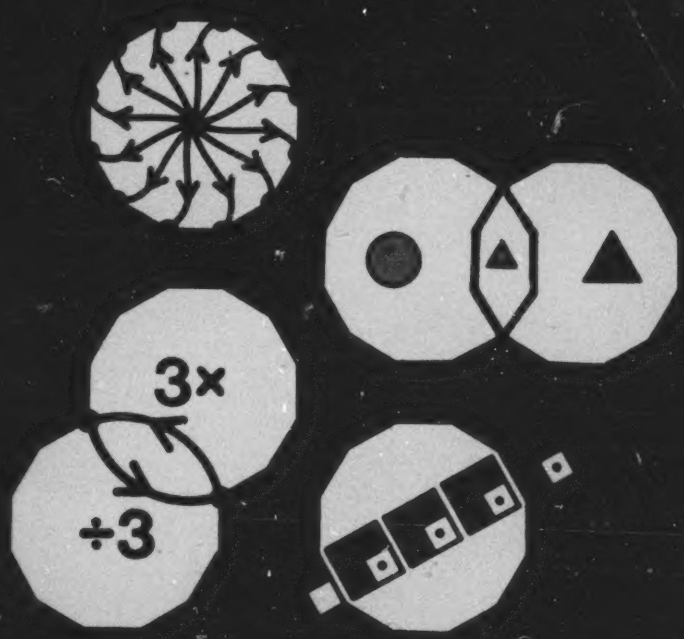

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Comprehensive School Mathematics Program
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PREFACE

CSMP Mathematics for the Upper Primary Grades (UPG) owes its existence to the coming together under the leadership of Burt Kaufman (CSMP Director) of a special group of people who have been concerned not only about the kind of mathematics that is being taught to children but also about the manner in which it is taught. This is even more important in the crucial early years of a child's life than in later years.

The CSMP elementary school curriculum is in large part the outgrowth of a great deal of active involvement with children in ordinary classroom settings, and its present form was strongly influenced by information gathered from classes participating in an Extended Pilot Test conducted nationwide during 1975-79. But its manner of presentation and its main pedagogical ideas and tools, which stand out from all other curricula, are based on the pioneering works of Georges and Frédérique Papy acknowledged here with deep gratitude.

Use of colored arrow diagrams* -

Georges Papy, Die Erste Elemente der Moderne Mathematik: Shriftenreihe zur Mathematik, Hefte 10-11 (Frankfurt and Hamburg: Otto Salle Verlag, 1962-63)
Mathématique Moderne, Vols, 1,2,3,5,6 (Brussels and Paris: Didier, 1964-1967)
Modern Mathematics 1,2 (London: Collier-Macmillan Ltd., 1968)

Frédérique and Georges Papy, L'Enfant et les Graphes (Brussels, Montreal and Paris: Didier, 1968)
Graphs and the Child (Montreal: Algonquin, 1970)

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* In 1959, Georges Papy introduced colored arrow diagrams into the teaching of mathematics at the elementary and the secondary school level. For this reason, they are sometimes referred to as "papygrams" in his honor.

Les Enfants de 4 Ans et le Langage des Graphes (Paris: Classiques Hachette, 1973)

Origin of the Papy Minicomputer -

Georges Papy, Minicomputer (Brussels: IVAC, 1968) in collaboration with Frédérique Papy

Use of graphs and the Papy Minicomputer with young children -

Frédérique Papy, Les Enfants et la Mathématique[*], Vols, 1, 2, 3, 4 (Brussels, Montreal and Paris: Didier, 1970-1976)

Mathematics and the Child 1, (Montreal: Algonquin, 1971)

By making this wealth of experience available to CSMP, Frédérique and Georges Papy have made an invaluable contribution to the production of this program. It must also be acknowledged that the fundamental idea of presenting mathematics to children via "pedagogical situations" is due to the mathematician and psychologist Caleb Gattegno.

Various prototype versions of the UPG Program dating back to 1974 were due to the efforts of former CSMP staff members José Agusti, Phyllis Klein, Edward Martin, and Christiane Vandeputte, and staff members Burt Kaufman and Frédérique Papy.

The large amount of information gleaned, thanks to the interest and concern of many teachers, in the Extended Pilot Tests since 1975 led to the revision of the program that culminated in this final version. During the 1977-78 school year, this final version of Part III was tried out by CSMP staff members Clare Heidema and Janis Schweitzer in West Richmond Elementary School, Maplewood-Richmond Heights School District, St. Louis, Missouri. In this effort they received valued collaboration and support of third grade teacher Dorothy Daily and the motivating responses of her students.

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[*] A series of books reporting on an experiment in teaching mathematics in grades 1 to 6 in Brussels, using the languages of strings, arrows, and the Papy Minicomputer. The reports of the first four years have been published by Didier; the reports of the last two years exist only in manuscript.

NOTE ON AUTHORSHIP

This printed copy of the final experimental version of CSMP Mathematics for the Upper Primary Grades, Part III, is the result of the work of the following people:

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Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades
Part III

General Introduction

CEMREL, Inc.

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SECTION ONE

NOTES TO THE TEACHER

WELCOME

You are about to teach the third semester (Part III) of CSMP Mathematics for the Upper Primary Grades (grades 2 and 3), perhaps for the first time. We wish you success in your teaching and hope that both you and your students will enjoy the activities suggested in this teacher's guide. It is the purpose of these Notes to the Teacher to help you use this book effectively, to describe the materials and to give suggestions for organizing the program, and to answer some frequently asked questions which may also be your questions about CSMP.

You will find, as you and your students take part in the activities during the year, that there is a definite CSMP viewpoint about elementary school mathematics, a viewpoint that we hope you will come to understand and to share. We believe that learning takes place when children react to interesting situations (sometimes called a "pedagogy of situations"*), whether real life or fantasy such as in stories or games. These situations involve children personally and allow arithmetic to take the form of "adventures in the world of numbers".

We believe that every child can and does learn something from each situation encountered in the program, sometimes suddenly and dramatically, sometimes latently. For this reason, CSMP teachers do not stick to one topic until it is "mastered". Indeed our experience indicates that true learning often stops when the purpose of a lesson is merely the mastery of a skill. Instead, we believe that learning is more a "spiral" process than a "straight line" process, that intuitive leaps play as big a role as acquiring small successive pieces of information, and that a variety of situations can provide the interrelated experiences through which we learn.

The Question and Answer section of these Notes to the Teacher gives more explanation of what we mean by a "pedagogy of situations" and a "spiral development", but you will probably learn their meanings only by experiencing them yourself as you teach the curriculum. Meanwhile we assure you that a large number of CSMP teachers across the country have come to understand and embrace the CSMP style of teaching and learning and are greatly pleased with the results.*

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* This phrase is due to the mathematician and psychologist, Caleb Gattegno.

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* See evaluation Report 2-A-1 in the CSMP Evaluation Report Series, CEMREL, Inc., St. Louis, Missouri.

PROGRAM DESCRIPTION

CSMP Mathematics for the Upper Primary Grades (UPG) is designed as a program for students in second and third grades. There are four parts to the curriculum, each corresponding to one semester's work. This is Part III of the four.

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson is an end in itself. It is neither intended nor expected that every student will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations from day to day rather than to continue one type of situation or topic until so-called mastery has occurred. This spiral development (see also Question 1 of Section Two) is reflected in the materials and scheduling of various topics in the curriculum. It is consistent with CSMP's pedagogy of situations since with such an approach, a child experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging. We hope you will keep in mind the spiral approach as you teach the CSMP curriculum.

1. Description of Materials

a) Teacher's Guide

This set of two volumes (five books) which is directed specifically to you, the teacher, is known as the "Teacher's Guide to CSMP Mathematics for the Upper Primary Grades, Part III". (By way of abbreviation, in what follows we shall refer to this collection simply as the "Teacher's Guide".) With the exception of this book, "General Introduction", each book of the Teacher's Guide addresses itself to a single area of the curriculum, and each of these areas is known as a "strand". The four strands with abbreviations given in parentheses are:

- World of Numbers - (N)

- Languages of Strings and Arrows - (L)
- Geometry and Measurement - (G)
- Workbooks - (W)

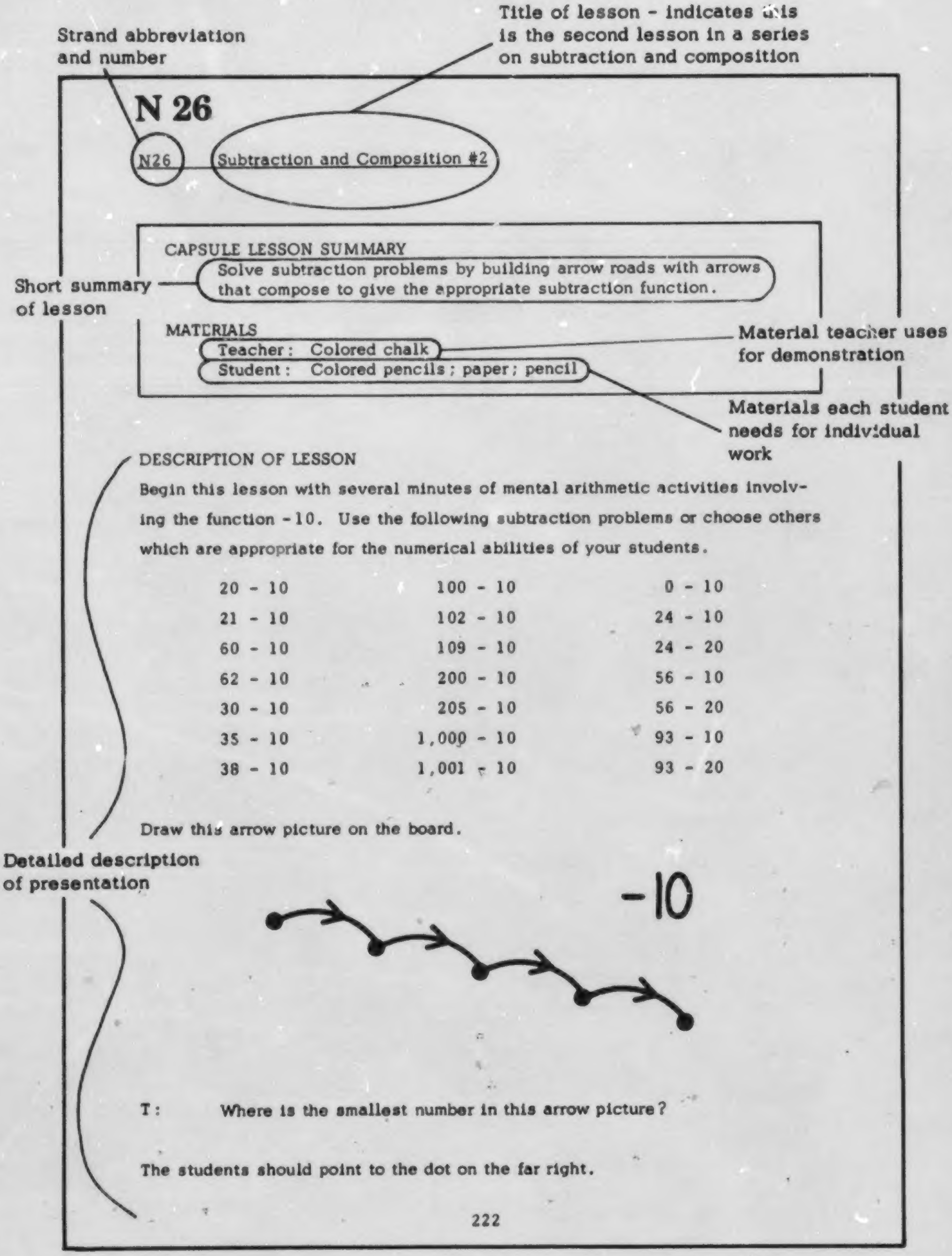
The five books comprising the Teacher's Guide are bound into two volumes, with a separation page inserted between the various books of a given volume. This volume contains both the General Introduction and the World of Numbers strand. The other volume contains the Languages of Strings and Arrows strand, the Geometry and Measurement strand, and the Workbooks strand. Each of these five books has its own table of contents and is paginated individually.

The strands have the following arrangement of material.

- A table of contents. This enables you to locate whatever you need in that particular strand.
- An introduction to the strand. This provides you with information explaining why certain material finds its place in that strand, what are the overall goals of study in that strand, and how this material fits in with that which is described in other strands.
- A series of lesson plans. These plans include Capsule Lesson Summaries, lists of needed materials for both teacher and student, and lesson descriptions which suggest possible classroom dialogue and indicate the use of worksheets and manipulative materials. The lesson descriptions also provide illustrations of diagrams which the teacher puts on the board or otherwise displays for the class and answer keys for worksheets or other independent work during the lessons.

In each of the strand books, N, L, G, and W, the lessons are consecutively numbered. Each lesson is designated to be used during one full class period. For example, Lesson N8 is the eighth lesson in the N (World of Numbers) strand. Since this lesson is surrounded by lessons from other strands, it might be taught on, say, Wednesday of the fourth week of the term and the next N lesson (N9) might be taught on Monday of the fifth week.

Each lesson is presented in a form that includes its strand and number, title, summary, materials, and description. For example, here is the first page from the twenty-sixth lesson in the N (World of Numbers) strand.



Title of lesson - indicates this is the second lesson in a series on subtraction and composition

Strand abbreviation and number

N 26

N26 Subtraction and Composition #2

CAPSULE LESSON SUMMARY

Solve subtraction problems by building arrow roads with arrows that compose to give the appropriate subtraction function.

Short summary of lesson

MATERIALS

Teacher: Colored chalk

Student: Colored pencils; paper; pencil

Material teacher uses for demonstration

Materials each student needs for individual work

DESCRIPTION OF LESSON

Begin this lesson with several minutes of mental arithmetic activities involving the function -10. Use the following subtraction problems or choose others which are appropriate for the numerical abilities of your students.

20 - 10
21 - 10
60 - 10
62 - 10
30 - 10
35 - 10
38 - 10

100 - 10
102 - 10
109 - 10
200 - 10
205 - 10
1,000 - 10
1,001 - 10

0 - 10
24 - 10
24 - 20
56 - 10
56 - 20
93 - 10
93 - 20

Draw this arrow picture on the board.

Detailed description of presentation

T: Where is the smallest number in this arrow picture?

The students should point to the dot on the far right.

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The organization of lessons in the teacher's guide for UPG-III, as for UPG-I and UPG-II, is a strand-to-strand presentation. By this we mean that on one day of the week a lesson is taught from a certain strand, and then on the next day of the week a lesson is taught from another strand, and so on. In this way we can achieve a spiraling effect and at the same time gain the benefit of organization of topics according to strands. For example, suppose you are interested in the way a topic in geometry develops in the third grade. With a day-to-day format you would need to search through the entire guide for geometry lessons, but with a strand-to-strand format all these lessons are found, in their sequential order, in one part of the guide, the G-strand. You can then read the lessons in their natural order to gain an overview of the Geometry strand in third grade (first semester). At the same time, the strand-to-strand arrangement allows you to anticipate what is coming on the next "spoke" of the spiral development of any topic simply by reading ahead in the strand book in which that topic is developed.

b) Teacher's Materials

In addition to the teacher's guide, there are some demonstration manipulative materials you will need. The following materials are available from CSMP and should be in your classroom set of materials.

- Demonstration Minicomputer Set
  (4 magnetic Minicomputer boards; 1 set of magnetic checkers)
- A-Block String Game Kit
  (2 sets of A-Blocks; 2 attribute posters; 2 sets of string game cards; magnetic material)
- Classroom Number Line
- UPG-III Poster Set
  (Two "World of Numbers" posters, one Language poster, one Workbook poster, and twelve Geometry posters)
- Poster Grid Sheets

- Two Colored Cubes (one blue and one red)
- One Set of ⑩-checkers
- Marble Set

Other materials needed for individual lessons are listed under MATERIALS at the beginning of each lesson. Usually such materials (those not in your classroom set) are readily available or can be easily prepared from usual classroom supplies.

c) Student's Materials

There are some individual materials that each student (each pair of students in the case of storybooks) will need. The following materials are available from CSMP and should be in your classroom set of materials.

- Worksheet Booklet for UPG-III (with perforated pages which can be torn off sheet by sheet as needed)
- Workbooks
  (six titles: "Which Road?", "A Festival of Problems #1", "Fishing for Numbers (Part III)", "Road Map", "A Festival of Problems #2", and "20? 100?")
- Story-Workbook from the CSMP LIBRARY ("Rollerskating 37")
- Storybooks from the STORIES BY FREDERIQUE Collection
  ("The Little Donkey" and "Singing Friends")
- Individual Minicomputer Set
  (2 sheets of Minicomputer boards; 1 sheet of punch-out checkers)
- Basketball Data and Graphs
- Centimeter Tape Measure

Other materials needed for a lesson are listed under MATERIALS at the beginning of the lesson. Materials not available from CSMP are readily available or can be easily prepared from usual classroom supplies.

Some teachers find it convenient to provide each student with a sturdy cardboard box for storing manipulative materials. Others find it more convenient to store all the materials of one kind together in a container of appropriate size and to distribute these materials to the students as they are needed. In any case, you should probably keep the worksheet booklets, workbooks, and storybooks in your storage area and distribute them as called for in the lessons.

Some storybooks in the STORIES BY FREDERIQUE collection are provided in your classroom set of materials although they are not used in any lessons of the UPG-III Program. There are two copies of each such storybook; we suggest that these be available in a reading corner for students to read whenever they wish.

2. Role and Use of Worksheets, Workbooks, and Storybooks

a) Worksheets

With some lessons there are worksheets for independent work. Worksheets are designated with numbers corresponding to the lessons with which they are used and with stars indicating the order in which they are to be done by the students. The stars also suggest an increasing level of difficulty and, thereby, provide for various abilities of students. IMPORTANT: It is imperative that you do not insist on every student finishing every worksheet before going on to the next lesson. The worksheets are designed to accommodate differences in students' interests and abilities. Some students will finish more worksheets than others during the allotted time. But the same ideas will emerge in new settings in later lessons, so that students will have other opportunities to work independently on the same topics.

b) Workbooks and Story-Workbooks

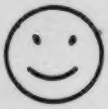
Part III of the CSMP Upper Primary Grades Program (UPG-III) includes six workbooks titled: "Which Road?", "A Festival of Problems #1", "Fishing for Numbers (Part III)", "Road Map", "A Festival of Problems #2", and

"20? 100?") and one story-workbook, "Rollerskating 37". With the exception of "Road Map", each is a thirty-two page booklet that reviews problems which have been encountered up to that point in the curriculum. Two lessons are scheduled for each thirty-two page workbook, and we suggest that all students be instructed to start at the beginning of the booklet and to work independently through as many pages as they can during the scheduled periods.

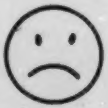
Ordinarily with a thirty-two page workbook the first ten pages of a workbook are easy enough to guarantee some success for all the students; the next ten to twelve pages are average level difficulty; and the remaining pages are more difficult, often challenging even your best students. We would estimate that about two-thirds of your students should correctly finish the first ten pages during the two scheduled lessons, one-third should finish the first twenty pages, and a few might finish all or most of the pages. Of course, these proportions will vary from class to class.

One class period is devoted to "Road Map"; two full class periods are devoted to each of the other workbooks. When two full class periods are allotted, you should collect the workbooks at the end of the first period and check some or all of the pages completed. Then when you return the workbooks at the beginning of the next workbook period, the students can correct their mistakes before going on with new problems.

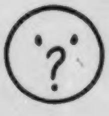
Notice that the cover page of each workbook includes spaces for marking your evaluation of the work on each page. One way to do this is to use these symbols:



Meaning "well done" or "correct"

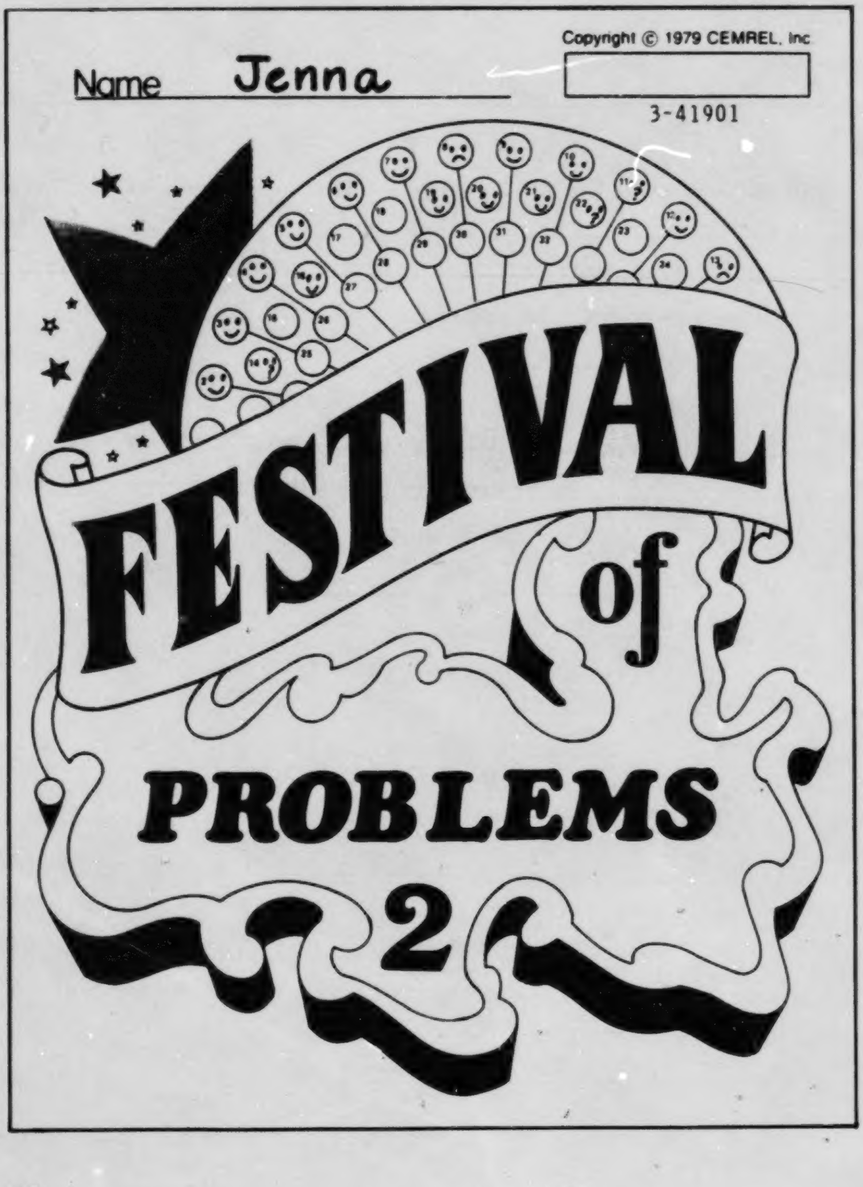


Meaning "something wrong, try again"



Meaning "did you forget this problem?"
or "do you need a hint?"

Here is the front cover of one student's "Festival of Problems #2" workbook after it was checked following the first lesson devoted to this workbook. This is the way Jenna's workbook was returned to her at the beginning of the second lesson. The empty bubbles numbered 15, 17, 18, and 23-32 indicate that she has not yet worked on those pages. Before Jenna begins on those pages during the second lesson, she should correct her mistakes on pages 8 and 13 and finish her work on pages 11, 14, and 22.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PROBLEMS
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IMPORTANT: Again we ask that you not require or even encourage every student to finish every problem in a workbook. If, at the end of the two lessons allotted for a workbook, some students have not completed as much of the workbook as you expect that they are capable of, you might suggest that they spend some extra time on it during the next adjustment day. It would not be useful to demand completion or "mastery" before permitting work to begin on the next workbook series.

After the second workbook period, and after you have checked the results, you might want to send the workbooks home with students. This could be one way to keep parents informed of their children's progress.

One of the story-workbooks from the CSMP LIBRARY, "Rollerskating 37" is used as the basis for two lessons in UPG-III. A story-workbook has both the flavor of a storybook and the advantages of a workbook. In these two lessons the students read "Rollerskating 37" collectively, stopping periodically to do some individual work.

c) Storybooks

Presently, there are twenty-four storybooks for young readers in the collection STORIES BY FREDERIQUE. Each story is presented in a colorful newsprint booklet and can be used for individual or collective reading at school or at home. The stories are independent of each other and for the most part need no teacher explanations. The titles of the books in the collection and the intended age range for each story are:

Ages 5 to 8

THE PLAYFUL NUMBERS
81 ROSES
I AM A VERY HAPPY BOY
ONE OUT OF SEVEN
THE HAPPY PUPPET
TWO BY TWO
THE LITTLE DREAMER
THE LITTLE DONKEY
THE MAGIC BOX
THE BABY IS BORN
THE WEIRD STORY OF 24
THE OLD SHOEMAKER

Ages 8 to 12

WHERE'S MY NOSE?
SINGING FRIENDS
DANCING FRIENDS
THE LIVING LINES
I AM NOT MY NAME
NABU WINS AN AWARD
THE SQUARE TRAP
SUMMER SCHOOL IN THE OLD DAYS

Ages 10 to 14

THE HIDDEN TREASURE
A VERY STRANGE NEIGHBORHOOD
ELECTION IN THE NUMBER WORLD
A VALENTINE MYSTERY

Two of these storybooks are used as the basis of lessons in the UPG-III Program. Young people can become personally involved in these stories; there are questions to answer, pictures to relate to the story, information to be found in the pictures, and new adventures to invent for the characters in the stories. They find that in the world of numbers there are emotions to be expressed and aesthetic senses to be developed just as in the world of people. Your students might enjoy rereading (or just looking at) a storybook many times after they have used it for some lessons. Other storybooks (additional titles from the STORIES BY FREDERIQUE collection have been included in your classroom set of materials although they are not used with lessons) could be made available for individual reading during free time.

3. Role and Use of Hand-Calculators

The electronic hand-calculator, if used creatively, can have a remarkably beneficial effect on the students' interest in and understanding of arithmetic. Its use can be a contributing factor in the improvement of their capabilities in that area. The reasons for this are many and varied, but among them are certainly the following:

- By removing the burden of mechanical computation from the children's shoulders, the calculator frees them to concentrate on the thinking that is a necessary part of the solution to any meaningful problem.

- The instant feedback provided by the calculator enables the children to indulge in making and testing conjectures, an area into which they would not otherwise venture because the quantity of computation involved would be overwhelming.

This pedagogical tool is particularly useful in the introduction of new numerical notions such as negative integers, decimal numbers, multiples of a given whole number, and so on, because through suitably chosen, rich situations these concepts can be made to arise in a natural way.

Two battery powered hand-calculators are recommended for UPG-III[*] and many different kinds are available from CSMP as a distributor for National Semi-Conductor, Inc. and Texas Instruments, Inc. Any of the products of these two companies can be obtained at discount prices.

Two hand-calculators are adequate for most lessons in UPG III, although there are two lessons in which it is recommended that each student have a hand-calculator. Encourage the students to bring calculators from home. The hand-calculators that are available from CSMP have algebraic logic and function keys which are permanently in a constant mode. You should check to be sure that any calculators brought in by students also have these characteristics.

- Algebraic logic. This means that the calculator responds to instructions given in the order in which they are usually written. To check that this is the case, press $\boxed{2}\ \boxed{+}\ \boxed{3}\ \boxed{=}$. If you have "5" on the display, then all is well.

- Constant mode. Some calculators have a switch so that the calculator can be set either in a constant mode or in a chain mode. If your calculator lacks such a switch, it may well be that the calculator has an automatic constant. For calculators with an automatic constant, the first number entered during multiplication is usually the constant, while the second number entered during the other three operations (addition, subtraction, division) is usually the constant.

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[*] Several UPG-III lessons require two hand-calculators in the class. If you already have calculators, be sure to check them for algebraic logic and constant mode (as explained on this and the following page).

To check your calculator for the constant mode operation of $\boxed{+}$ is a simple procedure. Press $\boxed{0}\boxed{+}\boxed{2}\boxed{=}\boxed{=}\boxed{=}$ and so on. If the number on the display is increasing by two at each depression of $\boxed{=}$, then $\boxed{+}$ is operating in a constant mode. Check the constant mode operation of $\boxed{-}$ in a similar fashion.

To check the constant mode operation of $\boxed{x}$ (where the first number entered will be the constant), press $\boxed{2}\boxed{x}\boxed{1}\boxed{=}\boxed{=}\boxed{=}$ and so on. If the number on the display doubles at each depression of $\boxed{=}$, then $\boxed{x}$ is operating in a constant mode.

While it would be possible to use calculators without these characteristics in the lesson, the adjustments that would be required in the instructions make the calculators unsuitable.

In several lessons, the hand-calculator is used to explore patterns in a sequence of numbers which appear on the display. For example, put 3 on the display and press $\boxed{+}\boxed{5}\boxed{=}\boxed{=}\boxed{=}$ and so on. The sequence 3, 8, 13, 18, 23, 28, ... should appear. Another example in which the positive multiples of some number are generated is to put 4 on the display and press $\boxed{+}\boxed{=}\boxed{=}\boxed{=}$ and so on. The sequence of positive multiples of 4 should appear; that is, 4, 8, 12, 16, 20, 24, 28, ... Even if your calculators have algebraic logic, it may be necessary for you to check that they operate in the way described in the lessons. If you find that your calculators do not operate in this manner, then try replacing (for example)

$\boxed{+}\boxed{5}\boxed{=}\boxed{=}\boxed{=}$ ... with $\boxed{+}\boxed{5}\boxed{=}\boxed{+}\boxed{5}\boxed{=}\boxed{+}\boxed{5}\boxed{=}$ ...

and

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SUGGESTIONS FOR ORGANIZING THE PROGRAM

1. Schedule of Lessons

The design and schedule of the lessons in this guide should be regarded as suggestions; they reflect the experiences of the CSMP staff in developing the curriculum. It is difficult for curriculum developers to predict which topics will be easy for your students and which lessons will require more or less time for your particular class. Only you will be able to judge that as you teach the lessons. Thus the program is not meant to be a rigid sequence of lessons to be followed exactly, but only to be a suggested guide.

The master schedule shown on pages 22 and 23 was developed as a one semester course for a hypothetical third grade class of average ability. It illustrates some important ideas about organizing the program. You will notice that several different strands are scheduled each week and that even within a strand the topics are interspersed. This was done to carry out the idea of a spiral curriculum. If you organized your own schedule, it is best to follow a similar pattern rather than to spend two or three full weeks, for example, "covering" one particular topic such as composition of functions or negative numbers. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Languages of Strings and Arrows strand, one day to the Geometry and Measurement strand, and one day to the Workbook strand.

In the master schedule, a block represents one full period of approximately 45-50 minutes duration. It may very well be, however, that you and your class will spend more than one period or, possibly, less than one period on some lessons. Exercise your own professional judgment in deciding when to stop one particular lesson and when or whether to take it up again. Just be sure you do not insist that every single child understands every single aspect of a lesson before you go on. It is better to stop a lesson at a point when students' interest is high and a problem remains unsolved than to teach so methodically that everyone in the class understands everything, but is completely bored.

Inside each block of the master schedule, the lesson of the day is designated in several ways. First, there is listed a number which indicates its suggested chronological order among the lessons in that strand. Hence, "N5" indicates that the lesson in question is suggested as the fifth lesson in the N (World of Numbers) strand. This identifying number also appears in the upper outside corner of each page in the description of that lesson, thus enabling a lesson to be located simply by flipping through the pages of the strand book. Second, you are told on which page of the strand book the description of the lesson in question begins. Third, you are given the title of the lesson.

Please bear in mind that the model schedule was designed for a hypothetical class with the usual range of abilities. In order to be successful with the program, you should adjust the schedule to the ability, maturity, and interests of your class. For example, you may wish to extend a lesson or use a workbook for an additional period beyond those scheduled.

2. Adjustment Days

From time to time an adjustment day appears in the schedule. Since we have intended that UPG-III would be the mathematics curriculum for the first semester of the third grade and a usual school semester is eighteen weeks, the schedule has been designed to indicate eighteen weeks of lessons. However, it is also usual for there to be days when no mathematics lesson is taught; e.g., parent-teacher conference days, field trip days, snow days, and so on. These adjustment days should account for such non-class days, and should also give you opportunities to do many kinds of things. You could use an adjustment day to review a lesson you feel needs review, or to complete an unfinished lesson, or to extend some interesting ideas from completed lessons. You could also use an adjustment day to give your students time to become more familiar with the manipulative materials in the program. The students may especially enjoy playing games such as the Number Line Game or the String Game, and an adjustment day could give them another opportunity to play these games. You may wish to use an adjustment day to do some mental arithmetic exercises, or to teach a topic in mathematics that you think would be especially interesting or useful to your class. For suggestions on ways to engage in mental arithmetic with your class, see the next section and Appendix B: MENTAL ARITHMETIC in the World of Numbers strand.

3. Supplementary Activities and Mental Arithmetic

Throughout the lessons there are many activities that children enjoy doing over and over in a variety of forms. For example, the String Game and the Number Line Game (to mention only two such activities) can be played whenever an extra time period is available. Of particular importance and enjoyment are the activities dealing with mental arithmetic; fully one-third of the lessons involve such activities. Other general suggestions about mental arithmetic are given in Appendix B to the World of Numbers strand. Teachers can invent clever variations of these mental arithmetic activities to be used often during a school day as adjuncts to the natural school routines. Some of the most obvious are: checking class attendance (How many present? How many should be here? How many absent?); the opening exercises (What day of the month is it? How many days in this month? How many days left?); distributing materials (How many students? Two pencils for each. How many pencils?).

Some teachers might want to do some supplementary drill work on basic number facts. We would suggest that the drill be kept to a minimum so as not to suggest to children that arithmetic is a chore.

SUGGESTED SCHEDULE OF LESSONS

Master Schedule for the CSMP
Upper Primary Grades, Part III

Week	World of Numbers	Languages of Strings and Arrows	World of Numbers	Workbooks	Geometry and Measurement
1	N1 Minicomputer Introduction #1 N p. 1	L1 The String Game #1 L p. 1	N2 Introductory Arrow Problems #1 N p. 14	W1 WHICH ROAD? (Lesson 1) W p. 1	ADJUSTMENT DAY
2	N3 Minicomputer Introduction #2 N p. 26	L2 Composition Games #1 L p. 10	N4 Roads #1 N p. 39	W2 WHICH ROAD? (Lesson 2) W p. 4	G1 Area #1 G p. 1
3	N5 Multiplication Problems N p. 44	L3 Two Lists L p. 19	N6 The Function +10 and Composition N p. 51	W3 The Empire State Building W p. 15	G2 Taxi-Distance G p. 13
4	N7 Multiplication and Addition Problems #1 N p. 58	L4 Multiples of 3 and 4 L p. 30	N8 Roads #2 N p. 66	W4 FESTIVAL OF PROBLEMS #1 (Lesson 1) W p. 27	G3 Length #1 G p. 20
5	N9 Eli the Elephant N p. 74	L5 Exchanging Names L p. 40	N10 Introductory Arrow Problems #2 N p. 84	W5 FESTIVAL OF PROBLEMS #1 (Lesson 2) W p. 32	ADJUSTMENT DAY
6	N11 Doubling and Halving on the Minicomputer N p. 92	L6 The String Game #2 L p. 49	N12 The Number Line Game N p. 101	W6 FISHING FOR NUMBERS (Lesson 1) W p. 41	G4 Area #2 G p. 28
7	N13 Minicomputer Tug of War #1 N p. 115	L7 Composition Games #2 L p. 55	N14 Roads #3 N p. 118	W7 FISHING FOR NUMBERS (Lesson 2) W p. 43	G5 Nine-square and Sixteen-square G p. 37
8	N15 Subtraction on the Minicomputer N p. 124	L8 Marble Game #1 L p. 63	N16 The Functions 2x and 1/2x N p. 131	W8 A Spinner Game #1 W p. 53	G6 Length #2 (Halloween Message) G p. 47
9	N17 Road Map of Nevada N p. 141	L9 Three Strings L p. 74	N18 +4 Arrow Roads #1 N p. 151	W9 A Spinner Game #2 W p. 61	ADJUSTMENT DAY

10	N19 3 x and 1/3 x on the Minicomputer N p. 158	L10 A Detective Story L p. 84	N20 +4 Arrow Roads #2 N p. 166	W10 ROAD MAP W p. 70	G7 Where Shall We Meet? #1 G p. 51
11	N21 Multiplication and Subtraction Problems N p. 173	L11 THE LITTLE DONKEY L p. 92	N22 Subtraction and Composition #1 N p. 181	W11 ROLLERSKATING 37 (Lesson 1) W p. 77	G8 Where Shall We Meet? #2 G p. 63
12	N23 10 x on the Minicomputer N p. 190	L12 Negative Numbers Game with Two Cubes L p. 96	N24 The Functions 3 x and 1/3 x N p. 199	W12 ROLLERSKATING 37 (Lesson 2) W p. 81	G9 Area #3 G p. 71
13	N25 Introduction to Decimal Numbers #1 N p. 212	L13 The String Game #3 L p. 104	N26 Subtraction and Composition #2 N p. 222	W13 FESTIVAL OF PROBLEMS #2 (Lesson 1) W p. 91	ADJUSTMENT DAY
14	N27 Introduction to Decimal Numbers #2 N p. 228	L14 Basketball Bar Graphs/Marble Game #2 L p. 109	N28 A 10 x and +1 Arrow Road N p. 238	W14 FESTIVAL OF PROBLEMS #2 (Lesson 2) W p. 97	G10 Length #3 G p. 83
15	N29 Multiplication and Addition Problems #2 N p. 249	L15 Changing Seats L p. 114	N30 Subtraction and Composition #3 N p. 256	W15 Who is Flip? W p. 109	G11 Area #4 G p. 90
16	N31 Road Map of Virginia N p. 260	L16 Counting Problems L p. 123	N32 Subtraction Roads #1 N p. 267	W16 20? 100? (Lesson 1) W p. 119	G12 Hiking Trails #1 G p. 94
17	N33 Subtraction Problems N p. 275	L17 SINGING FRIENDS L p. 131	N34 Multiplication and Composition N p. 282	W17 20? 100? (Lesson 2) W p. 126	ADJUSTMENT DAY
18	N35 Minicomputer Tug of War #2 N p. 291	ADJUSTMENT DAY	N36 Subtraction Roads #2 N p. 297	W18 Who are Tic and Tac? W p. 137	G13 Hiking Trails #2 G p. 101

SECTION TWO

QUESTIONS AND ANSWERS ABOUT CSMP

As you teach the CSMP curriculum or even as you prepare to teach it, there will be some basic questions that come to your mind. Very likely they are the same kinds of questions that parents and supervisors will ask about a curriculum that appears to be different from traditional programs. Perhaps some of the questions that concern you are answered here.

1. Q: WHAT IS THE SPIRAL APPROACH? HOW DOES IT WORK?

A: We learn through many interrelated experiences. But no experience, particularly a third grade math lesson, is an end in itself. With this in mind, CSMP introduces an idea in one lesson, then another in the next lesson, then another. Later the first idea reappears in a slightly new setting or in a slightly expanded version, then a second topic reappears, and so on. Each of the many times that a particular idea or area of study reappears, it is treated in a different manner and at a slightly higher level than the preceding time. This is called a spiral development.

Children learn at different times and at different rates. Only part of the learning of a given topic actually takes place during a lesson on the topic. In between the times when the topic occurs on the learning "spiral", the child is mentally digesting the idea, sometimes consciously, usually unconsciously. In this way the spiral development gives each student a new chance to "catch on" at each stage.

We find it highly effective to follow this kind of development rather than continuing on and on with a topic until so called "mastery" has occurred.

Sticking with the spiral idea requires an act of faith on your part, particularly if you have been used to teaching a given topic until you are satisfied that all (or at least most) of the class knows it "cold". Belief in the spiral approach implies that some lessons may be stoppe before some (or even a majority of) children appear to have caught on or in a lesson involving worksheets or workbooks, before everyone has successfully completed all the pages. It involves knowing that the topic will reappear again and again, and that varying degrees of understanding will come at varying rates and times. It entails a belief that an idea planted now may not sprout until much later, and that it is best not to force its development. All our experience to date (including five years of extended pilot testing of the curriculum) indicates that this approach really works.

2. Q: I UNDERSTAND HOW THE SPIRAL DEVELOPMENT WORKS, BUT HOW CAN I ASSESS MY STUDENTS' PROGRESS UNDER SUCH AN APPROACH?

A: We don't believe it is desirable or reasonable to try to evaluate student mastery of skills from day to day or even from week to week. On the other hand, teachers, parents, and the children themselves benefit from some monitoring of their progress and their involvement in the activities. Most teachers have their own methods for doing this with which they are comfortable, and we suggest that you continue to use your own monitoring methods. You will find that group interaction in the lessons and the individual worksheets and workbooks in the materials provide some valuable information. The spiral development in fact may help rather than hinder your long term assessment of students' progress.

3. Q: FOR WHAT SORT OF STUDENT IS THE CSMP ELEMENTARY CURRICULUM DESIGNED?

A: It is designed for all children in the schools. Each learning situation is designed to involve all students at their own levels in such a way that even the slowest can succeed in dealing with some aspects of the situation that challenge them. In fact, it has been found that such a "pedagogy of situations" (see Question 4) is particularly effective with the so-called "slow" learners, and recent work suggests it might be effective also with educable mentally handicapped children.

4. Q: BRIEFLY, HOW DOES CSMP DIFFER FROM A TRADITIONAL MATHEMATICS CURRICULUM?

A: The core content of CSMP is not revolutionary. The program aims to inculcate ideas and facts that have always been considered important in mathematics. The major differences are in method. Traditional teaching takes pride in disciplining students to be submitted to the usual paper and pencil algorithms. CSMP students are involved as well in many other real mathematical activities, with understanding at their level. For that purpose, CSMP employs new teaching aids such as non-verbal languages (see Question 7), which are themselves part of mathematics.

A pedagogy of situations is used to permit children to participate in the learning process. The students are faced with situations and are led by their reactions to important ideas in mathematics. Thereby, students begin to mathematize situations, a process which is of fundamental importance whenever mathematics is applied.

CSMP also makes use of a spiral approach rather than the mastery approach found in most mathematics curricula. (See Question 1)

5. Q: ISN'T THIS THE "NEW MATH"?

A: There is a current popular belief that all school mathematics today is either "new math" or "old math". Neither of these labels is well defined. "New math" usually denotes the programs that heavily emphasize the teaching of sets; that introduce technical mathematical language for concepts such as "commutativity", "associativity", "distributivity", and so on; and that teach "concepts" before "skills". The "old math" refers to programs of the "good old days": memorization of the multiplication tables and complete emphasis on paper and pencil drill in calculations.

The CSMP curriculum is definitely not the "new math" nor the "old math" by any definition. It uses no technical language, yet achieves the understandings sought by the "new math". It uses no repetitive drills, yet elicits the ability to calculate stressed by the "old math". But these acheivements are just by-products of a more pervasive goal: to help children use their curiosity, their intuition, their powers of reasoning to formulate and think about interesting problems and situations.

6. Q: ARE ALL THE TOPICS IN THE CSMP CURRICULUM REALLY MATHEMATICS?

A: Yes. Mathematics is much more than calculation with numbers, and its methods include more than paper and pencil algorithms. The CSMP curriculum is filled with experiences that directly and indirectly relate to the wide world of numbers, their interrelationships and the kinds of logical reasoning about these relationships that make mathematics an activity of interest and concern.

7. Q: WHY USE THE PAPY MINICOMPUTER AND THE LANGUAGES OF STRINGS AND ARROWS?

A: It is surprising that the language of strings was not used earlier and more extensively in the teaching of mathematics. Mathematicians have used strings for a very long time as they support classification notions so fundamental and natural that they go back to Aristotle. The scope of the string diagrams is broader than mathematics, the string diagram being a marvelous instrument for logical thinking. String diagrams permit the understanding of a situation at a glance; they generate math concepts and ideas through child-related discussion.

Arrow diagrams are probably the most important teaching aid in CSMP. They are used everywhere in our daily lives: signals, advertisements, directions, and so on. Why not use them in teaching mathematics? Arrows are a pictorial way to represent relations. Relations have always been important in mathematics, but in the past seemed to be something rather abstract, elusive, and mysterious. With arrows they become vivid, concrete, and easy to understand.

The Minicomputer allows very young children to become familiar with numbers and operations on them (especially large numbers and the more difficult calculations) before they are able to work with them on paper. Later, and more importantly, the Minicomputer is used as a vehicle for posing situations involving mental arithmetic and for examining the workings of the arithmetic operations. It provides experiences that either support or directly lend themselves to the development of usual algorithms.

8. Q: IF THE STUDENTS USE THE MINICOMPUTER, WILL THEY BE ABLE TO CALCULATE WITHOUT IT?

A: This question suggests that the only purpose of the Minicomputer in the CSMP curriculum is to perform calculations. To be sure, the Minicomputer does enable very young children to deal with "large" numbers and perform relatively difficult calculations long before they know the paper and pencil algorithms for the calculations. But very soon these experiences with the Minicomputer suggest how the paper and pencil algorithms come about, and from that moment the Minicomputer is no longer regarded as a calculating device. Later, for the remainder of the curriculum, the Minicomputer assumes its primary roles: as a source of very interesting problems and games, as an effective vehicle for stimulating mental arithmetic, as a teaching device for introducing more advanced topics such as the negative numbers and the decimal numbers, and, in general, as an alternative to the usual representation of numbers.

9. Q: WHAT IS THE CSMP POSITION ON HAND-CALCULATORS AND WHAT ROLE DO THEY PLAY IN THE PROGRAM?

A: Hand-calculators should be a part of any mathematics curriculum. As more and more children have hand-calculators, these tools will become incorporated in the elementary and the secondary school curricula in a natural way with uses very similar to those of the Minicomputer. In some cases, they may be used as a tool for mechanical computation where the difficulty of the calculation might turn an otherwise interesting problem into an exercise in drudgery. In other cases, they may be used as the context for posing situations leading to the discovery of important mathematical properties or providing opportunities for interesting exercises in mental arithmetic.

Hand-calculators are of particular benefit to "slow" learners. Many slow learners have mechanical difficulties rather than problems in thinking about a situation. The hand-calculator alleviates this problem.

Some lessons in the CSMP curriculum require only two calculators, while some lessons suggest that each student have one. Most often, however, lessons of this latter sort also include suggestions on how they may be modified since not every student may have a hand-calculator. In either situation, students with access to hand-calculators should always be encouraged to bring them to class.

10. Q: ARE THERE GOALS FOR EACH LESSON?

A: At the beginning of each lesson plan there is a CAPSULE LESSON SUMMARY, which describes briefly the activities and experiences included in the lesson. Indirectly these summaries suggest goals but, of course, the goals of the curriculum spread over whole series of lessons and cannot be broken into specific goals for individual lessons. If you read the introduction to the strand in which a lesson appears as well as the Capsule Lesson Summary for that lesson, we expect you will easily see the direction and intent of the lesson. We decline to predict for any one lesson what will happen in terms of student learning.

11. Q: SHOULD I SUPPLEMENT THE PROGRAM WITH MORE PRACTICE IN NUMBER FACTS?

A: There is a surprising amount of such practice throughout the program. It is implicit in the activities and is not in the form of practice for its own sake. If you still feel it is necessary to give supplementary practice, we suggest that you keep repetitive drill to a minimum so as not to endanger the students' interest in numerical situations.

12. Q: WHY IS THERE A LACK OF DRILL AND LITTLE EMPHASIS ON "SKILLS"?

A: CSMP believes that training for rote skill should never precede understanding. The avowed purpose of drill in arithmetic is to acquire the "skills", that is, the skills of calculation. Certain numerical skills are essential but, unfortunately for school children, the usual drill techniques for acquiring them are dull, stultifying, and sometimes even counterproductive. The CSMP curriculum appears to place little emphasis on "skills" because it does not resort to drill. But as a matter of fact, the essential skills of both mental and written calculation are learned almost inadvertently in response to the many situations where the ability to calculate is needed in order to respond to the situation: to win the number game, to identify the secret number of the detective story, or to build a number road. Calculation is always considered as a tool for doing interesting things, rather than as a chore done for its own sake.

13. Q: HOW DO STUDENTS LEARN TO USE MATHEMATICS IN PRACTICAL SITUATIONS?

A: For students of elementary school age, a situation that intrigues them and catches their imagination or their curiosity will certainly motivate them to learn whatever is necessary in order to deal with the situation. But situations that adults consider "practical" very often are outside the realm of interest of school children and do not provide motivation for learning. CSMP has found that students will indeed learn to use mathematics in situations they consider worthy of their attention. As they approach adulthood they will soon consider "practical" situations to be most worthy of their attention, but in the meanwhile, they are more interested in games, fantasy, puzzles, stories, and so on.

14. Q: HOW ARE TELLING TIME, MONEY, AND THE METRIC SYSTEM TREATED?

A: Telling time is not included in the CSMP program. It is our belief that telling time is more naturally a part of a language arts program than of a mathematics program, although it is frequently included in the mathematics curriculum. Since there are many excellent materials available to help you teach telling time, we urge you to teach this topic using those methods and materials you particularly like or that have proved successful to you in the past.

The topic of money is first introduced in the CSMP curriculum in situations involving combinations of coins. It is treated again in the Upper Primary Grades and the Intermediate Grades as a support for the teaching of the decimal numbers. Although money is not treated as a separate topic, these situations provide experience with our monetary system. Should the students need more exposure to this topic, teachers should feel free to develop supplementary materials in order to provide extra practice.

Metric units are used whenever there is cause to measure in standard units. Activities involving the metric system are begun in the First Grade Program and continue throughout the Geometry and Measurement strand of the Upper Primary Grades Program. We recommend that you do not ask the students to convert from the English to the metric system or vice versa (inches to centimeters, yards to meters, Celsius to Fahrenheit, grams to ounces, and so on). It would be confusing for children to deal with two systems simultaneously, especially at this early stage. Since the United States has not yet "gone metric", your students will still need to be familiar with the English system. We would suggest that you use whatever materials and methods you have found successful in the past for teaching this topic, but we repeat our recommendation that you not ask the students to convert from one system to the other.

15. Q: WHAT IS THE ROLE OF STORYBOOKS IN THE CURRICULUM?

A: The storybooks in the collection STORIES BY FREDERIQUE include twenty-four stories for young children, presented in colorful newsprint booklets. Two stories from this collection are used for lessons in UPG-III, with fifteen copies of each storybook provided in a classroom set of materials. (If your school can afford it, we recommend that you order enough extra copies to provide one for each child.) Other storybooks, two copies of each, are provided for supplementary reading. Each of the stories is portrayed by the mathematical language of dots, arrows, and strings which reinforce the ideas of the stories. Young readers can become personally involved in the stories; there are questions to answer, pictures to relate to the story, and new adventures to invent for the characters of the stories. They find that emotions and aesthetic senses are involved in the world of mathematics just as in the world of people.

16. Q: WHAT PROVISION IS MADE FOR INDIVIDUALIZATION IN THE CSMP PROGRAM?

A: In many of the lessons there are worksheets to be done individually by the students. Sometimes the worksheets are done at the beginning of a lesson, followed by a full group discussion of them. But ordinarily a topic is first developed in a full group setting, followed by individual student work on related worksheets. If "individualization" means a procedure for allowing students to work alone at their individual rates and levels, then the worksheets (and also the workbooks, discussed below) do provide for (partial) individualization. A series of worksheets for a given lesson offers a whole spectrum of levels from very simple to quite challenging. Students all start with the first worksheet (one-star) and proceed at whatever pace is appropriate and to levels according to their capabilities. The spiral approach accommodates this kind of work in the same manner as the full group work; that is, no worksheet is a prerequisite for the content of any succeeding lesson.

Eleven lessons during the first semester of the third grade are devoted to the six UPG-III workbooks. Each workbook is also designed to move through a spectrum of levels of difficulty. We believe that the worksheets and workbooks realize the best sense of "individualization".

17. Q: WHAT PROVISIONS ARE MADE FOR THE VARIETY OF ABILITIES IN A USUAL CLASS?

A: Every lesson includes something for all students; there are opportunities for the slowest students to participate in a meaningful way, and there are challenges for even the very brightest of students. This means that you should not exclude any students from a full group lesson. At the same time, you should not expect every student to understand everything during the course of the lesson. Furthermore, with the spiral development many situations are open-ended and allow students to experience them at levels as high as they are capable.

Evaluation has shown that students traditionally labeled "slow" are doing better in CSMP classes than in non-CSMP classes. We believe this is due to the spiral development, to full group work on mathematical activities, and to the presence in the CSMP curriculum of certain nonnumerical activities in which many supposedly "slow" students can excel.

Additionally, CSMP provides for a variety of ability levels in the student materials. Worksheets and workbooks offer problems in a range from very easy to quite challenging. Indeed, some of the "higher starred" worksheets and pages toward the ends of workbooks may be accessible only to your very best students.

In extreme cases, you might seriously consider the possibility of arranging for exceptionally bright (slow) students to take their math with a higher (lower) grade level CSMP class.

18. Q: I PREFER TEACHING SMALL GROUPS, BUT MOST OF THE LESSONS SEEM TO BE AIMED AT THE WHOLE GROUP. WHAT DO YOU SUGGEST?

A: If your school routine allows you to teach each lesson twice, once to each half of your class, that would be an alternative way to proceed. But if you do this, it might be wise to vary the composition of the groups from day to day so that you are not tempted to group your students according to your perception of their ability. Otherwise you would guarantee that half your students would not have as rich an experience as the other half, through no fault of their own. The purpose of the full group experiences is to allow interaction among all the children in every situation posed in the lessons. Students learn quite a bit from each others' reactions. You will be surprised to find that in some situations the key responses are made by students you had labeled "slow". We feel the spiral approach has maximum potential in full group teaching.

The above response might give the impression that CSMP teaching takes place only in full group settings. This is not the case. There is much time devoted to individual student work on worksheets and workbooks, and there are various games to be played. We also encourage you to arrange small group sessions whenever you have time, particularly with your slower students. Small group activities could supplement the full group lessons, but should not replace them. Extra small group work should not be simply for drill and for rehashing ideas, but rather should provide additional experiences.

19. Q: WHY IS A "DIALOGUE" STYLE USED AS FORMAT FOR THE LESSON PLANS? ARE TEACHERS EXPECTED TO FOLLOW THIS DIALOGUE EXACTLY?

A: Most of the lessons are written in considerable detail; step by step descriptions are frequently given, together with suggestions for the exact words a teacher might use (in the form of a possible dialogue between teacher and students). These details are given only for the teachers who feel the need for them. You should never feel obligated to follow any lesson word for word as it is written in the guide. By the same token, you should never insist that your students give the same responses as those given in the imaginary dialogues. Use your own creativity in drawing out students' responses as the situations suggest. Read the lesson plans very carefully beforehand to get an idea how the lessons are expected to proceed.

20. Q: IS IT NECESSARY TO FOLLOW THE SUGGESTED SCHEDULE EXACTLY?

A: This program is not meant to be a static sequence of exercises to be followed rigidly. The lessons in this guide are only suggestions based on our own experience teaching the materials. If this will be the first time you are teaching the program and you would like specific suggestions for organizing the lessons, we offer some guidelines and a suggested schedule of lessons. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate. It is probably best to follow a one-day, one-strand pattern when you organize your own schedule, rather than to spend two or three full weeks "covering" one particular topic. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Languages of Strings and Arrows strand, one day to the Geometry and Measurement strand, and one day to the Workbook strand.

Exercise your own professional judgment in deciding when to stop a particular lesson. It is better to stop a lesson when student interest is high than to teach so methodically that everyone in the class understands everything, but is completely bored. In order to be successful with this curriculum, you should adjust the schedule to the ability, maturity, and interests of your class.

21. Q: HOW DO I HANDLE STUDENTS WHO HAVE AN EXTENDED ABSENCE OR STUDENTS (NEW TO CSMP) WHO TRANSFER INTO MY CLASS DURING THE YEAR?

A: The spiral approach allows a student to miss one or two weeks of school without much "catch up" work needed. When the student returns, either you or classmates can help with any material that is deemed necessary.

If several months of classes have been missed, you might need to treat the student as you would a transfer student new to CSMP. Give the student a little extra attention during a lesson especially when the questions are review for most of the class.

In most cases, the differences between new and veteran students tend to fade rather quickly. One exception might be in lessons involving the Minicomputer. Veteran students who are unusually adept with the Minicomputer very often are good tutors for the transfer students in the use of this abacus.

It is probably best not to send worksheets and/or workbooks home to a child; without participating in the collective lessons that introduce the worksheets and workbooks, it would be very difficult for the student to proceed.

22. Q: HOW DO I HANDLE WHOLE CLASSES THAT ARE NEW TO CSMP?

A: CSMP does not assume that kindergarten and first grade students have any prior experience with CSMP. The Kindergarten and First Grade Programs will need no adaptation for classes that are new to CSMP.

Second grade classes that are new to CSMP will initially progress more slowly than veteran classes. The lessons at the beginning of the second grade program are designed to serve either as an introduction to CSMP languages for new classes or as a quick review for veteran classes.

Third grade classes that are new to CSMP will need to become fluent in the CSMP languages before beginning the lessons in UPG Part III. A special schedule has been prepared for these classes which consists of prerequisite lessons from a third grade entry supplement followed by a slightly modified UPG Parts III and IV program.

23. Q: WHAT SHOULD I DO WITH MY CLASS IF THEY DID NOT FINISH THE PREVIOUS YEAR'S SCHEDULE?

A: No special measures are required. Each level of the CSMP curriculum begins with introductory lessons. In addition, the spiral approach in the lessons permits students to participate at their own level of understanding, even if that level is not quite as high as it might have been if the previous year's schedule had been completed.

24. Q: HOW DO CSMP STUDENTS DO ON STANDARDIZED TESTS?

A: While CSMP disagrees with the philosophy underlying standardized testing, test results are available to us from several sources. The results from standardized tests indicate that CSMP classes generally do as well as or better than do non-CSMP classes. It should be pointed out that standardized testing and traditional programs are correlated, and that CSMP students have been exposed to a great deal of mathematics that is not taught in more traditional programs or that is not considered in the construction of the tests. There are also many unmeasurable or unmeasured areas such as enjoyment of mathematics, strategic thinking, organizational ability, and depth of understanding of the methods and uses of mathematics where we feel CSMP students have a definite advantage. Evaluation Report 2-A-3 in the CSMP Evaluation Report Series gives more detailed answers to this question. Copies are available upon request.

25. Q: WHAT PROBLEMS WOULD CSMP STUDENTS HAVE IF THEY RETURN TO A MORE TRADITIONAL PROGRAM IN THE NEXT SCHOOL YEAR?

A: There should be no problems for CSMP students who return to a traditional program. Test results indicate that CSMP students are at or above the level of non-CSMP students in material covered in traditional courses. In addition, CSMP students have encountered many situations that are not introduced in traditional programs nor tested for on standardized tests (see Question 24). In particular, their reasoning ability should be much sharper than their non-CSMP counterparts.

There are, of course, differences in timing as to when different programs introduce the standard algorithms and often CSMP's introduction will occur later than in some of the other programs. The reason for this is our belief that it is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying, and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

CSMP students usually can solve a problem before having learned the standard algorithm. If they enter a program that expects an algorithm they have not learned, they are still more than adequately prepared for the algorithm and should have no trouble quickly learning to use it.

CSMP introduces the usual addition algorithm in the second grade, one of the two standard subtraction algorithms in the third grade, the multiplication algorithm in the fourth grade, and the division algorithm in the fifth grade.

26. Q WHAT ARE SOME WAYS TO ORGANIZE MATERIALS AND MANAGE MY CLASS?

It is very important to organize your CSMP materials before school begins. Decide where you will store the materials once they are organized. Filing drawers, shelves, boxes, or closets are all satisfactory as long as you can easily get to the materials. Take them all out of their shipping boxes and spread them on several tables and/or desk tops in your classroom so they can be organized into the order you will be using them.

Workbooks: The teacher's guide is needed to find the order in which the workbooks are used. Locate the Workbook strand column in the Master Suggested Schedule of Lessons and collect all the workbooks in the order in which they are listed. After organizing the workbooks in order, place them in an upright position in your file drawer, box, shelf, etc., so that the first workbook to be used is at the front of the stack.

Worksheets: There is a separate worksheet booklet for each student and you should decide whether you will store these booklets and distribute them when a particular lesson calls for a worksheet, or whether you will let the students keep their own worksheet booklets in their desks.

Each worksheet can be removed from the booklet or left in depending on how you wish to manage worksheets. However, once a worksheet is removed from the booklet it cannot be replaced, so make sure students are careful about tearing off the correct sheet.

Manipulatives: Storage of each manipulative in a separate container, properly marked, is one way to make everything accessible. Another way is to provide a sturdy box (for example, a shoe box) for each student and to put individual manipulative materials in the box, properly labeled with the student's name.

Additional Materials: Be sure that you have plenty of high quality colored chalk, large sheets of unlined white paper, a set of felt-tip markers, and that your students have colored pencils, pens, or crayons.

Plan well ahead to be sure you have any needed materials that are not supplied by CSMP. Keep all teacher materials where they are easily accessible. You may find the guides easier to use if they are put in looseleaf binders; the books are perforated so that pages can be removed easily. The demonstration Minicomputer boards and the number line should always be on display in the room.

27. Q: SOME OF THE CSMP MATERIAL IS NEW TO ME. HOW DO I LEARN ENOUGH TO TEACH A CLASS?

A: Every CSMP teacher should have been trained by his or her coordinator. Kindergarten and first grade teachers receive a minimum of eighteen hours training, teachers of the Upper Primary Grades Program receive up to thirty-two hours training, and teachers of the Intermediate Grades Program receive up to fifty-six hours training. During the training sessions the CSMP content is taught to new teachers and suggestions are given for organization of their classes.

If the suggested training schedule has been followed, many lessons taken from the CSMP curriculum, or very similar lessons, have been presented. The training schedule also makes use of the spiral approach so that training sessions aim not only to teach CSMP mathematics but to familiarize teachers with the spirit and the pedagogy of the CSMP curriculum.

28. Q: WHAT SHOULD BE DONE WHEN THERE IS A SUBSTITUTE TEACHER?

A: Some school systems have included people who frequently substitute in their CSMP training program. When this is not the case, it would be very difficult for the untrained substitute to teach from CSMP materials. One solution is to arrange for another CSMP teacher to teach math to your class while the substitute teacher teaches another subject to that teacher's class. If that cannot be arranged and if you will only miss one or two days, you may find certain CSMP material, uncompleted workbooks for example, that the substitute can manage. Otherwise, use those days to cover material not included in the CSMP curriculum, such as telling time. If your absence will be an extended one, arrangements should be made with your coordinator to train a substitute, to rearrange schedules to put a CSMP teacher in your class, or to otherwise provide for your students.

29. Q: THE PROGRAM MAKES USE OF THE STUDENT'S CREATIVE FREEDOM HOW MUCH ROOM IS THERE FOR THE TEACHER'S INGENUITY?

A: CSMP not only allows for your ingenuity but depends on it. While the lessons may appear to be very rigid by presenting dialogue for teacher and student, it should be remembered that these are suggested questions, answers, and comments based on our experience with development classes. Even if you were to follow your part of the "script," your students are likely to offer responses which differ from those we have listed. It is the student's response that will determine the content and the direction of your next question or comment.

In a traditional program your creativity is called upon the first time you develop a particular lesson, and to a lesser extent, as you revise lessons in succeeding years. There is little room for spur-of-the-moment creativity once a lesson begins. The CSMP curriculum, by creating a free and open situation emphasizing the student's creative freedom, continually presents you with unanticipated student response and calls on your ingenuity at all times.

30. Q: HOW CAN I EXPLAIN CSMP TO PARENTS?

A: The best way would be for your coordinator to organize a workshop for interested parents. The parents can then see the languages of the program in use and have some of the same experiences their children have had. Workshops can be of almost any length but anything short of two hours does not allow for much familiarity with the material. In such a workshop we would suggest that the language of the Minicomputer be explained with particular emphasis on its use in presenting situations where mental arithmetic is called upon and in developing algorithms (for example, physically carrying a checker from the ones' to the tens' board in the "8 + 2 = 10" play, paralleling "carrying" in the paper and pencil operation). A lesson using the storybook "Where's My Nose?" provided a good example of the power of the language of strings. A nonnumerical example, such as the brother-sister graph, and a numerical example, such as building a road from 2 to 23 using +5 and -2 arrows, will provide an introduction to the language of arrows. The lessons that are the favorites of the children would be excellent for a presentation to parents as well.

You might also extend an invitation to parents to visit your class during the math period. Many times such an invitation leads eventually to a closer parent-teacher relationship, with the parent offering voluntary services as an aide.

You will find a letter to parents at the end of this section which may be reproduced and sent home with the student.

TO THE PARENTS OF CSMP STUDENTS

Your child is participating in the Comprehensive School Mathematics Program's (CSMP) elementary school curriculum, developed under a contract with the National Institute of Education. While the program cannot be fully explained in a brief note, we hope that this letter will give you some feeling for the substance of your child's mathematics education.

The goals of the CSMP K-6 program includes a development of the standard number systems and their operations (including an early introduction to the integers, rational numbers and decimal numbers) interwoven with a development of geometry and measurement. On an equal footing with the above, the CSMP program includes a strong component of probability, statistics, and combinatorics which extends throughout grades 2-6. Powerful pedagogical devices are used, such as several nonverbal languages. These make the unifying ideas of the program accessible to very young children.

These languages include:

1) The Language of Strings. This "language" mimics the basic methods of collecting and classifying data applying to the problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of set.

2) The Language of Arrows. This graphical language models the process of comparing and analyzing sets and operations on them. That is, it deals with the mathematical notions of relations and functions including, but not restricted to, ordering relations and the numerical processes of adding to, subtracting from, multiplying by, and sharing equally among.

3) The Language of the Papy Minicomputer. The Papy Minicomputer, an extremely effective paper abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to a multitude of numerical algorithms, including the standard algorithms for the basic numerical operations.

Among other things, these languages allow for the development of the basic operations on numbers in a natural way based on previous experiences with the languages. They also provide a context in which situations arise that call for the repeated use of arithmetic operations without resorting to drill exercises, which most children find boring.

The content is learned in an atmosphere of constant applications utilizing a "pedagogy of situations".* These vary from simple story situations in the early primary grades to challenging applications and nontrivial simulations of real world problems in the upper grades.

One of the tenets of CSMP's development philosophy is that mathematics is a unified whole and should be learned as such. As a result, along with CSMP's concern with the variation in ability levels in the typical heterogeneous class, the curriculum is structured in a spiral format throughout the K-6 program. The content is completely sequenced in spiral form so that each student is brought into contact with each area of content continuously throughout the program. This consciously avoids atomizing the content and mastering each bit before continuing to the next. Rather, each student spirals through repeated exposures to the content, building interlocking experiences of increasing sophistication.

Another tenet of CSMP's development philosophy, based on years of experimentation in the classroom, is that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation, and individual independent experiences. Individualization throughout the K-6 program is achieved through the independent work on workbooks and worksheets of varying levels of difficulty. Due to the spiral nature of the curriculum, each student can engage in a workbook without losing touch with the progress of the class while at the same time working to the limits of his or her ability.

---

* Inspired by the work of the mathematician and psychologist, Caleb Cattegno.

A feature unique to the CSMP program is a collection of storybooks written by Frédérique Papy (STORIES BY FREDERIQUE) and a collection of story-workbooks (THE CSMP LIBRARY). These probe the affective domain of learning and bring children into contact with areas of mathematics until now unexplored by them.

CSMP does not have its own system of student assessment and progress reporting. While standardized testing indicates that CSMP students develop the "basic skills" as early or earlier than do non-CSMP students, we hope you will not be obsessed with your child's development of these "skills". Each child is an individual and should not be judged against norms but on what he or she can and does accomplish. Many extremely important areas--development of self-confidence, ability to think and reason, etc.--are very difficult to measure and grade. If you have any questions regarding the progress of your child, his or her teacher is in the best position to answer those questions.

THE CSMP STAFF

THE WORLD OF NUMBERS

Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

Part III

The World of Numbers

CEMREL, Inc.

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THE WORLD OF NUMBERS

INTRODUCTION

Numbers are among the most important things that mathematics (at all levels) is about. Mathematicians are interested in numbers just as astronomers are interested in stars, botanists in plants, and sociologists in the organization and dynamics of human societies. Surely everyone agrees that one of the primary goals of any elementary school mathematics curriculum should be to introduce students to the world of numbers -- to give them the opportunity to become familiar with numbers, their properties, and the relations between them. The ability to calculate with numbers is an important part — but not the only part — of being familiar and comfortable with numbers. The World of Numbers strand of the CSMP Upper Primary Grades Program is designed to provide the students with a wide variety of challenging experiences with numbers so that gradually they will become not only familiar, but comfortable with numbers; they will, so to speak, get to know numbers on a "first name basis".

The Minicomputer and the World of Numbers

The CSMP Upper Primary Grades program uses the Papy Minicomputer* as a support for the positional system of numeration; for calculations and estimation; for number patterns and mental arithmetic; and for modeling the basic operations involving whole numbers, integers, and decimal numbers. Although the Minicomputer can be seen as a tool for calculation and as a device to help students learn routine methods for calculations, its more exciting use is as a vehicle for posing interesting problems which challenge a child's intellectual curiosity about numbers and for presenting situations which dually encourage strategic thinking and reinforce numerical skills. The lessons which make use of the Minicomputer are often intended to be explorations into the world of numbers.

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* See also Questions 7 and 8 of Section Two, Questions and Answers About CSMP, of the General Introduction and APPENDIX A: STANDARD CONFIGURATION AND STANDARD MINICOMPUTER MOVES.

Standard Algorithms of Arithmetic

CSMP seeks to develop basic numerical skills as well as an understanding of the underlying mathematical ideas. We are fully in agreement with the thesis that, along with the growth of understanding of the world of numbers, there must be a concomitant growth of familiarity and facility with numbers and operations on them. But facility should not be confused with understanding; they are partners in the growth of mathematical maturity. A balanced growth of each must be maintained, neither being sacrificed for the other.

Students must eventually learn mechanical algorithms for the basic operations (addition, subtraction, multiplication, division). However, premature presentation of these algorithms may actually stunt a student's ability to develop alternative algorithms, to do mental arithmetic, or to estimate.

Consider the problem of calculating "194 - 57". A third grader may have difficulty performing a paper and pencil subtraction algorithm. An easy way to proceed is to subtract 60 from 194 and then to add 3 (194 - 60 = 134 and 134 + 3 = 137). To insist on a mechanistic response to such a problem would be to encourage inefficiency and might also inhibit the development of the flexibility necessary for problem solving. On the other hand, a rich array of situations in which students interact with numbers provides them with opportunities to gain the necessary facility with standard algorithmic procedures while retaining the openness required to respond creatively to new situations in the world of numbers.

Numerical Relations

One of the main aims of the World of Numbers strand is to familiarize students with numbers by studying explicitly and in a variety of contexts relations between numbers. (For more general comments about mathematical relations, see the introduction to the Languages of Strings and Arrows strand). Arrow diagrams represent relations in a simple, suggestive, and pictorial way--usually more conveniently than the same information could be given in words or other symbols.

Students are brought into contact with an assortment of challenging situations, many of which would be totally inaccessible to them were it not for the arrow diagrams. The problems and activities of this strand include many examples: solving linear equations presented in terms of arrows, studying iterated processes and patterns in sequences of numbers, tackling problems which may have many solutions or no solution, estimating or testing that a solution is reasonable, and exploring properties of operations on numbers.

In summary, what is most important in the study of numbers is to confront the students with a variety of problems and situations which capture their interests, challenge their abilities to reason, and stimulate their curiosities about numbers--and, at the same time, provide them with tools for coping with these situations and problems.

N1 Minicomputer Introduction #1

CAPSULE LESSON SUMMARY

Review the value of the squares on the Minicomputer. On the Minicomputer represent numbers less than or equal to 8,000.

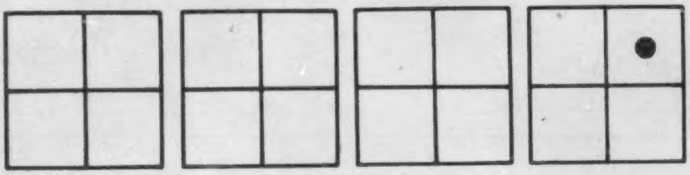
MATERIALS

Teacher: Minicomputer set[*]
Student: Worksheets N1*, **, ***, and ****; pencil

DESCRIPTION OF LESSON

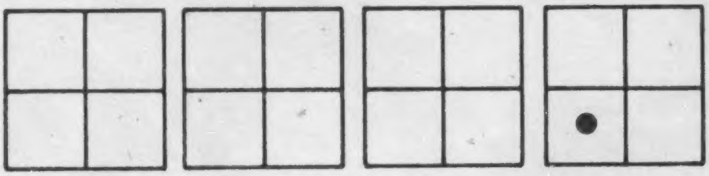
Display four Minicomputer boards.

T: This is the number 4 on the Minicomputer.



To prevent confusion, always remove the checkers from the Minicomputer before asking the next question.

T: Who can put 2 on the Minicomputer?



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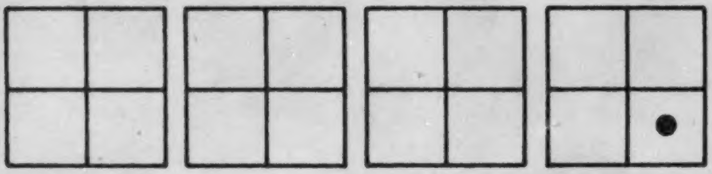
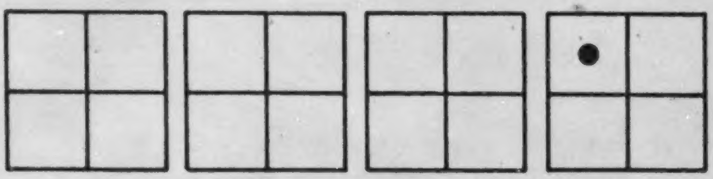
[*] A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers.

N1

T: Who can put 1 on the Minicomputer?

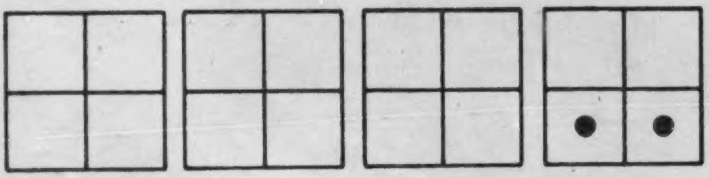


T: Who can put 8 on the Minicomputer?



Very quickly review the placement of the checker for 1, 2, 4, and 8.

T: I'll give you two checkers. Can you put 3 on the Minicomputer?



T: Can you put 5 on the Minicomputer using exactly two checkers?

. . . 6 using exactly two checkers?

. . . 9 using exactly two checkers?

How many regular checkers do you need to put 7 on the Minicomputer? (at least three)

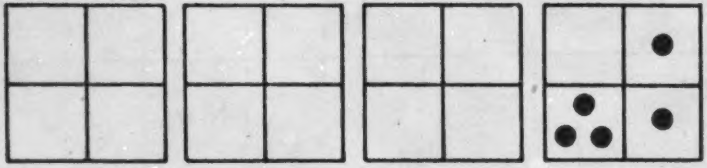
Ask someone to put 7 on the Minicomputer using exactly three checkers.

Exercise 2

Gradually put these checkers on the Minicomputer:

- put three checkers on the red square; pause;
- put one checker on the purple square; pause;
- put one checker on the white square.

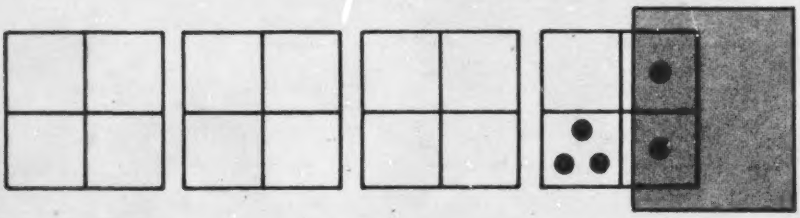
This gives your students a chance to figure out mentally what number it is.



T: Which number is on the Minicomputer? (11)

No explanation is necessary if everybody gets the right number. If someone is wrong, you might use this procedure: cover part of the board with a piece of paper to focus the class' attention on certain checkers and gradually uncover the full set of checkers.

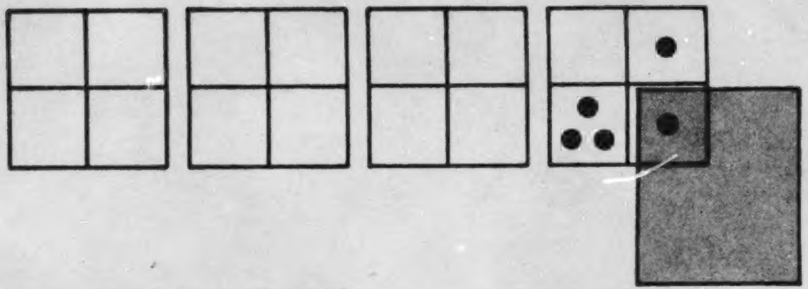
Cover the purple and the white square of the ones' board.



T: What number is this? (6)

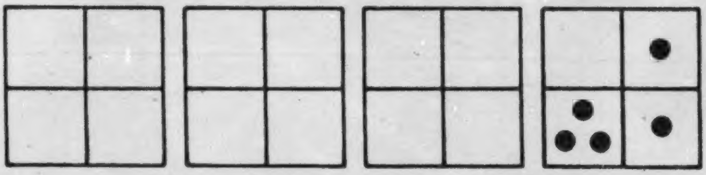
N1

Uncover the purple square of the ones' board.



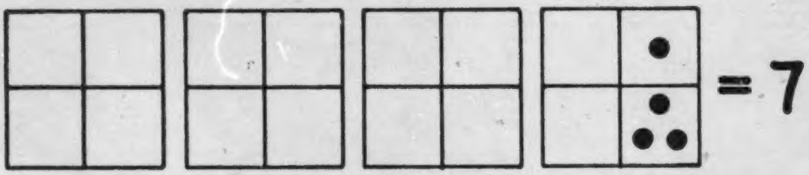
T: . . . and 4 more? (10)

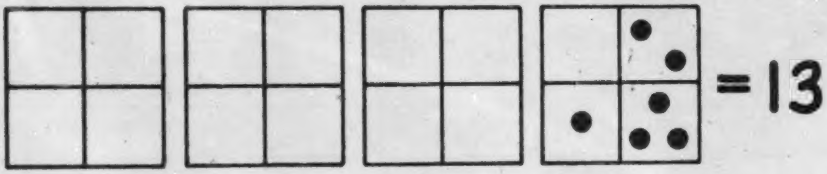
Uncover the ones' board completely.

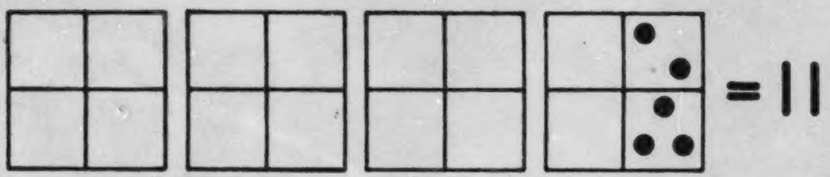


T: . . . and 1 more? (11)

Repeat this activity with other configurations; some possibilities are suggested below and on the next page. Ask the students to record their answers on a piece of paper. Walk around the room and observe many of the students' answers before asking a student to answer aloud.



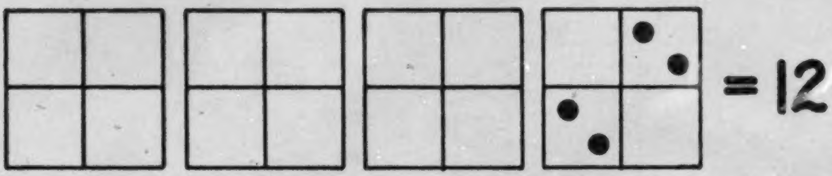


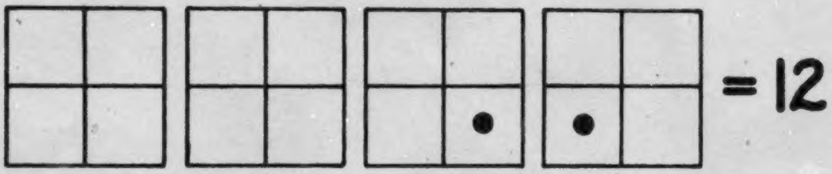


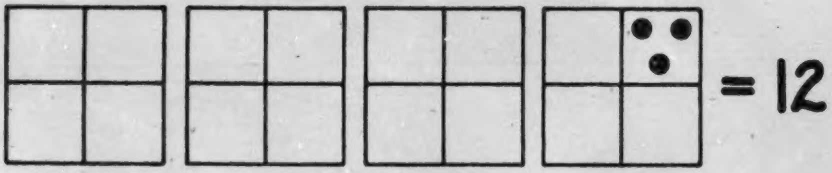
Exercise 3

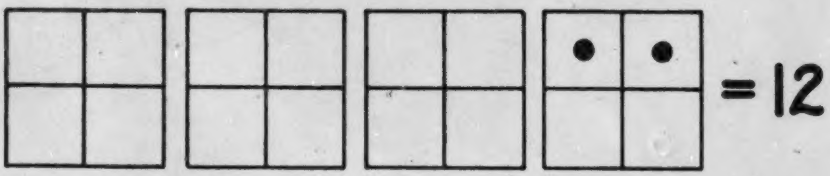
T: Who can put 12 on the Minicomputer?

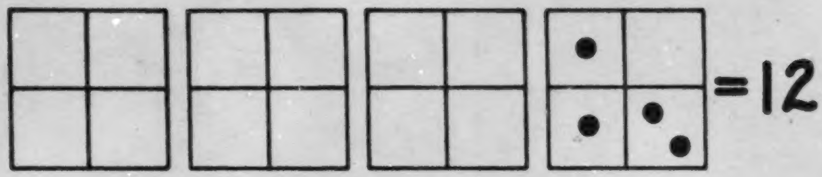
Let students suggest a variety of ways of putting 12 on the Minicomputer. Some of the many possibilities are shown below and on the next page.

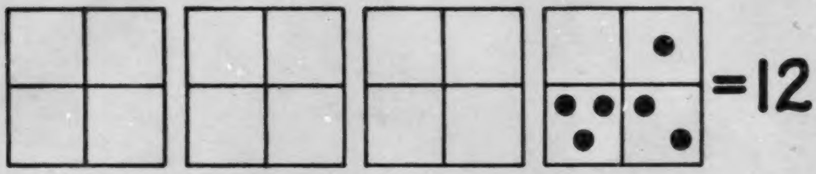






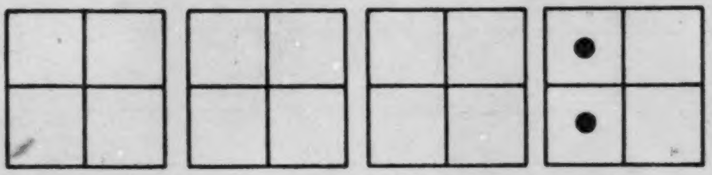






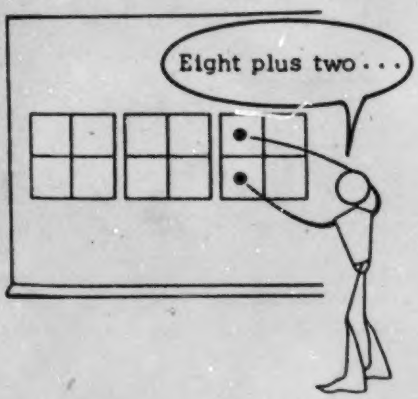
Exercise 4

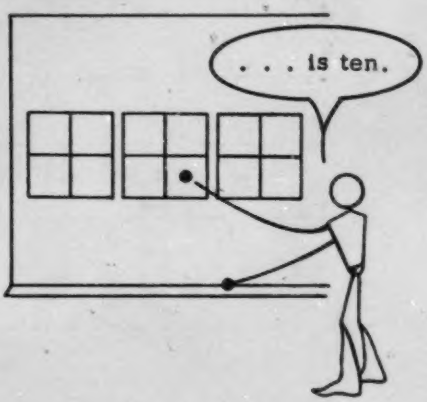
Put one checker on the 8-square and one on the 2-square.



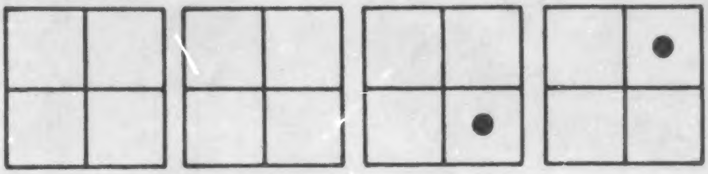
T: What number is on the Minicomputer? (10)

Pick up the checker on the brown square with one hand and the checker on the red square with the other. Then put one of these checkers on the 10-square and take the other checker away (put it in the chalk tray). As you are making the play, say "8 + 2 = 10."

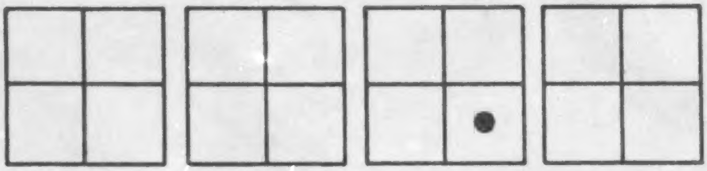




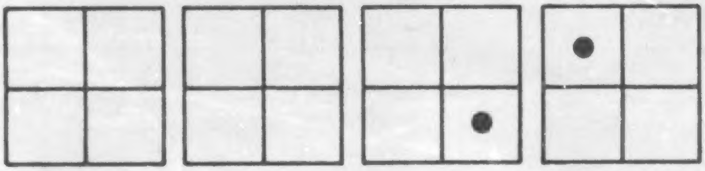
T: This is a way to put 10 on the Minicomputer using just one checker. I'll give you one more checker. Can you put 14 on the Minicomputer?



Remove the checker on the ones' board.



T: With one more checker, can you put 18 on the Minicomputer?



T: Who would like to put a secret number on the Minicomputer?

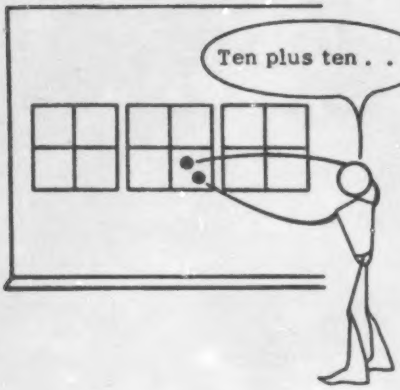
Whisper a whole number less than 20 to a volunteer. When that number has been put on the Minicomputer, call on another student to say which number it is. Repeat this activity several times.

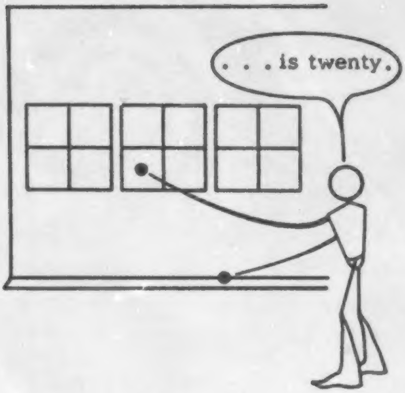
Ask someone to put 20 on the Minicomputer using two checkers. If no one volunteers, put two checkers on the 10-square.

T: There is a way to put 20 on the Minicomputer using just one checker.

N1

Make the play and say "10 + 10 = 20."

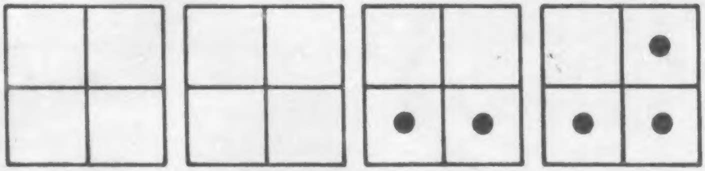




Remove the checkers and continue with other numbers.

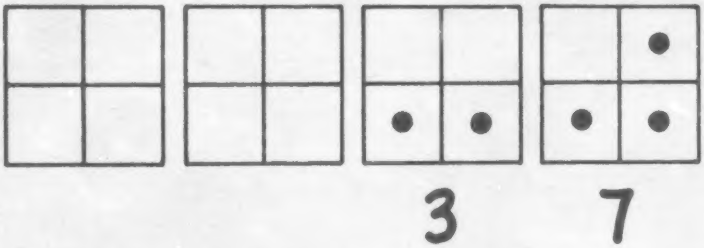
T: Who can put 25 on the Minicomputer? 27? 30?

Put this configuration on the Minicomputer.

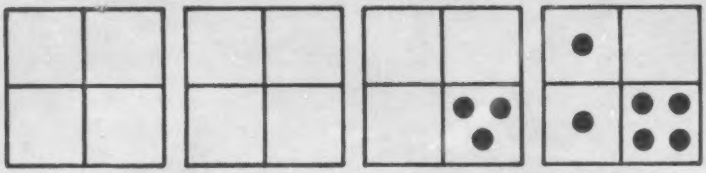


T: What number is this? (37)

Write "3" below the tens' board and "7" below the ones' board.



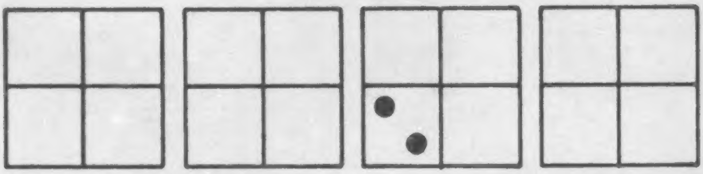
Put this configuration on the Minicomputer.



T: What is this number? (44)

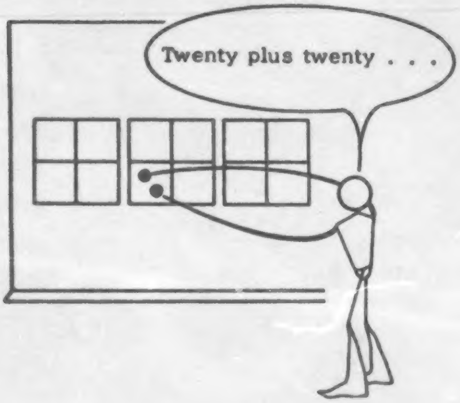
If the students have difficulty, make the "8 + 2 = 10" play and then repeat the question.

Put two checkers on the 20-square.

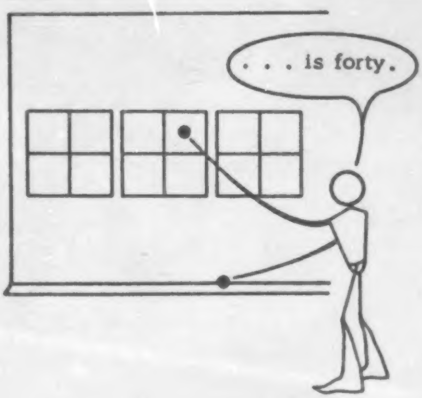


T: What number is this? (40)

Make the play yourself and say, "20 + 20 = 40."



Twenty plus twenty . . .



Remove the checker from the 40-square.

Move a checker back and forth very quickly from the 1-square to the 10-square. Each time you move the checker, ask the class which number is on the Minicomputer: "one; ten; one; ten; and so on."

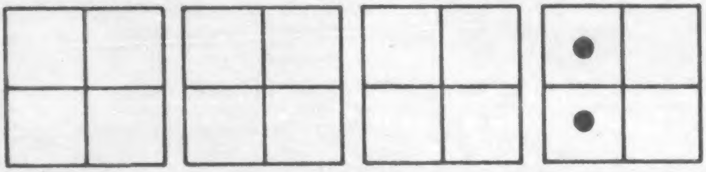
Repeat this activity with 2 and 20; 4 and 40; and 8 and 80.

T: Who would like to put a secret number on the Minicomputer?

Whisper a whole number less than 100 to a volunteer. When that number has been put on the Minicomputer, call on another student to say which number it is. Repeat this activity several times.

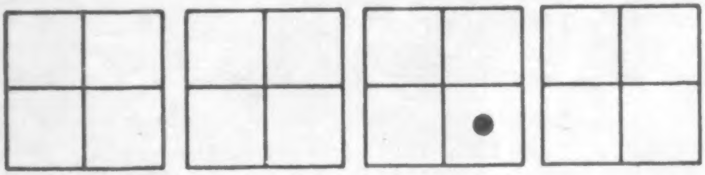
Exercise 5

Put this configuration on the Minicomputer.

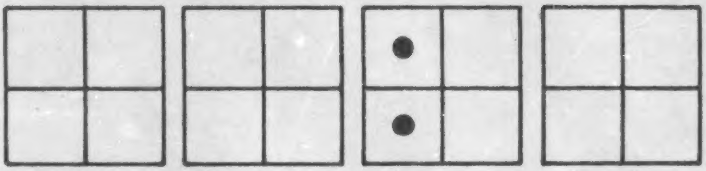


T: What number is this? (10)
Who can put 10 on the Minicomputer using only one checker?

Encourage the volunteer to say "8 + 2 = 10" as the play is made.



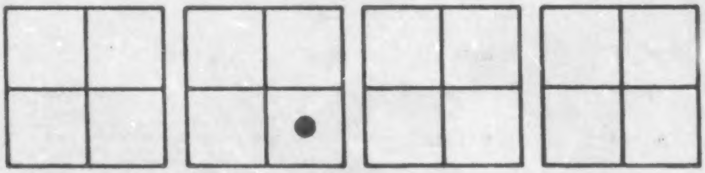
Put this configuration on the Minicomputer.



T: What number is this? (100)

Who can put 100 on the Minicomputer using only one checker?

Encourage the volunteer to say "80 + 20 = 100" as the play is made.

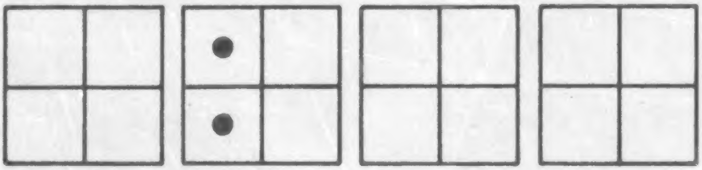


Move a checker from the white square of the ones' board to the white square of the tens' board to the white square of the hundreds' board. Each time you move the checker, ask the class what number is on the Minicomputer. Repeat this activity with the red, purple, and brown squares of the Minicomputer.

T: Who would like to put a secret number on the Minicomputer?

Whisper a whole number between 100 and 200 to a volunteer. When that number has been put on the Minicomputer, call on another student to say which number it is. Repeat this activity several times.

Put this configuration on the Minicomputer.

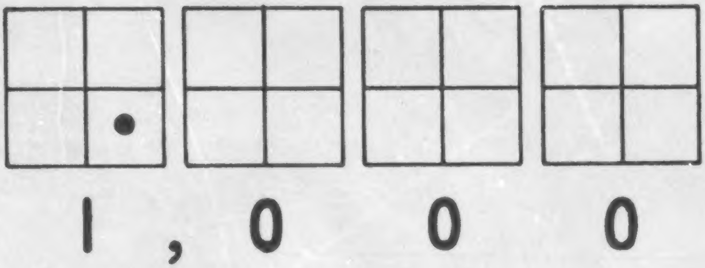


T: What number is this? (1,000)

Who can put 1,000 on the Minicomputer using only one checker?

Encourage the volunteer to say "800 + 200 = 1,000" as the play is made.

T: Who can write "1,000" below the Minicomputer?



Move a checker from the white square on the ones' board to the white square of each of the other three boards of the Minicomputer. Each time you move the checker, ask the class what number is on the Minicomputer. Repeat this activity with the red, purple, and brown squares of the Minicomputer.

Exercise 6

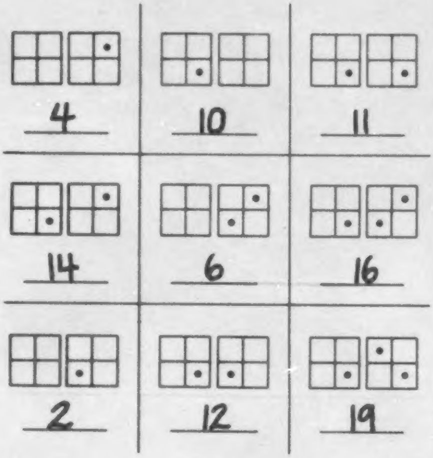
Give a student four checkers to put anywhere on the first three boards of the Minicomputer. You may want to stop a student who begins to put several checkers on the 8-square, 80-square, or the 800-square of the Minicomputer as it would be difficult for your students to calculate the number. Ask the students to write on pieces of paper what number is on the Minicomputer. Check some of their answers and then ask a student to write the number below the Minicomputer.

Worksheets N1*, **, ***, and **** are available for individual work.

Name

N 1 *

What number is on the Minicomputer?



4 10 11

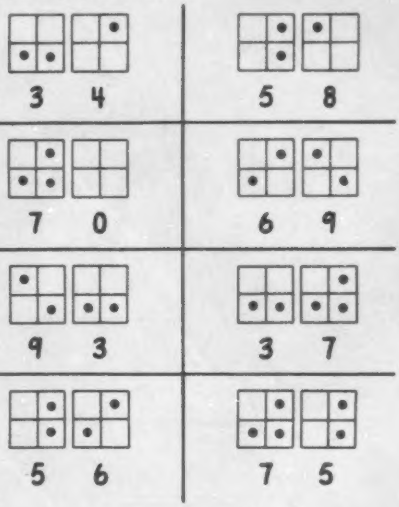
14 6 16

2 12 19

Name

N 1 **

Put these numbers on the Minicomputer.



3 4 5 8

7 0 6 9

9 3 3 7

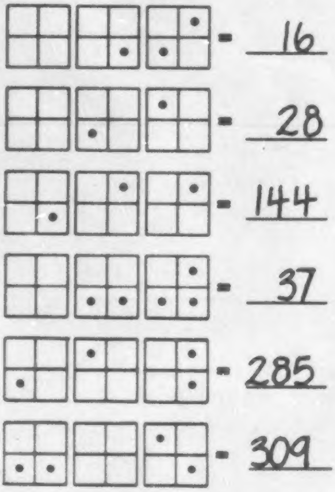
5 6 7 5

Standard configurations are given here; however, many other solutions are possible.

Name

N 1 ***

What number is on the Minicomputer?



= 16

= 28

= 144

= 37

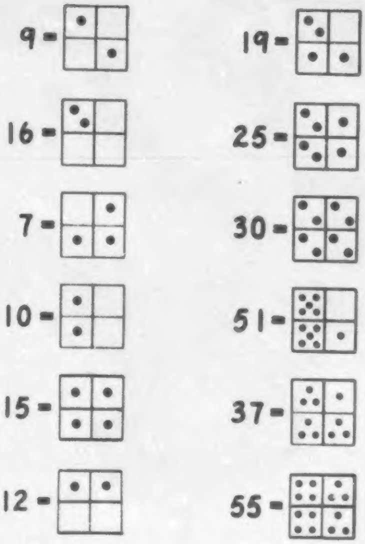
= 285

= 309

Name

N 1 ****

Put each number on the Minicomputer.



9 = 19 =

16 = 25 =

7 = 30 =

10 = 51 =

15 = 37 =

12 = 55 =

Many solutions are possible.

N2

N2 Introductory Arrow Problems #1

CAPSULE LESSON SUMMARY

Discover that:

- five +2 arrows has the same effect as one +10 arrow;
- ten +2 arrows has the same effect as one +20 arrow;
- five +4 arrows has the same effect as one +20 arrow;
- ten +4 arrows has the same effect as one +40 arrow.

Given the middle number of a road consisting of eight +5 arrows, decide what its starting and ending numbers are. Then locate and draw +20 arrows in the same picture.

MATERIALS

Teacher: Colored chalk
Student: Paper; pencil

DESCRIPTION OF LESSON

Exercise 1: Mental Arithmetic

Begin this lesson with about five minutes of mental arithmetic activities. Mental arithmetic activities should move quickly.

T: 1 + 3 = . . . ? (4)
10 + 30 = . . . ? (40)
100 + 300 = . . . ? (400)

2 + 5 = . . . ? (7)
20 + 50 = . . . ? (70)
2,000 + 5,000 = . . . ? (7,000)

2 x 3 = . . . ? (6)
2 x 30 = . . . ? (60)
2 x 300 = . . . ? (600)

2 x 6 = . . . ? (12)
2 x 60 = . . . ? (120)
2 x 600 = . . . ? (1,200)

S: 12 hundred.

T: What is another name for 12 hundred?

If necessary tell the students that 12 hundred is the same number as one thousand, two hundred. Ask for volunteers to write "1,200" on the board.

T: 4 + 3 = . . . ? (7)
40 + 30 = . . . ? (70)
4 million + 3 million = . . . ? (7 million)

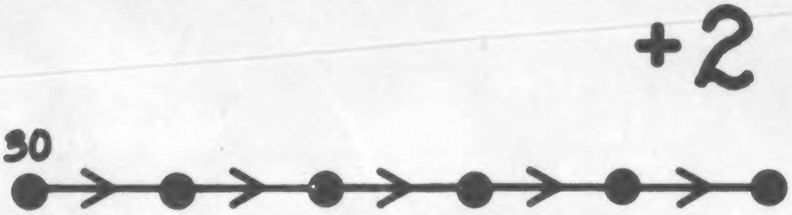
2 + 6 = . . . ? (8)
20 + 60 = . . . ? (80)

8 + 6 = . . . ? (14)
80 + 60 = . . . ? (140)

Continue this activity with calculations appropriate for the abilities of your students. Try to involve all the students by alternating easy and difficult calculations.

Exercise 2

Draw this arrow picture on the board.



T: Suppose an arrow road consists of five +2 arrows. If its starting number were 30, what would its ending number be? Write it on a piece of paper.

N2

Look at many of the students' answers.

T: Many of you have written "40". Can you explain why?

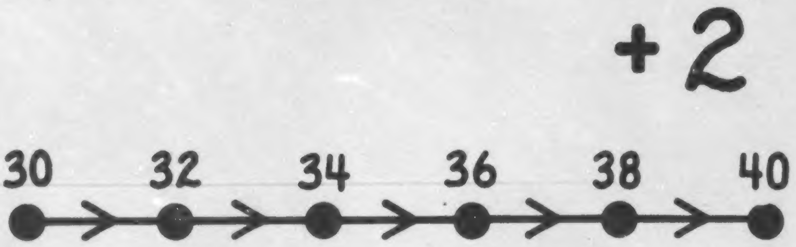
S: Because 30 + 2 + 2 + 2 + 2 + 2 = 40.

S: I counted on my fingers.

S: I started at 30 and counted by twos.

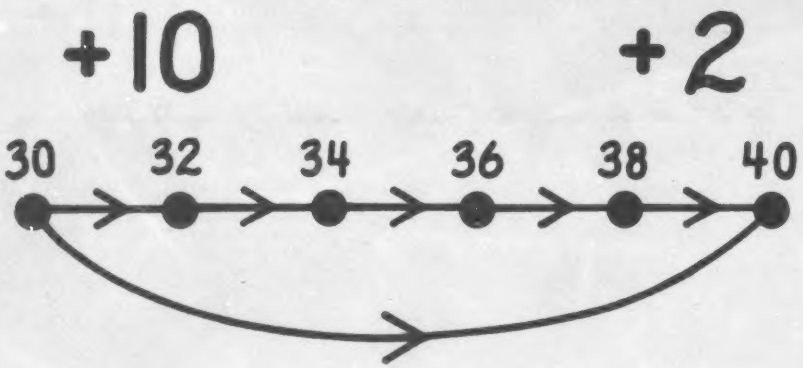
S: 5 x 2 = 10 and 30 + 10 = 40.

Ask for a volunteer to label all the dots in the arrow picture.



T: How many +2 arrows did I draw? (Five)
What number is 5 x 2? (10)
Five +2 arrows is the same as one +10 arrow.
Where can we draw a +10 arrow in this arrow picture without drawing any more dots?

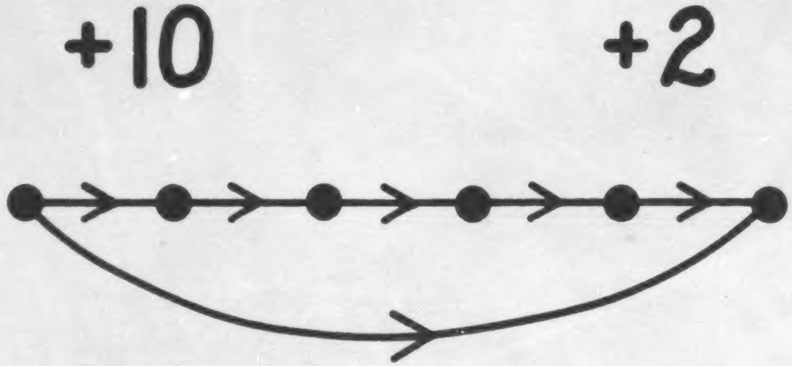
Ask a volunteer to draw the +10 arrow in blue. If necessary, draw the +10 arrow yourself. Write "+10" in blue near the arrow picture.



Put your left forefinger on the dot for 30. Trace the +2 arrow road with your finger; then trace the +10 arrow as you say:

T: Five +2 arrows is the same as one +10 arrow.

Erase the labels for the dots.



Point to the dot on the left.

T: If this dot were for 20, what would the ending dot be for? How do you know?

S: 30, because 20 + 10 = 30.

Continue this activity with the following or similar starting numbers which are appropriate for the abilities of your students. Corresponding ending numbers are shown in parentheses. Try to include all the students by alternating difficult and simple calculations. Occasionally, ask students to explain how they calculated an answer. Do not label the dots in the arrow picture.

50	(60)	1,000	(1,010)
51	(61)	43	(53)
10	(20)	59	(69)
12	(22)	0	(10)
100	(110)	95	(105)

Label the starting dot "32".

T: Suppose an arrow road consists of ten +2 arrows. If its starting number were 32, what would its ending number be? Write it on a piece of paper.

Look at many of the students' answers. You may need to repeat the question.

T: Many of you think the ending number would be close to 50. How can we find the exact answer?

S: We could count by twos.

Ask a volunteer to show how to find the ending number by counting by twos. If necessary, demonstrate this yourself. You can keep track of how many twos have been added by holding up a finger for each time you add two.

T: The ending number of the road would be 52. Is there another way to find this ending number?

Perhaps one of your students will suggest that there are ten +2 arrows. 10 x 2 = 20 and 32 + 20 = 52, so the ending number would be 52.

T: We are thinking about a +2 arrow road with ten arrows.
What number is 10 x 2? (20)
If we drew one arrow from the starting number to the ending number, what could that arrow be for? How do you know?

S: The arrow could be for +20, because 10 x 2 = 20.

NOTE: This arrow could also be for "I am less than you" or several other relations, but at this time we are only interested in the +20 relation.

T: Ten +2 arrows is the same as one +20 arrow because 10 × 2 = 20. (Point to "32".) 32 + 20 = 52.

Erase "32" and relabel the starting dot "67".

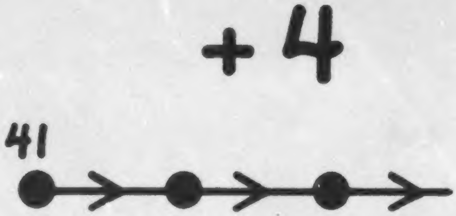
T: If the starting number were 67, what would the ending number be? Remember the road has ten +2 arrows.

S: 87.

Erase the arrow picture.

Exercise 3

Draw this arrow picture on the board.



T: Suppose an arrow road consists of five +4 arrows. To show that this is not all of the road, I didn't draw an ending dot here (point to the third arrow). If the starting number of this road were 41, what would its ending number be? Write your answer on a piece of paper.

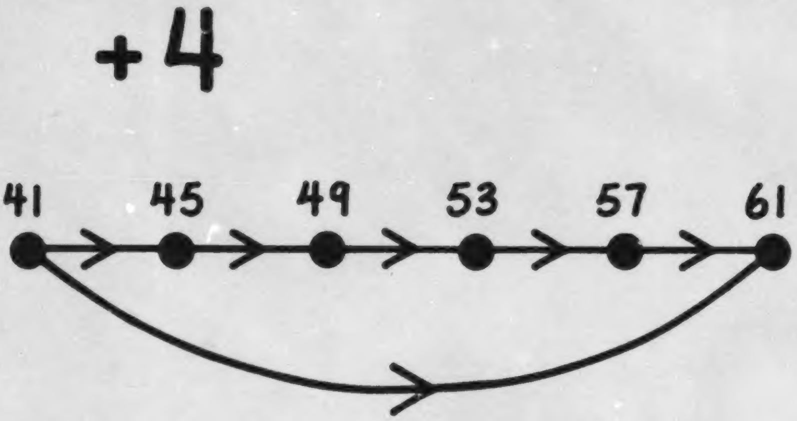
Look at many of the students' answers. Make sure that your students understand that there are five arrows in the road.

T: Some of you think the ending number would be 61. Can you explain why?

S: I counted by fours from 41.

S: 5 × 4 = 20 and 41 + 20 = 61.

Invite someone to complete the arrow picture quickly. Draw a red arrow from 41 to 61 yourself.

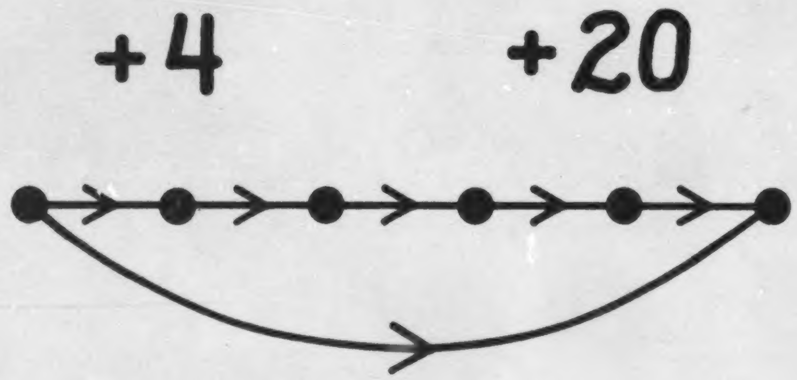


T: What could this red arrow be for? How do you know?

S: It could be for +20; 41 + 20 = 61.

S: It could be for +20, because 5 × 4 = 20.

Write "+20" in red near the arrow picture. Erase the labels for the dots.



Point to the dot on the left.

T: If this dot were for 10, what would the ending number be? How do you know?

S: 30, because 10 + 20 = 30.

Continue this activity with the following or similar starting numbers which are appropriate for the abilities of your class. Corresponding ending numbers are given in parentheses. Try to include all of the students by alternating simple and difficult calculations. Occasionally ask students to explain how they calculated an answer. Do not label the dots in the arrow picture.

15	(35)	100	(120)
60	(80)	69	(89)
27	(47)	1,000	(1,020)
51	(71)	83	(103)
20	(40)	1,000,000	(1,000,020)
21	(41)	98	(118)

Point to the dot on the right.

T: If this dot were for 25, what would the starting dot be for?

S: 5.

T: How do you know?

S: 5 + 20 = 25.

S: 25 - 20 = 5.

Label the dots for 25 and 5. Draw a green arrow from 25 to 5.

N2

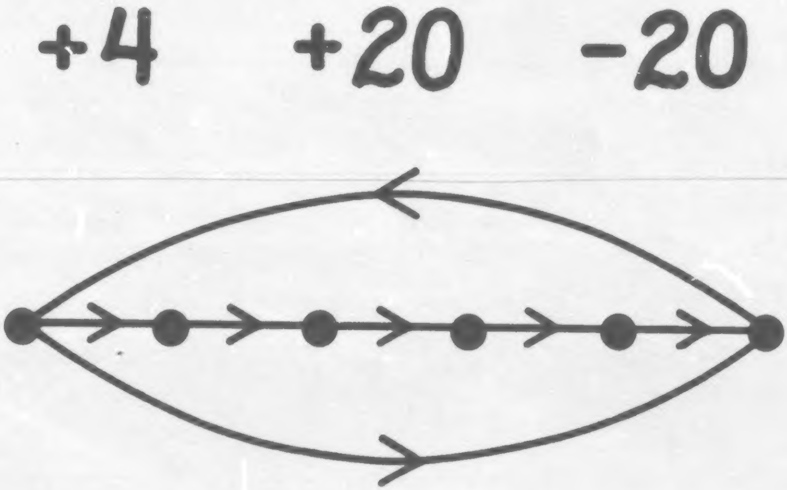
T: What could this green arrow be for? How do you know?

S: - 20. The red arrow is for + 20, so this arrow could be for - 20.

Trace the + 20 arrow and then the - 20 arrow.

T: This arrow is for + 20, so the return arrow is for - 20.

Write "- 20" in green near the arrow picture and erase the labels for the dots.



Point to the dot on the right.

T: If this dot were for 100, what would the starting dot be for? How do you know?

S: 80, because 100 - 20 = 80.

Continue this activity with the following or similar ending numbers which are appropriate for the abilities of your class. Corresponding starting numbers are given in parentheses. Occasionally ask students to explain how they calculated an answer. Do not label the dots in the arrow picture.

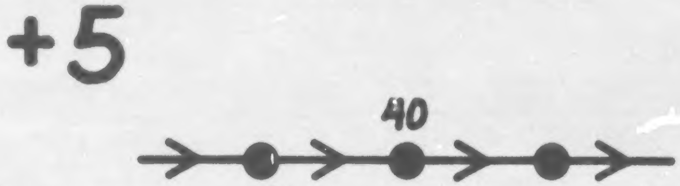
89	(69)	271	(251)
40	(20)	871	(851)
51	(31)	20	(0)
70	(50)	1,020	(1,000)
170	(150)	104	(84)
270	(250)	204	(184)

Vary this activity by asking for the ending number of a road which starts at 57 and which consists of ten +4 arrows. (97)

Erase the board before going on to Exercise 4.

Exercise 4

Draw this arrow picture on the board.



T: 40 is the middle number of a +5 arrow road that has exactly eight arrows. Before completing the arrow picture, can you predict the starting number and the ending number of this road? Write them on a sheet of paper.

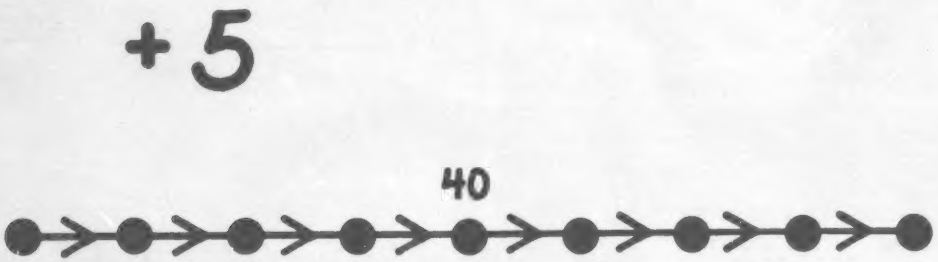
Look at many of the students' answers. Point to the dot for 40.

T: If 40 is the middle number of a road which has exactly eight arrows, how many arrows are there to the right of 40? How many arrows are there to the left of 40?

N2

Allow the students to change their answers now if they wish.

Ask a student to draw the missing arrows or draw them yourself.



T: What is the ending number of this arrow road? (60)
How do you know?

S: I started at 40 and counted by fives.

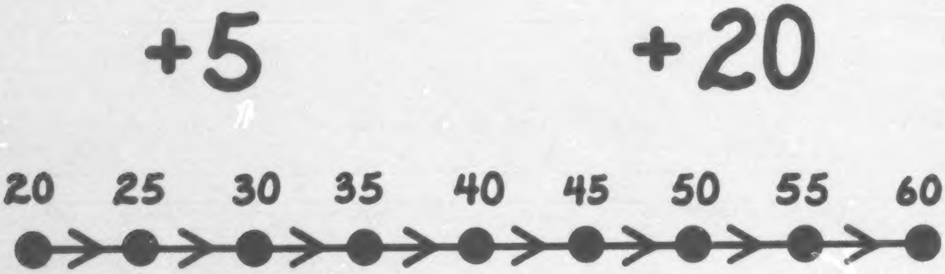
S: 4 x 5 = 20 and 40 + 20 = 60.

T: What is the starting number of this arrow road? (20)
How do you know?

S: 4 x 5 = 20 and 40 - 20 = 20.

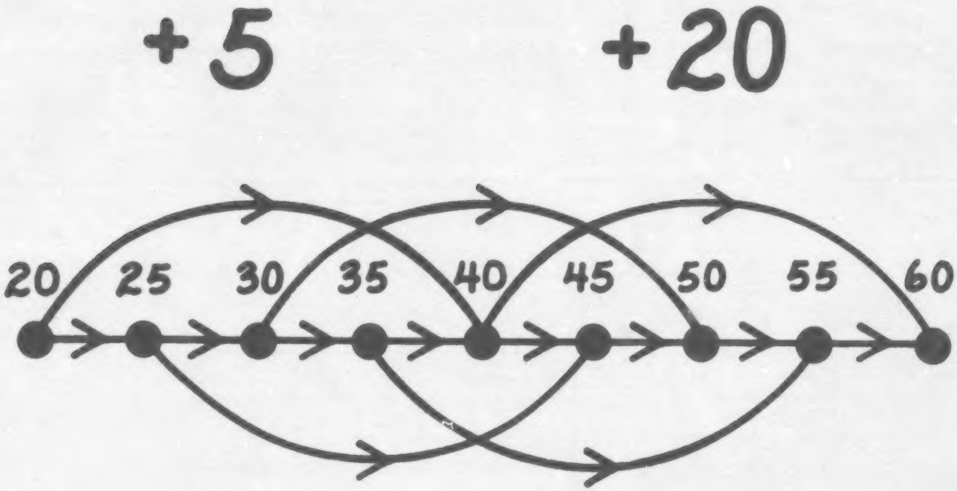
S: I counted backward by fives from 40.

Invite a student to label all the dots in this arrow picture. Write "+20" in blue near the arrow picture.



T: Without drawing any more dots, where can we draw some +20 arrows?

Let students locate and draw the +20 arrows in this picture. Emphasize that one +20 arrow has the same effect as four +5 arrows.


+5
+20
20
25
30
35
40
45
50
55
60

N3 Minicomputer Introduction #2

CAPSULE LESSON SUMMARY

Review the plays on the Minicomputer and the value of the squares. Explore the effect of moving various checkers in a configuration on the Minicomputer — in each case, does the numerical value increase, decrease, or stay the same? Use the Minicomputer to do subtraction and multiplication calculations.

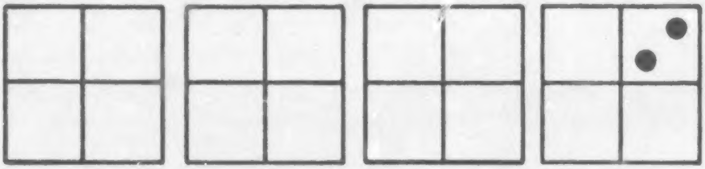
MATERIALS

Teacher: Minicomputer set
Student: None

DESCRIPTION OF LESSON

Exercise 1

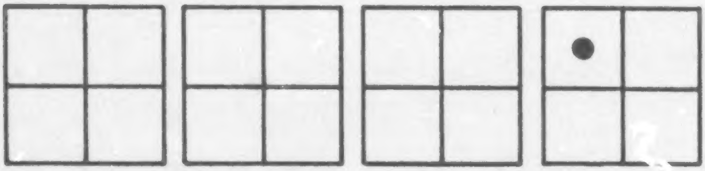
Put this configuration on the Minicomputer.



T: What number is on the Minicomputer? (8)

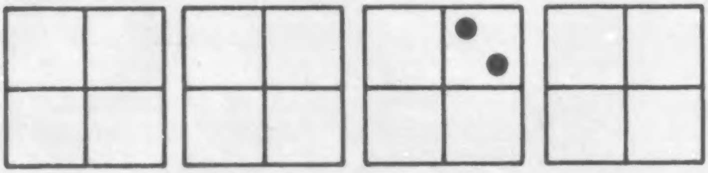
Could we put on 8 using only one checker?

Invite a student to put 8 on the Minicomputer using exactly one checker.



Demonstrate the "8 = 4 + 4" and "4 + 4 = 8" plays yourself.

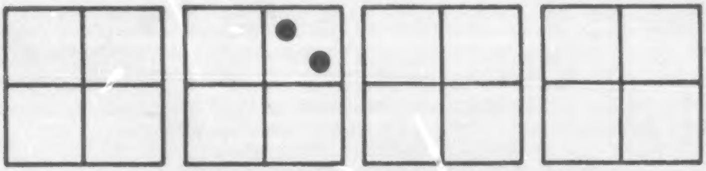
Put this configuration on the Minicomputer.



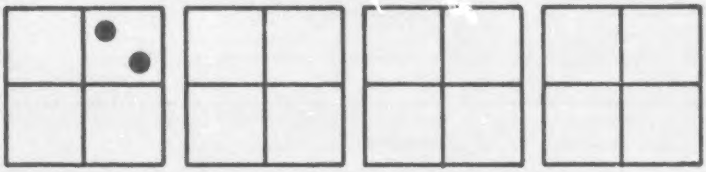
T: What number is on the Minicomputer? (80)

Ask for a volunteer to make a play so that 80 will be on the Minicomputer with only one checker. As the play is made, the student should say, "40 + 40 = 80".

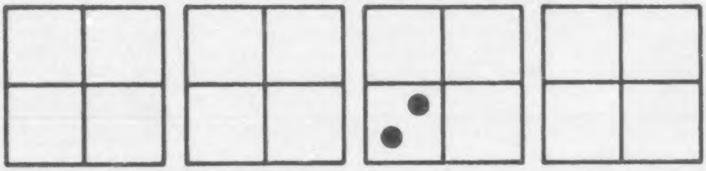
Repeat this activity with the following configurations.



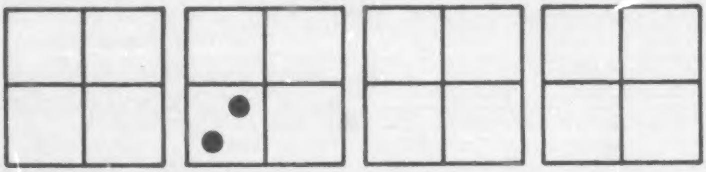
= 800



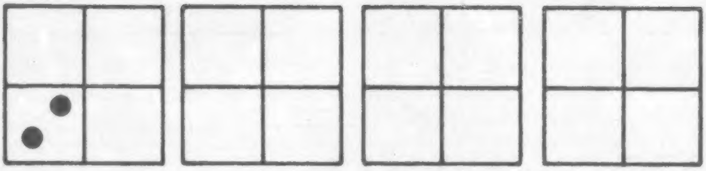
= 8,000

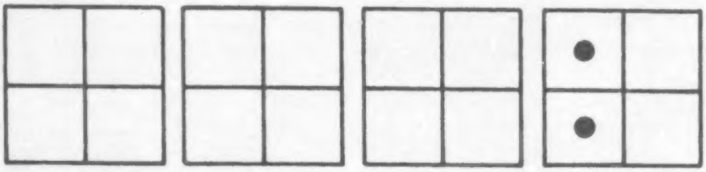


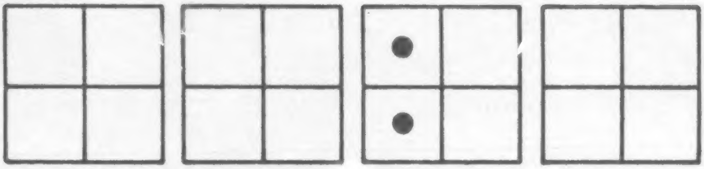
= 40

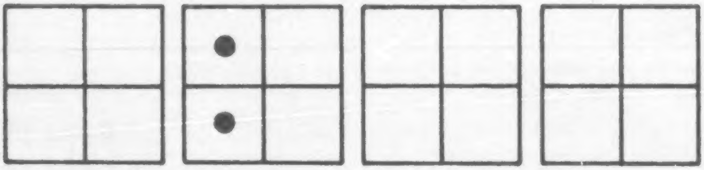


= 400









Exercise 2

Move a checker from one square to another very quickly to review the value of the squares. Each time you move the checker, ask the class which number is on the Minicomputer.

Review the standard configurations of the following numbers. Move the checkers very quickly from one board to another, each time asking which number is on the Minicomputer.

5; 50; 500; 5,000

3; 30; 300; 3,000

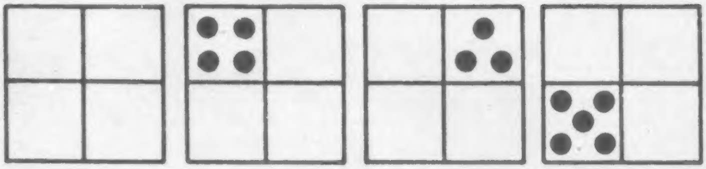
9; 90; 900; 9,000

6; 60; 600; 6,000

7; 70; 700; 7,000

Exercise 3: Transforming a Number

Put this configuration on the Minicomputer.



Guide the students to make a closer and closer estimate of the number on the Minicomputer. A possible dialogue is given here.

T: For the moment don't try to calculate this number exactly.
Is this number more than 100? (Yes)
How do you know?

S: There is a checker on the 800-square.

T: Is this number more than 800? (Yes)
How do you know?

S: It is more than 800 because there are four checkers on the 800-square.

T: We know this number is more than 800. Is it more than 1,000? (Yes)
Can you tell us why?

S: 800 + 800 = 1,600 and 1,600 is more than 1,000.

T: Is this number more than 2,000?
How do you know?

A student pointing to the two pairs of checkers on the 800-square says:

S: This pair of checkers is more than 1,000 and this other pair of checkers is more than 1,000; so the number must be more than 2,000.

T: We know this number is more than 2,000. Is it also more than 3,000? How do you know?

It is possible that a student will be able to explain and to convince the class why this number is more than 3,000. In any case, your class should estimate the number to be larger than 2,000 and possibly larger than 3,000.

T: We don't know exactly which number is on the Minicomputer, but we do know that it is more than 2,000 (3,000).

Write these words on the board.

Larger
Same
Smaller

T: I am going to move, remove, or add some checkers. Each time tell me if the number on the Minicomputer is larger than, smaller than, or the same as the number before.

Move a checker from the 800-square to the 80-square. Point to each of the three words in turn as you say:

T: If you think this number is larger than the number we had, hold up your hand.
If you think the number is the same, hold up your hand.
If you think the number is smaller than the number we had, hold up your hand.

Repeat the move from the 800-square to the 80-square very obviously if many students do not know that the new number is smaller than the previous one.

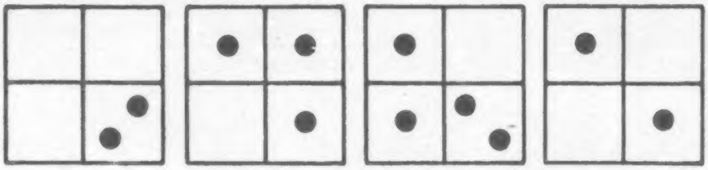
Ask a student who indicated that the new number is smaller than the original number to explain why. Accept any reasonable explanation. Continue this activity with the following moves or similar ones.

- Change the color of a checker on the 2-square. (Same)
- Make a "2 + 2 = 4" play. (Same)
- Move a checker from the 80-square to the 200-square. (Larger)
- Move a checker from the 40-square to the 80-square. (Larger)
- Make a "800 = 400 + 400" play. (Same)

Continue this activity with a slight variation. Each time you change the number on the Minicomputer, ask the class how much more or less the new number is than the previous one.

- Move a checker from the 2-square to the 4-square. (2 more)
- Move a checker from the 4-square to the 8-square. (4 more)
- Make a "800 + 200 = 1,000" play. (Same)
- Move a checker from the 2-square to the 10-square. (8 more)
- Move a checker from the 2-square to the 1-square. (1 less)
- Move a checker from the 40-square to the 20-square. (20 less)
- Move a checker from the 4-square to the 10-square. (6 more)
- Move a checker from the 40-square to the 100-square. (60 more)
- Move a checker from the 400-square to the 1,000-square. (600 more)

You should have this configuration on the Minicomputer.



NOTE: If you don't have this configuration on the Minicomputer, you can continue transforming the number until you obtain it or you can adjust the following dialogue.

T: We have a new number on the Minicomputer.
Don't try to calculate this number now. Is it more than 1,000?
How do you know?

S (pointing to the two checkers on the 1,000-square): This number is more than 2,000.

T: We know this number is more than 2,000. Is it also more than 3,000?

S: Yes, 800 + 400 is more than 1,000 and there are two checkers on the 1,000-square.

T: Is this number more than 4,000?

It is possible that a student will be able to explain why this number is less than 4,000. In this case your class would conclude that the number on the Minicomputer is between 3,000 and 4,000.

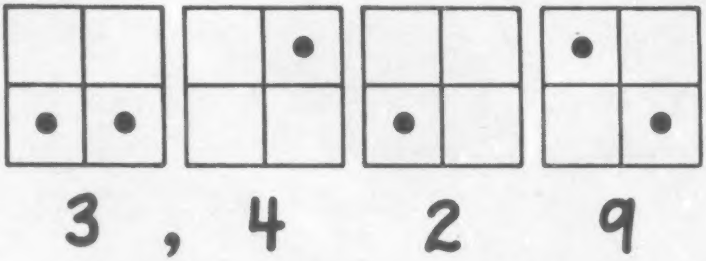
T: We don't know exactly which number is on the Minicomputer, but we do know that it is more than 3,000. What do you think this number is?

Record several guesses on the board. If any student guesses a number which is less than 3,000, remind the student that we know that this number is more 3,000 and do not record the guess.

T: How could we make this number easier to read?

S: Make some plays.

Invite students to make plays on the Minicomputer. Continue until you obtain standard configuration of the number. Ask a student to write the number below the Minicomputer.



Allow the stuuents to determine which guess was the closest. Erase the board and remove all the checkers from the Minicomputer.

Exercise 4: Subtraction Problem

Write this problem on the board.

337 - 214

T: What number is 337 - 214?

Accept several guesses and list them on the board.

T: How could we calculate "337 - 214" on the Minicomputer?

S: Put 337 on the Minicomputer and then take off 214.

Ask a student to put 337 on the Minicomputer. Here we assume that the standard configuration for 337 is put on the Minicomputer.

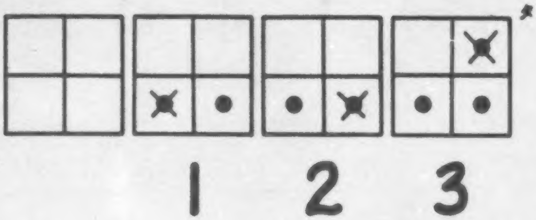
T: Who can take 214 off the Minicomputer?

The student should remove the checkers on the 200-square, 10-square, and 4-square.

T: What number is on the Minicomputer now?

S: 123.

Complete the number story on the board.



337 − 214 = 123

Let the students decide which guess was the closest. Erase the board and remove the checkers from the Minicomputer.

Exercise 5: Multiplication Problem

Write this problem on the board.

2 × 496

T: What number is 2 × 496?

Accept several guesses and record them on the board.

T: How can we calculate "2 × 496" on the Minicomputer?

S: Put 496 on the Minicomputer twice and then make plays.

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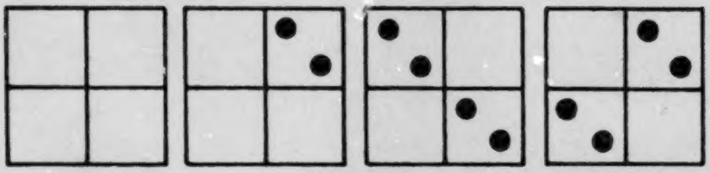
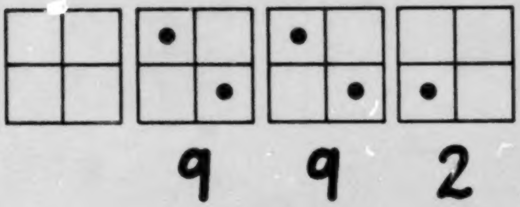
* An "X" through a checker indicates that the checker has been removed from the Minicomputer.

Ask a student to put 496 on the Minicomputer twice.



S: We need to make some plays.

Invite students to make plays until you obtain standard configuration of the number. Ask a student to write the number below the Minicomputer and to complete the number story.

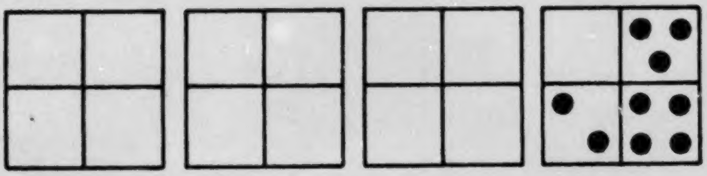


2 × 496 = 992

Let the students decide which guess was the closest. Erase the board and remove the checkers from the Minicomputer.

Exercise 6

Put the following configuration of checkers on the Minicomputer slowly, allowing the students to determine the number mentally.



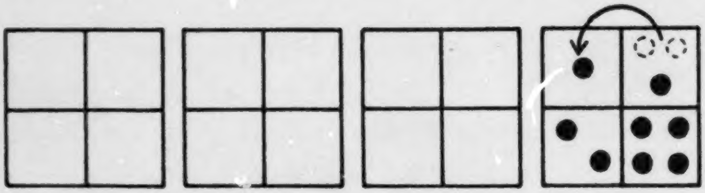
T: What number is this? (20)

T: How many checkers have I used to show 20 on the Minicomputer?

(Nine)

Can you put 20 on the Minicomputer using only eight checkers?

If the students do not respond, suggest that they make a play. Any forward play will solve the problem. For example,



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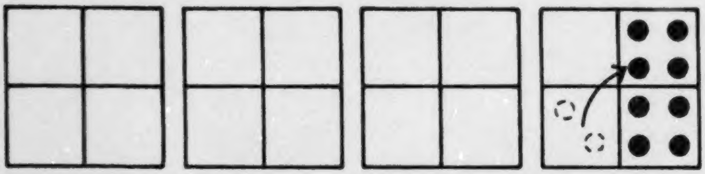
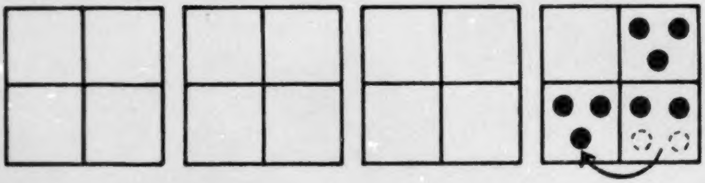


or



Other answers are also possible.

T: This is 20 with eight checkers.

Can you put 20 on the Minicomputer using only seven checkers?

Continue by representing the number 20 on the Minicomputer using exactly six checkers and then using exactly five checkers.

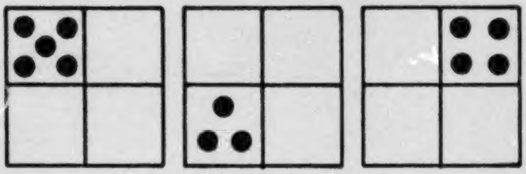
If the lesson is moving quickly and the students remain interested, you may wish to continue with Exercise 7.

Exercise 7

Display three Minicomputer boards. Give a student twelve checkers as you say:

T: Put these checkers on the Minicomputer wherever you want.

Suppose the student chooses this configuration.



T: Is this number more than 100? How can you tell?
Is it more than 1,000? How can you tell?
Is it more than 2,000? How can you tell?
Is it more than 3,000? How can you tell?

Allow the students to guess which number is on the Minicomputer and record their guesses on the board.

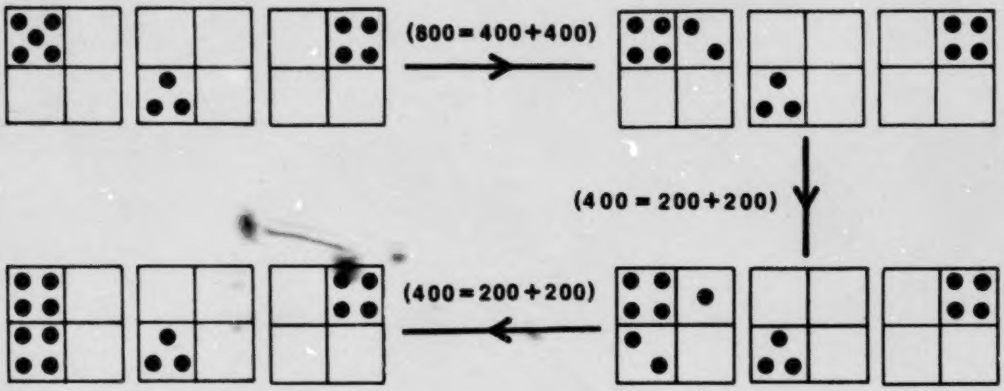
T: How can we make the number easier to read?

S: Make some plays.

Invite students to make plays and encourage them to say which plays they make. You may need to initiate backward plays the first few times they are needed. Display a fourth Minicomputer board if the configuration is for a number larger than 999.

NOTE: The configuration shown in the last illustration has five checkers on the 800-square. Since in this case there is no way to get checkers on the

200-square by making forward plays, some backward plays are needed. For example, this sequence of plays . . .



. . . sets up the checkers so that "800 + 200 = 1,000" plays can be made. A fourth board is needed.

When standard configuration for the number is obtained, ask a student to record the number below the Minicomputer. Determine which guess was the closest.

N4 Roads #1

CAPSULE LESSON SUMMARY

Find 2 x a number and possibly use the answer to find 3 x the same number. Find ways to put 25¢ postage on a letter using only 2¢ stamps and 3¢ stamps. Show the various solutions in arrow pictures.

MATERIALS

Teacher: Colored chalk

Student: Paper; colored pencils*

DESCRIPTION OF LESSON

Exercise 1: Mental Arithmetic

Begin this lesson with a few minutes of mental arithmetic involving the functions 2 x and 3 x. One example of a possible sequence of calculations is given here.

T: What number is 2 x 7? (14)
If 2 x 7 = 14, what number is 3 x 7? (21)
How do you know?

S: I added 7 to 14.

T: What number is 2 x 5? (10)
If 2 x 5 = 10, what number is 3 x 5? (15)

What number is 2 x 10? (20)
If 2 x 10 = 20, what number is 3 x 10? (30)
How do you know?

S: I just added 10 to 20.

---

* Crayons may be substituted for colored pencils in any lesson.

In the same manner, continue with these calculations or similar ones. Answers are given in parentheses.

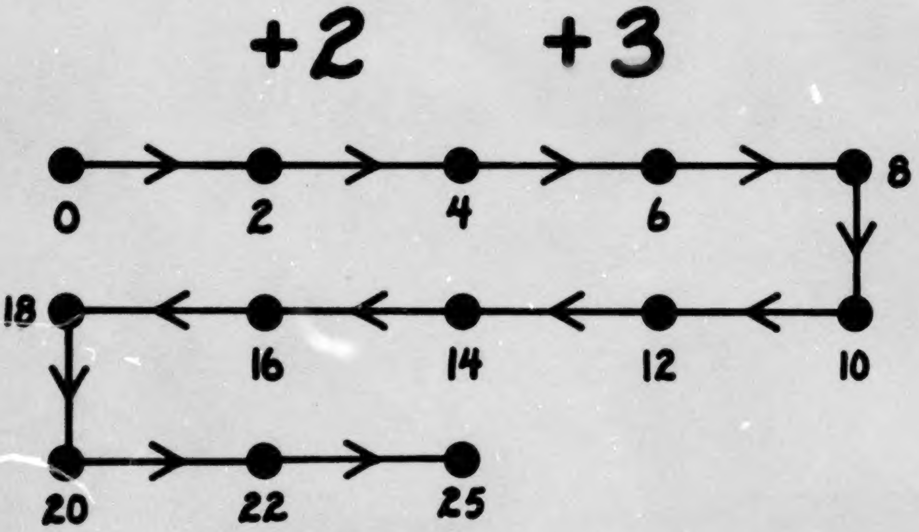
2 x 6 (12)   2 x 9 (18)   2 x 3 (6)
3 x 6 (18)   3 x 9 (27)   3 x 3 (9)

Exercise 2

Tell the following or a similar story to your class. Choose one of your students to star in this story.

T: Lori has a letter which needs 25¢ postage. She couldn't find a 25¢ stamp, but she did find several 2¢ stamps and 3¢ stamps. Can Lori put 25¢ postage on her letter by using 2¢ stamps and 3¢ stamps?

You may need to repeat the story and the question to be sure the students understand. Allow the students to work independently until one of them discovers a way for Lori to put 25¢ postage on her letter. Ask the student to share his or her solution with the class. If the student used an arrow picture to solve the problem, put it on the board; otherwise, ask the student how you could show his or her solution in an arrow picture. For example, using eleven 2¢ stamps and one 3¢ stamp could be shown in an arrow picture as in the next illustration.



T: In an arrow picture, show another way Lori could put 25¢ postage on her letter.

Encourage the students to use the same colors for their +2 and +3 arrows as in the arrow picture on the board. Ask each student who finishes an arrow picture either to put it on the board or to look for a different solution. Continue until several arrow pictures with different numbers of each type of arrow are on the board.

Point to one of the arrow pictures on the board.

T: If Lori solves her problem this way, how many 2¢ stamps would she put on her letter?

The answer will vary according to which arrow road you are discussing. For the purpose of this dialogue we will assume the arrow road has two +2 arrows.

S: Two.

T: How can you tell from this picture?

S: There are two +2 arrows in the arrow picture.

T: If she uses two 2¢ stamps, how many 3¢ stamps should she use?

S: Seven.

T: How can you tell from this arrow picture?

S: There are seven +3 arrows.

T: Would this give Lori exactly 25¢ postage? (Yes)
How many stamps would she use altogether? (Nine)

Record this information in a table.

2¢ stamps	3¢ stamps	total number of stamps
2	7	9

T: What number is 2 x 2? (4)

What number is 3 x 7? (21)

Write "4 + 21" on the board.

T: What number is 4 + 21?

S: 25.

Complete the number story on the board.

$$4 + 21 = 25$$

T: This one way Lori can put 25¢ postage on her letter.

Continue this activity until you have discussed all the arrow pictures on the board. If several arrow pictures have the same number of each type of arrow, do not record the information in the table more than once. Very likely your students will find all four solutions.

2¢ stamps	3¢ stamps	total number of stamps
2	7	9
5	5	10
8	3	11
11	1	12

T: What is the largest number of stamps (2¢ and 3¢) she could use?

S: Twelve — eleven 2¢ stamps and one 3¢ stamp.

T: What is the smallest number of stamps (2¢ and 3¢) she could use?

S: Nine — two 2¢ stamps and seven 3¢ stamps.

If your students did well with this activity, continue with the following situation.

T: Imagine you have a package which needs 50¢ postage. You have 3¢ stamps, 5¢ stamps, and 7¢ stamps. How many of each type stamp do you need to put on the package?

Encourage the students who finish quickly to find more than one solution.

N5 Multiplication Problems

CAPSULE LESSON SUMMARY

Practice reading numbers on the Minicomputer. Find 2× a number and possibly use the answer to find 3× a number. Given a configuration in which the checkers are in groups of three, figure out which number it represents by considering each board separately and by doing mental calculations; then check the answer by making plays. Calculate "2 × 45" and "2 × 512" on the Minicomputer.

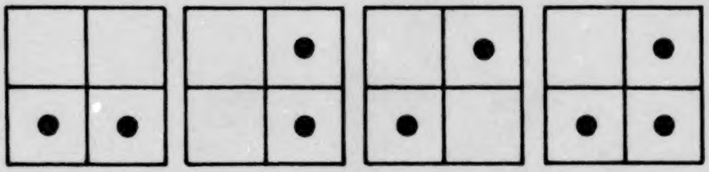
MATERIALS

Teacher: Minicomputer set
Student: Minicomputer set[*]; paper; pencil

DESCRIPTION OF LESSON

Exercise 1

Put this configuration on the Minicomputer.



T: What number is on the Minicomputer? Write it on your paper.

Look at many of the students' answers. Then ask a student to write the number below the Minicomputer and to say the number aloud. Invite the students to explain how they know this number is 3,567. If an explanation suggests a calculation, record the calculation on the board. A sample dialogue is given here.

S: I added 3,000; 500; 60; and 7.

---

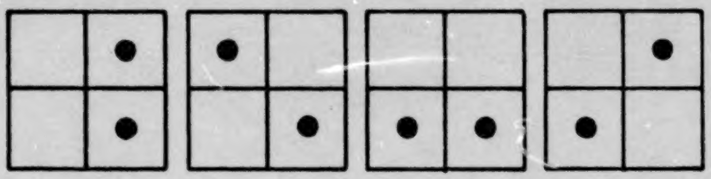
[*] A student's Minicomputer set consists of two individual Minicomputer sheets (each having two boards) and a sheet of individual checkers.

Write this addition problem on the board.

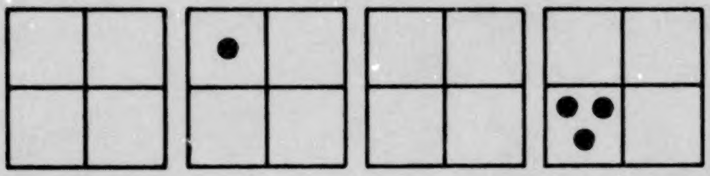
$$\begin{array}{r} 3{,}000 \\ 500 \\ 60 \\ +\quad 7 \\ \hline 3{,}567 \end{array}$$

S: I just wrote "3", "5", "6", and "7" because those are the numbers on the Minicomputer boards.

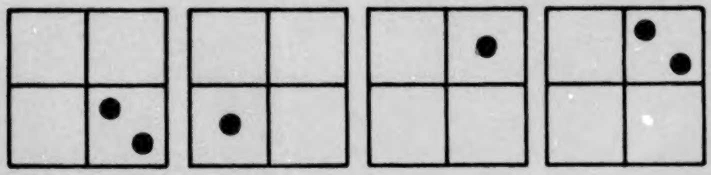
Repeat this activity with these configurations. After a number is written below the Minicomputer, invite students to make any plays needed to put the number in standard configuration.



= 5,936



= 806



= 2,248

Exercise 2 : Multiplication Problems

Begin this exercise with a short mental arithmetic activity involving the functions 2x and 3x. An example of such an activity is given here.

T: What number is 2 x 6? (12)

2 x 6 = 12; so what number is 3 x 6? (18)

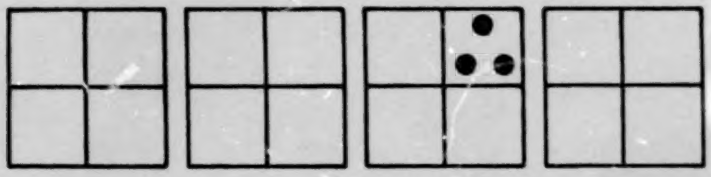
Right, 2 x 6 = 12 and 12 + 6 = 18, so 3 x 6 = 18.

What number is 2 x 4? (8)

What number is 2 x 40? (80)

2 x 40 = 80; so what number is 3 x 40? (120)

After about five minutes of mental arithmetic, put this configuration on the Minicomputer.



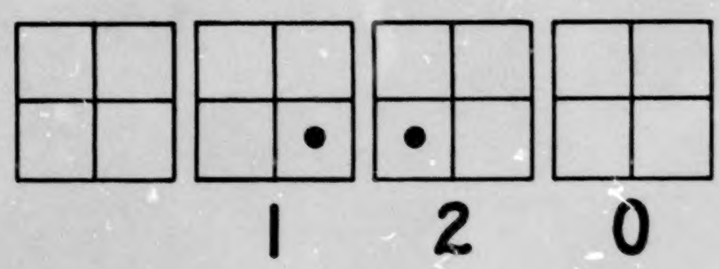
T: What number is this? How do you know?

S: 120, because 3 x 40 = 120.

T: Is this the way we usually put 120 on the Minicomputer?

S: No, we need to make some plays.

Invite students to make plays on the Minicomputer until the standard configuration for 120 is obtained. A backward play (40 = 20 + 20) is needed; if necessary, make this play yourself. Ask a student to write the number below the Minicomputer.

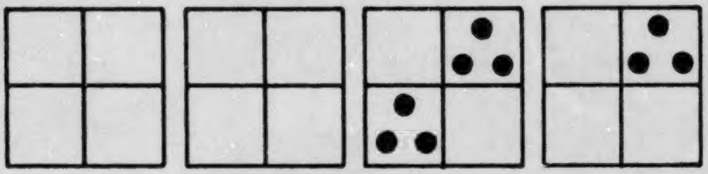


Remove the checkers from the Minicomputer, erase the board, and write this problem on the board.

$$3 \times 64$$

T: What number is 3 x 64? Write it on your paper.

Allow the students a few minutes to calculate the answer. Invite a student to put 3 x 64 on the Minicomputer.



$= 3 \times 64$

T: What calculation are we doing on the ones' board?

S: "3 x 4".

T: What number is 3 x 4?

S: 12.

Write "3 x 4 = 12" above "3 x 64".

$$3 \times \ 4 = 12$$
$$3 \times 64$$

T: What calculation are we doing on the tens' board?

S: "3 x 20" and "3 x 40".

S: "3 x 60".

T: What number is 3 x 60?

S: 180.

If necessary, prompt the students by asking what number is 3 x 6.

Record "3 x 60 = 180" on the board.

$$\begin{array}{l} 3 \times 60 = 180 \\ 3 \times \ \ 4 = \ \ 12 \\ 3 \times 64 \end{array}$$

T: 3 x 60 = 180 and 3 x 4 = 12, so what number is 3 x 64?

S: 192.

$$\begin{array}{l} 3 \times 60 = 180 \\ 3 \times \ \ 4 = \ \ 12 \\ 3 \times 64 = 192 \end{array}$$

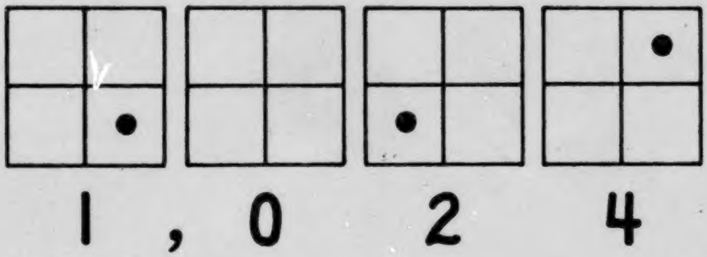
Encourage several students to explain how they calculated the answer. Ask for volunteers to make plays on the Minicomputer until the standard configuration for 192 is obtained. You may need to help the students make the necessary backward plays.

Exercise 3: Individual Minicomputers

Each student should have two Minicomputer sheets (four boards) and a set of checkers. Ask the students to put 45 on their Minicomputers. Write "2 × 45 = $\square$" on the board and ask the students to calculate "2 × 45" on their Minicomputers. Many students will be able to do the calculation mentally, but ask them to do it on their Minicomputer also. Help any students who are having difficulty making plays. If your class would benefit, calculate "2 × 45" on the demonstration Minicomputer and ask the students to follow by making the same plays on their individual Minicomputers. Conclude that 2 × 45 = 90 and complete the number story.

$$2 \times 45 = \boxed{90}$$

Repeat this activity with "2 × 512". Emphasize that since the hundreds' board is empty, we write a "0" in the hundreds' place.



Conclude that 2 × 512 = 1,024.

N5

Erase the board and write these problems or others which are appropriate for your students' abilities on the board.

2 × 107 =

3 × 73 =

2 × 134 =

3 × 625 =

4 × 59 =

5 × 216 =

4 × 260 =

3 × 187 =

Ask the students to copy these problems on their papers and to solve as many of them as they can. Encourage all students to calculate the answers on their Minicomputers, but permit them to use other methods as well if they wish.

N6 The Function + 10 and Composition

CAPSULE LESSON SUMMARY

Using the hand-calculator, start with a number other than 0 and count by tens. Discover that three + 10 arrows has the same effect as one + 30 arrow. Extend the picture until it consists of six + 10 arrows and locate all possible + 30 arrows.

MATERIALS

Teacher: Colored chalk; two hand-calculators*
Student: Paper; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Give the calculators to two students who have had some experience with them before, either at home or at school. Let the class hear the instructions you are giving to the students with calculators. Those students and the ones close to them can verify what numbers appear on the display.

T: Listen carefully to my directions. Turn on the calculators. What number is on the display?

S: 0.

T: Start with "0" on the display. Press $\boxed{7}$. What number is on the display now?

S: 7.

Record "7" near the top of the board.

T: Now press $\boxed{+}\ \boxed{1}\ \boxed{0}\ \boxed{=}$.
What number is on the display ?

---

* See ROLE AND USE OF HAND-CALCULATORS on page 14 of the General Introduction.

S: 17.

Record "17" below "7" on the board.

T: Press $\boxed{=}$ again. What number is on the display?

S: 27.

Record this result in the list on the board.

7
17
27

T: What are the hand-calculators doing?

S: They are counting by tens.

T: How do you know?

S: 7 + 10 = 17 and 17 + 10 = 27.

T: What number will we get if we press $\boxed{=}$ again? (37)
Press $\boxed{=}$. What number is on the display?

S: 37.

Record the result in your list on the board.

T: What number will we get next? (47)
Press $\boxed{=}$. What number is on the display?

S: 47.

Record this result in the list on the board.

T: Does anyone see a pattern?

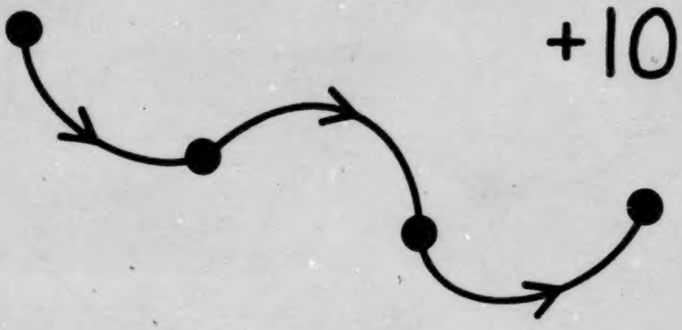
Encourage students to comment on the list of numbers on the board. Continue this activity until the list includes 127. Ask again for a pattern. Erase the board and ask the students with calculators to clear the display.

Repeat this activity starting with a number which does not end in "7", such as 23. Occasionally ask the class to predict the next two or three numbers which will appear on the display.

Erase the board and collect the calculators.

Exercise 2

Draw this arrow picture on the board.



T: Where is the largest number in this arrow picture?

The students should point to the dot on the far right. Point to that dot yourself as you ask:

T: How do you know that this number is the largest one in this arrow picture?

N6

S: The blue arrows are for +10. When you add 10 to a number, you get a larger number.

T: Where is the smallest number in this arrow picture?

The students should point to the dot on the far left.

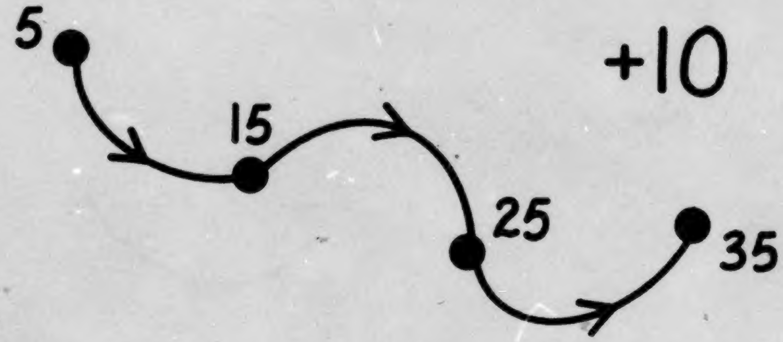
Label the left most (starting) dot "5".

T: If this number were 5, what would the ending number be? Write it on a piece of paper.

Look at many of their answers.

T: Let's label the dots and find out what the last number is.

Ask for a volunteer to label the dots in the arrow picture.



Ask the students who correctly predicted that the ending number would be 35 to explain how they calculated the answer. Erase the labels of the dots and then label the starting dot "18".

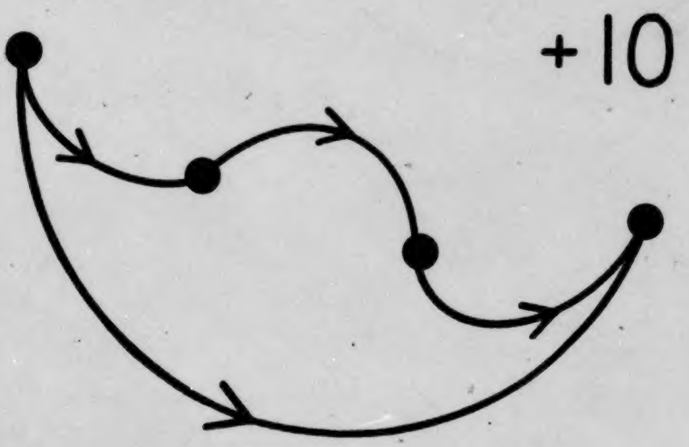
T: If this number were 18, what would the ending number be? Write it on a piece of paper.

Walk around and observe many answers. If there are several students who do not know the correct answer (48), ask someone to label the dots in the arrow picture on the board. Ask the students who predicted that the ending number would be 48 to explain how they calculated the answer. Erase the labels of the dots before continuing.

With a faster pace, give a starting number and ask the class for the corresponding ending number. Choose starting numbers which are appropriate for the abilities of your students, and try to alternate easy and difficult calculations in order to involve many of your students. Frequently ask students how they calculated an answer. Perhaps some of your students will observe that they can add 30 to the particular starting number. Suggestions for starting numbers are given below; corresponding ending numbers are given in parentheses.

0	(30)	51	(81)	1,000	(1,030)
9	(39)	100	(130)	29	(59)
38	(68)	105	(135)	92	(122)
80	(110)	74	(104)	1,000,000	(1,000,030)

Draw an arrow in red from the starting dot to the ending dot.



T: What could this red arrow be for?

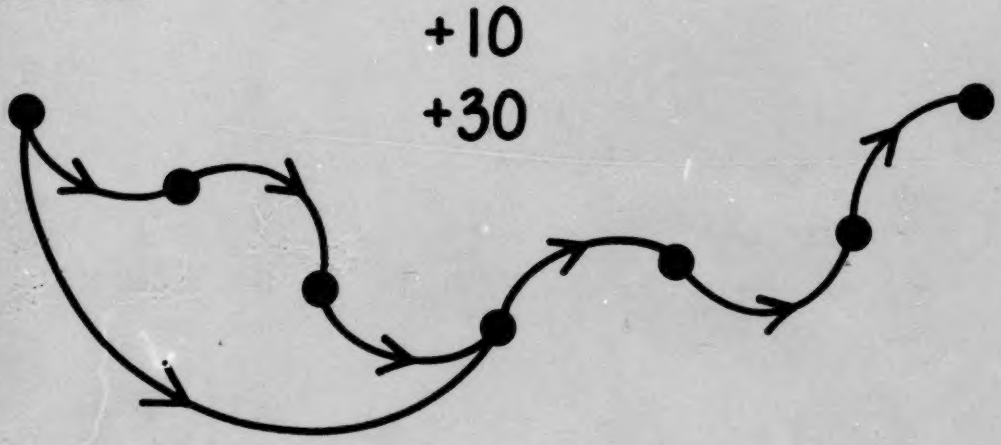
S: +30, because 3 × 10 = 30.

S: +30, because there are three +10 arrows.

S: The ending number is always 30 more than the starting number.

If no one knows that the red arrow could be for +30, label the starting dot "0" and ask for a volunteer to label the other dots. Ask the students again what the red arrow could be for and then erase the labels of the dots.

Write "+30" in red near the arrow picture and extend the picture to include six blue arrows.

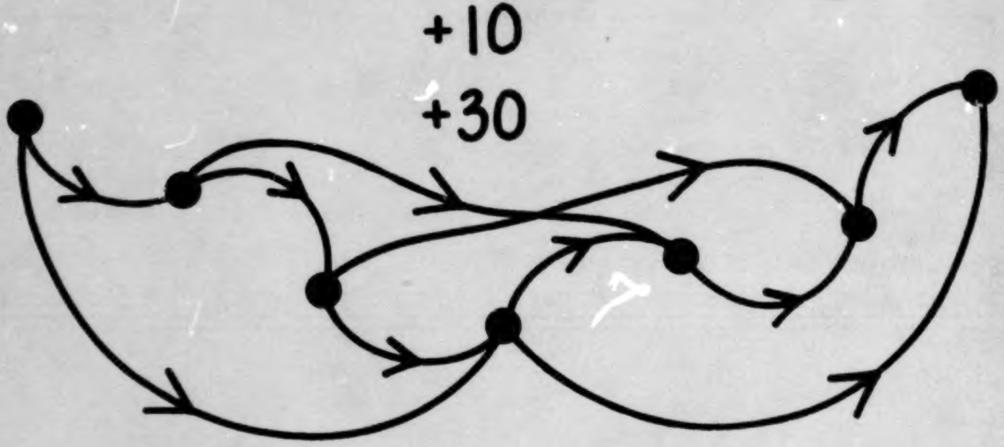


T: Where could we draw another +30 arrow?

Ask a volunteer to trace a +30 arrow. If it is correctly traced, ask the student to draw the arrow.

If students have difficulty locating +30 arrows, label the starting dot "61" and invite someone to label the other dots. Ask again where another +30 arrow could be drawn.

Continue until all the possible +30 arrows are drawn.



Distribute colored pencils and unlined paper. Ask the students to build their own +10 arrow road starting with any whole number between 0 and 10. Encourage all students to continue their +10 arrow road until the ending number is more than 200. Suggest to students who finish quickly that they draw +30 arrows between dots already in their arrow pictures.

N7 Multiplication and Addition Problems #1

CAPSULE LESSON SUMMARY

Calculate "5 x 47" and "3 x 356" with the help of the Minicomputer. Decide how many students voted in an election knowing the tallies on the three options. Do other similar addition calculations.

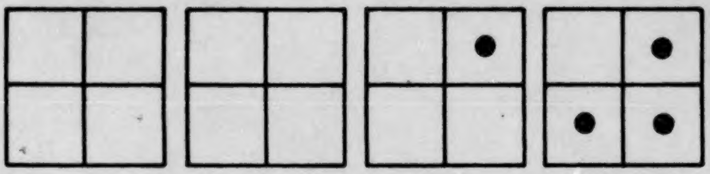
MATERIALS

Teacher: Minicomputer set
Student: Paper; pencil

DESCRIPTION OF LESSON

Exercise 1: Multiplication Problems

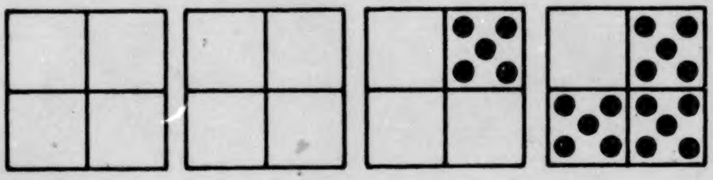
Put this configuration on the Minicomputer.



T: What number is on the Minicomputer?

S: 47.

Add checkers so that this configuration is on the Minicomputer.



T: Now how many times is 47 on the Minicomputer?

S: Five.

Write the calculation on the board.

$$5 \times 47$$

T: Let's calculate "5 x 47". What calculation are we doing on the tens' board?

S: "5 x 40".

T: What number is 5 x 40?

S: 200.

If no one knows that 5 x 40 = 200, guide the students to this conclusion by asking them to calculate "5 x 4" and then "5 x 40". Another method is to ask for "2 x 40", "3 x 40", "4 x 40" and "5 x 40" successively.

Record "5 x 40 = 200" above "5 x 47".

$$5 \times 40 = 200$$
$$5 \times 47$$

T: What calculation are we doing on the ones' board?

S: "5 x 4" and "5 x 2" and "5 x 1".

S: "5 x 7".

If necessary, suggest "5 x 7" yourself.

T: 5 x 7 = . . . ?

S: 35.

T: How do you know?

S: I counted by fives.

Record "5 x 7 = 35" on the board.

$$\begin{array}{l}5 \times 7 = 35 \\ 5 \times 40 = 200 \\ 5 \times 47\end{array}$$

T: Now it is easy to calculate "5 x 47". Write the answer on a piece of paper.

Look at many students' papers before asking someone to give the answer aloud. Ask for methods of doing the calculation.

Complete the number story.

$$\begin{array}{l}5 \times 7 = 35 \\ 5 \times 40 = 200 \\ 5 \times 47 = 235\end{array}$$

Repeat this activity to calculate "3 x 356".

Exercise 2: Addition Algorithm

Tell this story or a similar one to your students.

T: The students at an elementary school had an election to decide where the school picnic would be held. 278 students voted to have the picnic at the zoo.

Write "278" on the board.

T: 436 students voted to go to the lake for the picnic.

Write "436" on the board to one side of "278".

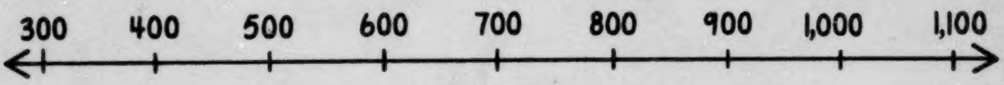
T: And 129 students voted to have the picnic at an amusement park.

Write "129" on the board.

278 **436** **129**

T: Which picnic site is the most popular with these students? (The lake)
Which picnic site is the least popular? (An amusement park)

Draw this number line on the board.



T: About how many students voted in the election?

Ask several volunteers to show approximately where the answer would be on the number line and to explain why they think it would be close to that location.

T: On your paper calculate the number of students who voted in this election.

While the students are working, write the problem on the board.

$$\begin{array}{r} 278 \\ 436 \\ +\,129 \\ \hline \end{array}$$

Look at many of the students' papers and then ask a student with the correct answer to solve the problem at the board and to explain each step. Go over the algorithm again yourself. A sample dialogue follows.

T (pointing to the numbers in the ones' column): First we add the numbers in the ones' column. What number is 8 + 6? (14) . . . and 9 more? (23) How shall we record "23"?

S: Write "3" and carry "2".

T: Why do we write "3" in the ones' column and "2" in the tens' column?

S: Because 23 is two tens and three ones.

Write the "2" and "3" in their appropriate places.

$$\begin{array}{r} 2\overset{2}{7}8 \\ 436 \\ +\,129 \\ \hline 3 \end{array}$$

T: Next we add the numbers in the tens' column. What calculation do we need to do?

S: "2 + 7 + 3 + 2".

T: 2 tens + 7 tens + 3 tens + 2 tens.
What number is 2 tens + 7 tens? (90)
. . . +30? (120)
. . . +20? (140)
How shall we record "140"? Why?

S: Write "4" below the tens' column and "1" at the top of the hundreds' column. 140 is one hundred and four tens.

Write the "1" and the "4" in their appropriate places.

$$\begin{array}{rccc} & {}^{1} & {}^{2} & \\ & 2 & 7 & 8 \\ & 4 & 3 & 6 \\ + & 1 & 2 & 9 \\ \hline & & 4 & 3 \end{array}$$

T: Next we add the numbers in the hundreds' column. What calculation do we need to do?

S: "1 + 2 + 4 + 1".

S: "100 + 200 + 400 + 100".

T: What number is 100 + 200? (300)
. . . +400? (700)
. . . +100? (800)
How shall we record "800"?

N7

S: Write "8" in the hundreds' column.

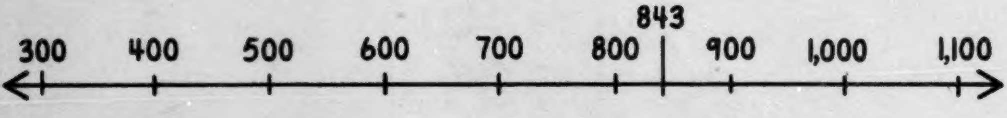
Write the "8" in its appropriate place.

$$\begin{array}{r} {\scriptstyle 1\,2\,} \\ 278 \\ 436 \\ +\,129 \\ \hline 843 \end{array}$$

T: How many students voted in the election? (843)

About where is 843 on the number line?

Ask a volunteer to draw and label a mark for 843 on the number line.



Write these problems on the board and ask the students to solve them individually. Answers are in the boxes.

$$\begin{array}{r} 235 \\ +423 \\ \hline \boxed{658} \end{array} \qquad \begin{array}{r} 378 \\ +414 \\ \hline \boxed{792} \end{array} \qquad \begin{array}{r} 546 \\ +\,66 \\ \hline \boxed{612} \end{array} \qquad \begin{array}{r} 2{,}349 \\ +3{,}723 \\ \hline \boxed{6{,}072} \end{array}$$

Help students who are having difficulty. If some students finish quickly, write a second set of problems on the board. Again, answers are in the boxes.

$$\begin{array}{r} 528 \\ 140 \\ +264 \\ \hline \boxed{932} \end{array} \qquad \begin{array}{r} 2{,}307 \\ 645 \\ +4{,}089 \\ \hline \boxed{7{,}041} \end{array} \qquad \begin{array}{r} 5{,}328 \\ 1{,}659 \\ +3{,}184 \\ \hline \boxed{10{,}171} \end{array}$$

N8 Roads #2

CAPSULE LESSON SUMMARY

When playing checkers with his grandfather, Pedro wins 3¢ or loses 2¢ depending upon the outcome of a game. Decide what Pedro's money situation could be after playing four games if he starts with no money. Describe the possibilities in arrow pictures. Pose related problems.

MATERIALS

Teacher: Colored chalk
Student: Paper; pencil; colored pencils

DESCRIPTION OF LESSON

Tell the following or a similar story to the class.

T: A boy named "Pedro" likes to play checkers with his grandfather. Each time Pedro wins, his grandfather gives him 3¢. Each time his grandfather wins, Pedro gives him 2¢.

Record this information on the board.

Pedro wins: Pedro gets 3¢

Grandfather wins: Pedro gives him 2¢

T: Pedro starts with no money. What could happen in four games? Draw an arrow picture to show Pedro's possible wins and losses. What kind of arrows will you use?

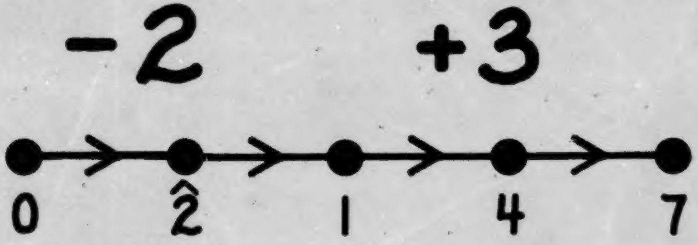
S: -2 arrows and +3 arrows.

Distribute paper and colored pencils to the students. Write a key for the arrows on the board and ask the students to copy it on their papers.

−2 +3

Observe students' work. Find students with roads which have different ending numbers and ask them to copy their pictures onto the board.

Discuss several of the arrow pictures on the board. For example, to consider this road. . .



ask questions such as the following:

- How many games did Pedro win? (Three)
- How many games did Pedro lose? (One)
- How much money would Pedro have after the four games? (7¢)
- What does $\hat{2}$ here tell us? (Pedro owes his grandfather 2¢ after the first game.)

After looking at several pictures, suggest organizing the information in a table.

T: Let's put all this information in a table. What is the best Pedro could do after four games of checkers?

S: Pedro could win all four games and receive 12¢.

N8

Draw this table on the board.

Pedro wins	Grandfather wins	Pedro's money situation
4	0	12¢

T: What is the worst Pedro could do after four games of checkers?

S: Pedro could lose all four games and end up owing his grandfather 8¢.

Record this information in the table, leaving several lines of space between the entries.

Pedro wins	Grandfather wins	Pedro's money situation
4	0	12¢
0	4	$\hat{8}$¢

Complete as much of this table as possible by referring to the arrow pictures the students drew on the board. Record the information as in the table below so that the patterns will be obvious to the students. If your students have not found one of the solutions, the order in which the information is recorded should be sufficient to prompt their discovery of it.

Pedro wins	Grandfather wins	Pedro's money situation
4	0	12¢
3	1	7¢
2	2	2¢
1	3	$\hat{3}$¢
0	4	$\hat{8}$¢

NOTE: A student might suggest the possibility of a tie. You should point out to the class that in checkers it is not possible to have a tie; someone always wins if the game is finished.

T: Do you see any patterns?

The students should easily see the patterns in the first two columns. You may need to ask specifically for a pattern in the last column.

T: How much more is 12 than 7? (5)
How much more is 7 than 2? (5)
How much more is 2 than $\hat{3}$? (5)
How much more is $\hat{3}$ than $\hat{8}$? (5)

Refer to the number line if the students have difficulty answering these questions.

You may wish to ask why Pedro will have 5¢ more if he wins one game more. Commend any student who suggests that it is 5¢ more because 3 + 2 = 5. Do not point this out yourself.

NOTE: When Pedro wins a game rather than losing it, he gains 3¢ from his grandfather but also keeps 2¢ he would otherwise have given to his grandfather if he had lost.

Erase the arrow pictures and the table.

T: One afternoon Pedro and his grandfather played several games of checkers. When they finished, Pedro had won exactly 8¢. Is this possible? Draw an arrow picture to show how Pedro could win 8¢.

Write this information on the board.

Pedro won 8¢

You may need to restate this problem several times before all the students understand the task.

T: Draw an arrow picture to find out how many games Pedro might have won and how many games Pedro might have lost to end up with exactly 8¢.

Allow five to ten minutes for individual work. Ask the first student who finds a solution to put it on the board.

If after several minutes of individual work, no one has used the idea of building a road from 0 to 8 using +3 and -2 arrows, you should guide them as in the following dialogue.

T: Before they first started playing checkers, Pedro had no money.

Draw a dot on the board and label it "0".

T: When they stopped playing checkers. Pedro had won exactly 8¢. He started with 0¢ and gradually won 8¢.

Draw another dot and label it "8".

T: Each time Pedro wins, we will draw one red arrow.

Write "+ 3" in red on the board.

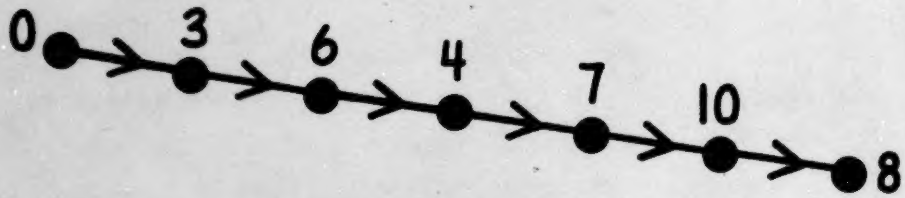
T: Each time Pedro loses, we will draw one blue arrow.

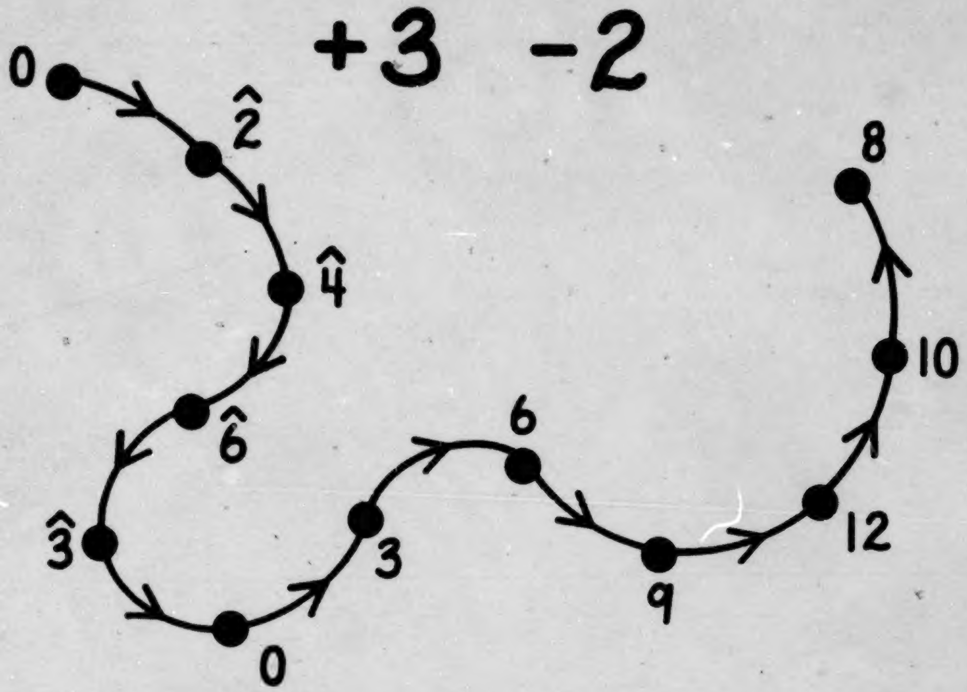
Write "- 2" in blue on the board.

T: Using just these two kinds of arrows build a road from 0 to 8 to show how Pedro could win exactly 8¢.

Observe the students' work. Find students with different roads and ask them to put their arrow pictures on the board. There are many solutions; two are given here (below and on the next page).

+3 -2





NOTE: To show the sequence of events in this situation, we allow more than one dot to have the same label.

Discuss a few of the arrow pictures on the board, asking in each situation for how many wins and losses for Pedro and for the total number of games played.

T: Have we found all the possibilities?

Encourage discussion. There are an infinite number of solutions to this problem, but it is not necessary that your students realize this.

If your students are interested, you may wish to propose the following problem.

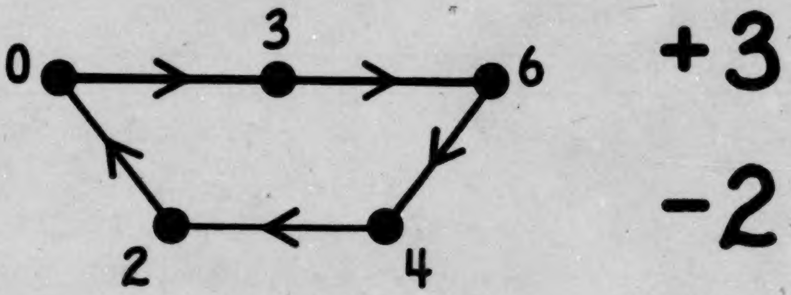
T: One day, Pedro and his grandfather were playing checkers again. After several games, they had both won as much money as they had lost; so Pedro was back to 0¢. How could this have happened?

Allow some minutes for the students to explore this new problem, then continue with a collective discussion.

T: How could Pedro and his grandfather break even.

S: It could happen after five games if Pedro wins twice and his grandfather wins three times.

Ask students who find solutions to describe their arrow pictures. Copy one arrow picture on the board; for example,



NOTE: There are many other arrow roads from 0 back to 0 with +3 and -2 arrows. Perhaps one of your students will build a road with ten or even fifteen arrows before returning to 0.

T: If Pedro and his grandfather are even after five games, could they be even again if they continue playing.

S: After five more games they could be even again.

Perhaps a student will observe that they could break even after every fifth game and this could continue forever; so they could be even after five games, ten games, fifteen games, and so on.

N9 Eli the Elephant

CAPSULE LESSON SUMMARY

When Eli awakes, five of his thirteen peanuts are missing. Clarence has played a trick on him! Decide that Clarence could have added five magic peanuts to Eli's bag or he could have taken five regular peanuts out. Conclude that $13 - 5 = 13 + \widehat{5}$. Using the Minicomputer, extend this experience by showing that $154 - 89 = 154 + \widehat{89}$. Practice adding a negative number to a positive number on individual Minicomputers.

MATERIALS

Teacher: Minicomputer set
Student: Minicomputer set

DESCRIPTION OF LESSON

Ask a student to explain, for the benefit of any new students to the program, who Eli the Elephant is[*]. Be sure the student explains what happens when a regular peanut meets a magic peanut (both peanuts disappear).

Ask the students who Clarence the Crocodile is. The students should tell you that Clarence likes to trick Eli the Elephant and sometimes he changes the number of peanuts in Eli's bag while Eli is asleep.

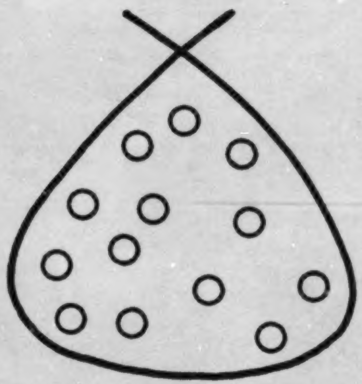
Exercise 1

T: One day Eli becomes very tired after a long walk in the jungle and he decides to take a nap. Before he goes to sleep, Eli counts his peanuts very carefully and finds that there are thirteen peanuts in his bag.

---

[*] Eli is a very hungry elephant whose favorite food is peanuts. Eli always keeps a little bag of peanuts with him and whenever he goes walking he gathers peanuts. Sometimes Eli picks some magic peanuts which look very much like regular peanuts. They are called magic peanuts because whenever a magic peanut meets a regular peanut, they both disappear.

Draw this picture on the board.



T: While Eli is sleeping, Clarence the Crafty Crocodile comes by. When Clarence sees that Eli is asleep, he opens Eli's bag of peanuts and does something. Then Clarence closes Eli's bag and quietly goes away. When Eli wakes up, he opens his bag and counts the peanuts. Eli discovers that there are only eight peanuts in his bag. What do you think Clarence the Crafty Crocodile did?

Allow the students to discuss this problem. Two possibilities should be suggested: Clarence could have removed five peanuts from Eli's bag, or Clarence could have put five magic peanuts into the bag.*

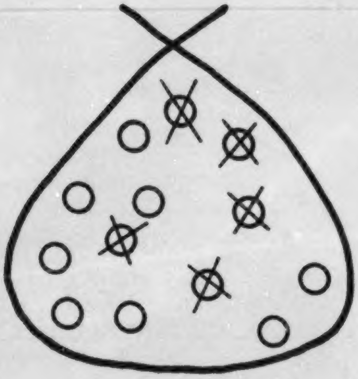
When each of these two possibilities is suggested, illustrate it with a picture of Eli's bag of peanuts. For one of these illustrations, use the picture already on the board.

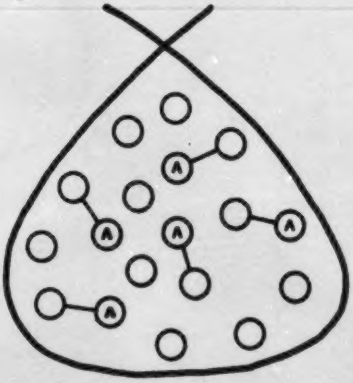
---

* There are other more complex possibilities which may be suggested; for example, Clarence could have removed four regular peanuts and added one magic peanut. Accept such correct suggestions, but for the purpose of this lesson concentrate on these two possibilities: Clarance removed five regular peanuts or Clarence added five magic peanuts.

N9

When a student suggests that Clarence could have taken peanuts out of Eli's bag, cross out five of the peanuts and write "13 - 5 = 8" below the bag. When a student suggests that Clarence could have put five magic peanuts into Eli's bag, draw five magic peanuts " Ⓐ ", indicate that five regular and five magic peanuts disappear, and write "13 + $\hat{5}$ = 8"* below the bag. Arrange the pictures so they are side by side on the board.





$13 - 5 = 8$

$13 + \hat{5} = 8$

Emphasize that we don't know what Clarence did when he opened Eli's bag because removing five regular peanuts and adding five magic peanuts have exactly the same effect. Write this number story on the board.

$$13 + \hat{5} = 13 - 5 = 8$$

Erase the board.

T: Another day Eli is walking in the jungle and he gathers 154 regular peanuts. Eli becomes very tired and decides to take a nap. What do you think happens while Eli is sleeping?

S: Clarence comes by and plays another trick on Eli.

---

* Read "13 + $\hat{5}$ = 8" as "thirteen regular peanuts plus five magic peanuts is the same as eight regular peanuts."

T: You're right! Clarence comes by. When Eli awakes, he is very hungry so he opens his peanut bag. Eli counts his peanuts and discovers that 89 peanuts are missing. What do you think Clarence the Crafty Crocodile did?

The students might tell you that Clarence could have removed 89 peanuts from Eli's bag or that he could have put 89 magic peanuts into Eli's bag. As each possibility is suggested, write the appropriate expression on the board.

$$154 + \widehat{89}$$

$$154 - 89$$

T: How can we find out how many peanuts Eli has left in his bag?

If a student suggests drawing a picture of Eli's bag, you should accept this as a good, but not a very practical idea because of the number of peanuts involved. If no one suggests using the Minicomputer, suggest it yourself. Point to the calculations on the board and ask which problem they would like to do first; follow the preference of the majority.

To calculate "154 - 89", invite a student to put 154 on the Minicomputer and ask another to take off 89. When students protest that they need to make some backward plays first, ask a student to indicate where checkers need to be for 89. Usually the response will be the standard configuration for 89.

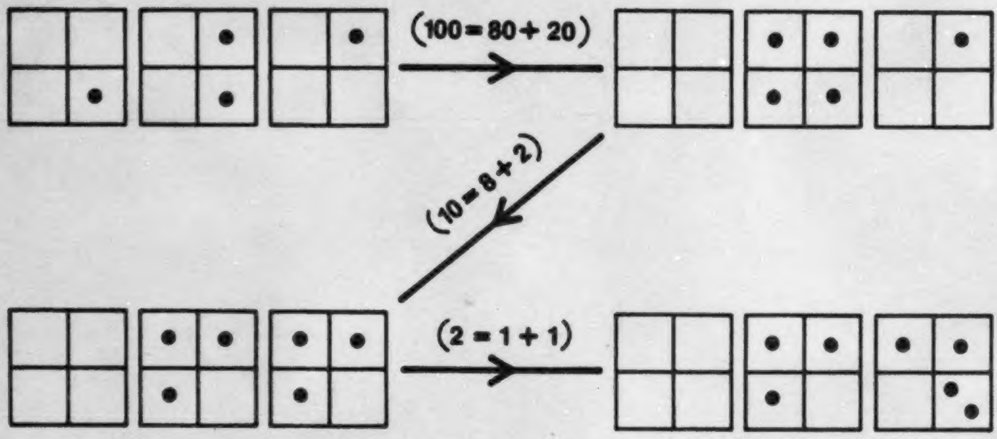
T: Who can make a backward play to help us get checkers for 89?

Ask volunteers to tell you which play they intend to make before actually making it. This will allow you to discourage plays which do not help get the checkers in position to subtract 89.

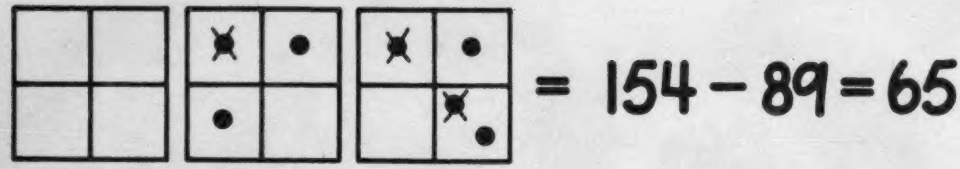
N9

Whenever a play is made which puts a checker in position for the subtraction, mention this to the class and indicate that they are getting closer to the goal.

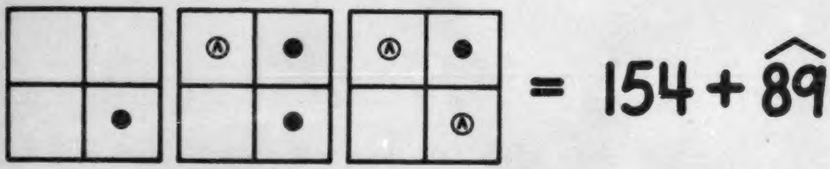
A possible sequence of plays is shown below.



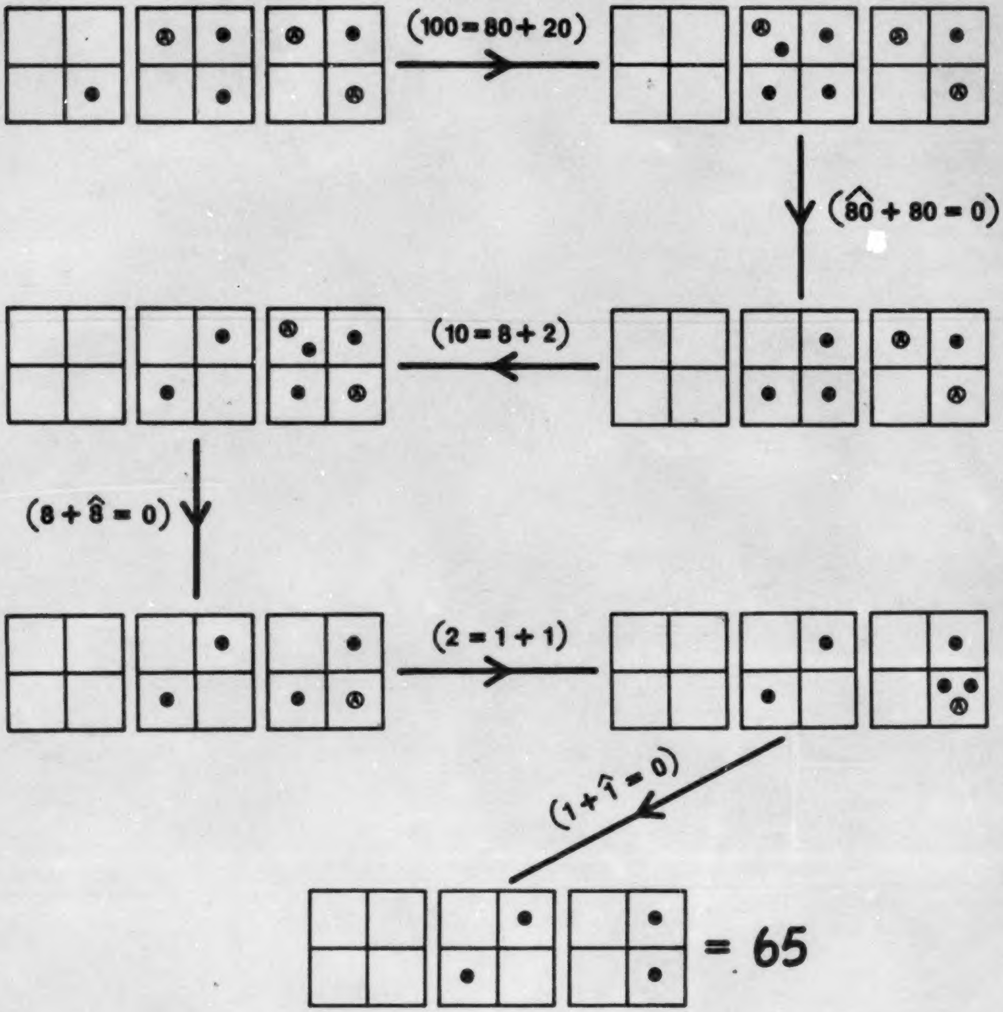
Now it is possible to remove checkers for 89. Invite a student to do so.



To calculate "154 + $\widehat{89}$", ask one student to put 154 on the Minicomputer and another to put on $\widehat{89}$.



The students should suggest backward plays. If necessary, ask the class what plays they could make which would result in positive and negative checkers on the same square. There are many possible sequences of plays; one of which is shown below. Watch closely as students make plays; a student might suggest incorrectly that $\hat{8} = \hat{4} + 4$ when the correct play is $\hat{8} = \hat{4} + \hat{4}$.



After doing each of the two calculations on the Minicomputer, complete the appropriate number story.

Emphasize that we don't know what Clarence did when he opened Eli's bag because removing 89 regular peanuts and adding 89 magic peanuts have exactly the same effect.

$$154 - 89 = 154 + \widehat{89} = 65$$

T: Which calculation was easier to do on the Minicomputer?

Very likely your class will conclude that it is easier to add $\widehat{89}$ than to subtract 89 because the negative checkers provide a visual reminder which guides the students when they are making plays.

Erase the board and remove the checkers from the Minicomputer.

Exercise 2

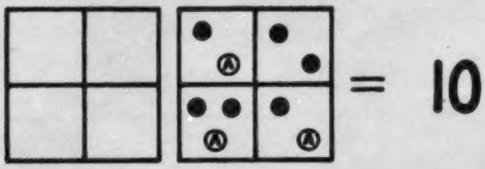
Each student should have two Minicomputer boards (one sheet) and some checkers. Display two Minicomputer boards.

T: Put 100 on your Minicomputer.

Invite a few students with different configurations for 100 to show them on the demonstration Minicomputer.

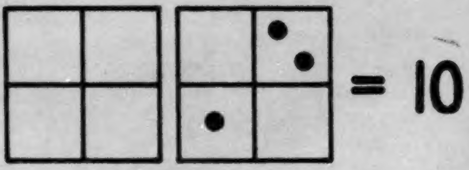
T: Put 10 on your Minicomputer using at least one negative checker.

Invite a few students with different configurations for 10 to show them on the demonstration Minicomputer. Briefly check each configuration with the class. A possible dialogue is given here.



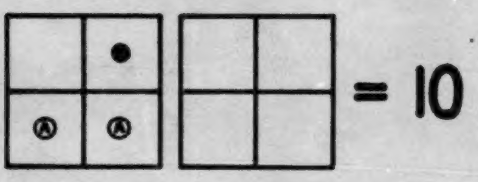
T: Anthony claims that this number is 10. Do you agree?

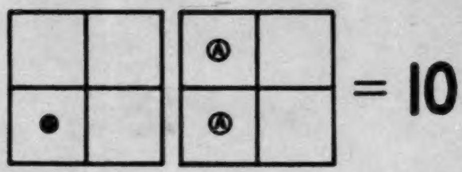
S: We can take off some checkers. Then it will be easier to read.



S: 4 + 4 + 2 = 10. The number on the Minicomputer is 10.

If your students enjoy this activity, ask them to put 10 on their Minicomputers using exactly one regular checker and two negative checkers. There are two possible solutions.

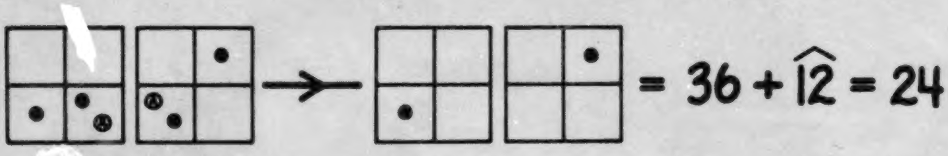




Write this problem on the board.

$$36 + \widehat{12}$$

Ask the students to calculate "$36 + \widehat{12}$"[*] on their (individual) Minicomputers. Invite a student who does the calculation correctly to do it again on the demonstration Minicomputer. Students having difficulty should watch closely.



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[*] Read "$36 + \widehat{12}$" as "thirty-six plus negative twelve."

N9

Continue this activity with calculations which are appropriate for the abilities of your students. Occasionally, after making a calculation, write the corresponding subtraction problem on the board. For example, after the students determine that 36 + $\widehat{12}$ = 24, write this sbutraction problem on the board.

$$\begin{array}{r} 36 \\ -12 \\ \hline 24 \end{array}$$

Calculations of varying difficulty are listed below and on the next page.

*

No plays are needed.

$$59 + \widehat{18} \qquad 37 + \widehat{25}$$

$$96 + \widehat{14} \qquad 63 + \widehat{42}$$

* *

One backward play is needed.

$$48 + \widehat{20} \qquad 43 + \widehat{22}$$

$$24 + \widehat{10} \qquad 57 + \widehat{23}$$

$$39 + \widehat{14} \qquad 30 + \widehat{28}$$

* * *

More than one backward play is needed.

$46 + \widehat{21}$

$59 + \widehat{24}$

$95 + \widehat{43}$

$63 + \widehat{18}$

$70 + \widehat{26}$

$80 + \widehat{37}$

NOTE: Of course, the plays needed to do each of these problems depend upon the configurations put on the Minicomputer. We assume that all numbers are put on the Minicomputer in standard configurations in stating the number of backward plays needed.

N10 Introductory Arrow Problems #2

CAPSULE LESSON SUMMARY

Count by threes and then multiply 3 by various numbers as a mental arithmetic activity. Consider pieces of arrow roads and ask if they could be parts of the same arrow road.

MATERIALS

Teacher: Colored chalk
Student: Paper; pencil; colored pencils

DESCRIPTION OF LESSON

Exercise 1: The Function +3

Call on several students to count by threes starting at 0 and continuing to 30 unless an error is made.

T: What number is 3 x 3?

S: 9.

T: How do you know?

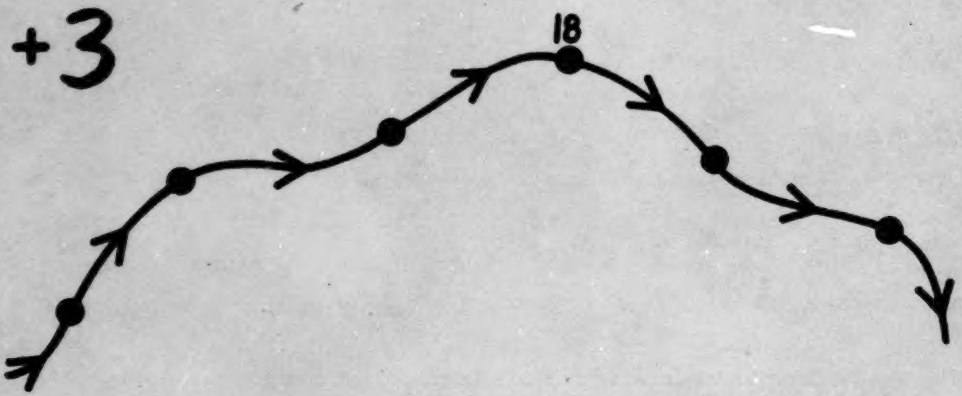
S: 3 + 3 = 6 and 6 + 3 = 9.

Continue this activity with the following calculations; answers are shown in parentheses. Occasionally ask students to explain how they calculated an answer.

5 x 3 (15)
6 x 3 (18)
8 x 3 (24)
4 x 3 (12)
10 x 3 (30)
9 x 3 (27)
12 x 3 (36)
20 x 3 (60)
30 x 3 (90)

NOTE: A student may opt to calculate "3 x 20" ("20 + 20 + 20") rather than counting by threes to calculate "20 x 3". The same is true for calculating "30 x 3".

Draw this arrow picture on the board.



Point to the appropriate arrows as you say:

T: To show this arrow road goes on and on in both directions, I have drawn an arrow without a starting dot and an arrow without an ending dot.

Point to the dot for 18.

T: This dot is for 18. Which dots are for numbers larger than 18?

Students should indicate the dots to the right of 18.

T: How do you know?

S: The red arrows are for +3; adding 3 to a number gives a larger number.

T: Which dots are for numbers smaller than 18?

Students should indicate the dots to the left of 18.

T: How do you know?

S: If you go backward on a +3 arrow road, the numbers get smaller.

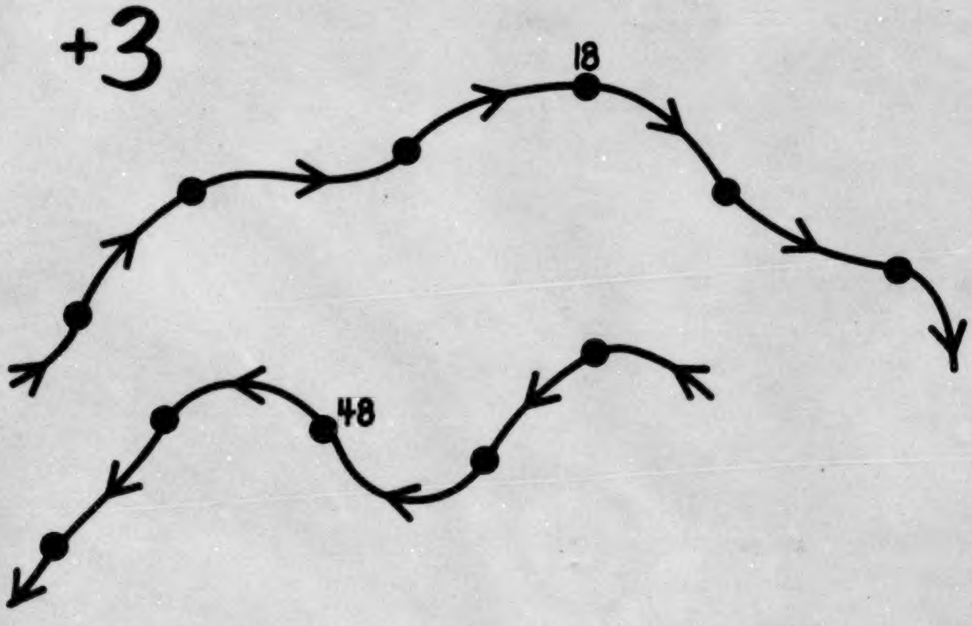
N 10

T: What are some of the other numbers on this +3 arrow road which meets the number 18?

Accept several suggestions. Any multiple of 3 is correct.

NOTE: You can determine easily if a number is a multiple of 3 by summing its digits. If the sum of the digits is a multiple of 3, then the number is also a multiple of 3. For example: Since 7 + 2 = 9 and since 9 is a multiple of 3, 72 is also a multiple of 3. We know that 200 is not a multiple of 3 because 2 + 0 + 0 = 2 and 2 is not a multiple of 3. This is for your information only.

Below the first picture, draw part of a +3 arrow road meeting the number 48.



T: This is part of the +3 arrow road which meets the number 48. Which of these dots are for numbers larger than 48?

Students should indicate the dots to the left of 48.

T: Which of these dots are for numbers smaller than 48?

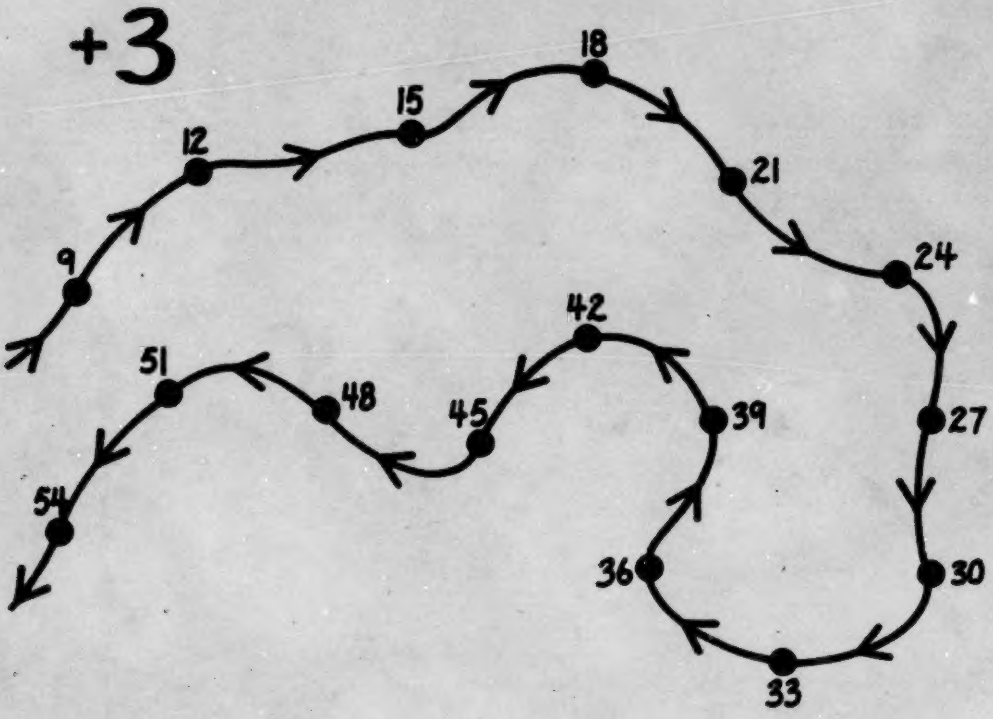
Students should indicate the dots to the right of 48.

T: What are some of the numbers on the +3 arrow road which meets the number 48?

Accept several suggestions. Any multiple of 3 is correct.

T: Could it be that these are parts of the same arrow road? Why or why not?

Encourage the students to express themselves. Then ask the students to draw a +3 arrow road which meets 18 and to extend it until they know for sure whether or not it meets 48. After students have worked independently for five to ten minutes, invite some to extend the picture on the board to show that 18 and 48 are indeed on the same +3 arrow road.



Trace the arrow road from 18 to 48.

T: 18 and 48 are on the same +3 arrow road. How many +3 arrows do we need to go from 18 to 48?

S: Ten.

T: If I drew an arrow from 18 to 48, what could this arrow be for?

S: +30.

T: How do you know?

S: Ten +3 arrows is the same as one +30 arrow.

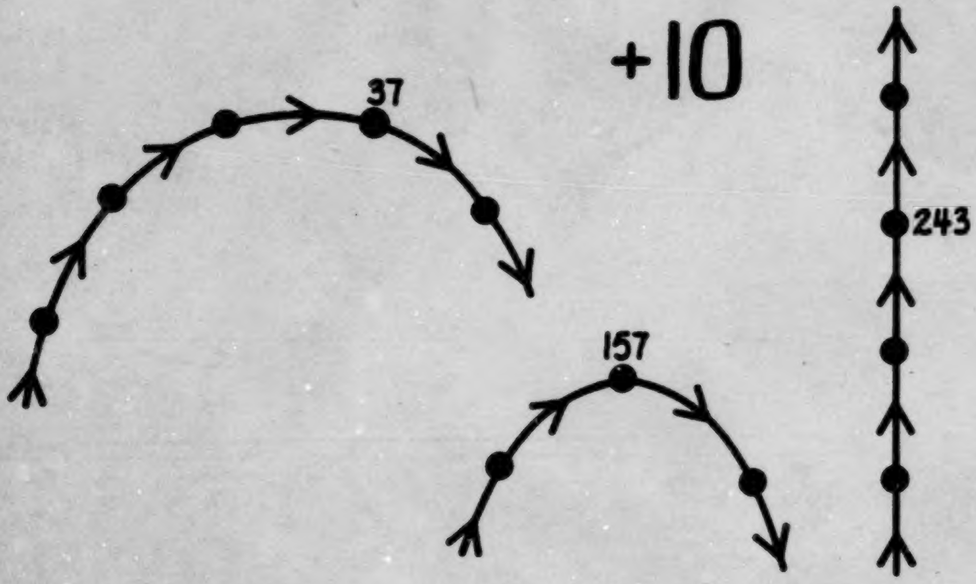
S: 18 + 30 = 48.

Erase the board before going on to Exercise 2.

Exercise 2: The Function +10

Ask students to count by tens starting at 0, at 1, at 5, and at 8. Emphasize the pattern that in each case the last digit remains constant.

Draw this arrow picture on the board.



T: Do you think these could all be parts of the same +10 arrow road?

S: 37 and 157 are on the same +10 arrow road because both numbers end in 7.

S: The +10 arrow road which meets 243 only meets positive numbers which end in 3, so 243 can't be on the same arrow road as 37 or 157.

If your students are not sure that 37 and 157 are on the same +10 arrow road and that 243 is on another +10 arrow road, ask the students to draw a +10 arrow road which meets 37 and to extend it until they know whether or not it meets 157 and 243. When a student has extended a +10 arrow road past 243, draw it to the class' attention and conclude that 157 is on the same +10 arrow road as 37 but 243 is not.

T: How many +10 arrows do we need to go from 37 to 157? How do you know?

S: We need twelve arrows. I counted by tens from 37 to 157.

S: 37 + 100 = 137, so we need ten +10 arrows to go from 37 to 137 and then two more +10 arrows to take us to 157. 10 + 2 = 12.

T: If I drew an arrow from 37 to 157, what could this arrow be for? How do you know?

S: +120, because 37 + 120 = 157.

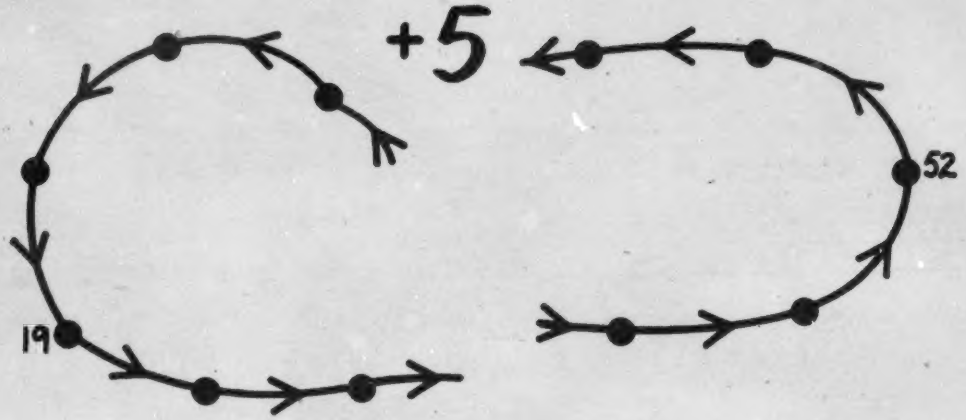
T: Twelve +10 arrows is the same as one +120 arrow.

Erase the board before going on to Exercise 3.

N 10

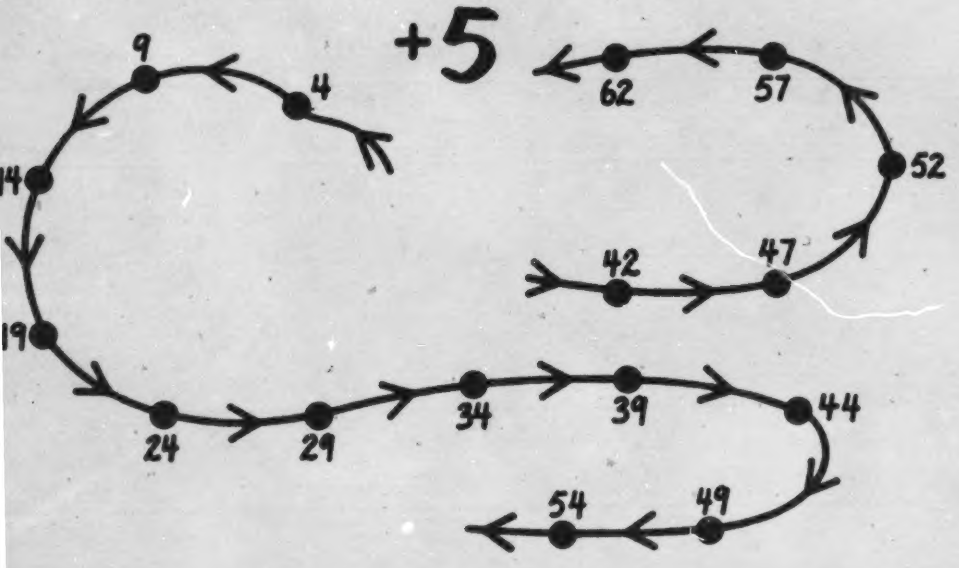
xercise 3: The Function +5

raw this arrow picture on the board.



: Do you think that these are parts of the same +5 arrow road?

ncourage the students to express their opinions. Then ask them to draw a +5 rrow road which meets 19 and to extend it until they know whether or not it eets 52. Allow time for independent work. Then invite a student to label the ots in the picture on the board and to extend the arrow road meeting 19 until is clear that 52 is skipped.



Conclude that 19 and 52 are not on the same + 5 arrow road.

T: Do you see any patterns in this arrow picture on the board?

S: The numbers in the arrow road which meets 19 all end in 4 or 9.

S: The numbers in the arrow road which meets 42 all end in 2 or 7.

Invite a student to count by fives starting at 4. If the student has difficulty, point to the + 5 arrow road which starts at 4. Ask for volunteers to count by fives starting at 1, at 2, and at 3.

N 11

N11 Doubling and Halving on the Minicomputer

CAPSULE LESSON SUMMARY

Given a configuration of checkers on the Minicomputer, decide which number it represents. Do some mental arithmetic exercises involving doubling and halving. Consider a configuration on the Minicomputer for some number in which the checkers are grouped in pairs and in which both checkers in each pair are on the same square. First recognize what one-half of the number is and then decide which number it is. Record the situation in an arrow picture. Using the Minicomputer, calculate "$\frac{1}{2}$ x 1,978".

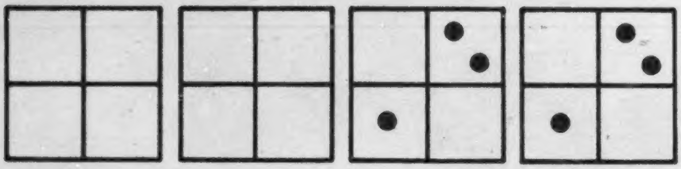
MATERIALS

Teacher: Minicomputer set; colored chalk
Student: Paper; pencil

DESCRIPTION OF LESSON

Exercise 1

Put this configuration on the Minicomputer.



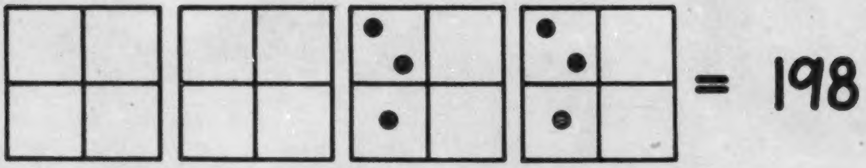
T: What number is on the Minicomputer? Write it on your paper.

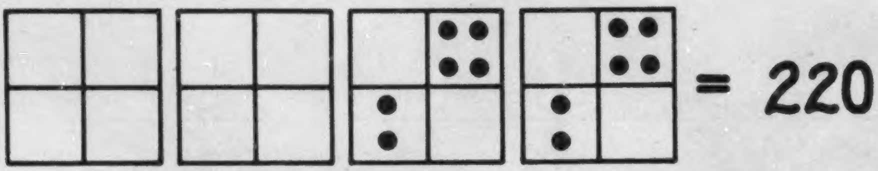
Look at many of the students' answers.

T: What number is on the ones' board? (10)
What number is on the tens' board? (100)
What number is this? (110) How do you know?

Let several students tell which calculations they did. If several students are uncertain that this number is 110, ask for a volunteer to make plays on the Minicomputer until the standard configuration for 110 is obtained.

Repeat this activity with these configurations.





Exercise 2

Ask your students to do some mental arithmetic calculations which involve halving and doubling. A sequence of problems is suggested below; answers are given in the boxes. Do not write the problems on the board. Try to involve all the students by varying the difficulty of the calculations. Occasionally ask students to explain how they calculated an answer.

$2 \times 20 = \boxed{40}$

$\frac{1}{2} \times 40 = \boxed{20}$*

$2 \times 25 = \boxed{50}$

$\frac{1}{2} \times 50 = \boxed{25}$

$2 \times 30 = \boxed{60}$

$\frac{1}{2} \times 60 = \boxed{30}$

$2 \times 45 = \boxed{90}$

$\frac{1}{2} \times 90 = \boxed{45}$

$2 \times 36 = \boxed{72}$

$\frac{1}{2} \times 72 = \boxed{36}$

$2 \times 50 = \boxed{100}$

$\frac{1}{2} \times 100 = \boxed{50}$

$2 \times 51 = \boxed{102}$

$\frac{1}{2} \times 102 = \boxed{51}$

$2 \times 49 = \boxed{98}$

$\frac{1}{2} \times 98 = \boxed{49}$

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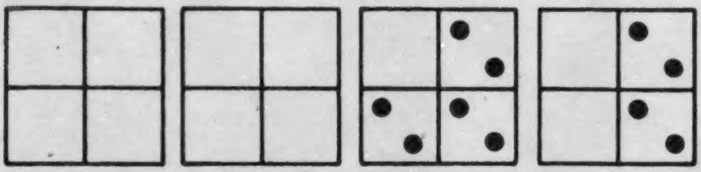
* "Read "$\frac{1}{2} \times 40$" as "one-half of forty."

Exercise 3 :

T: I am going to put a number on the Minicomputer. When you think you know which number it is, write it on your paper.

Gradually put this configuration on the Minicomputer:

- put two checkers on the 40-square ; pause ;
- put two checkers on the 20-square ; pause ;
- put two checkers on the 10-square ; pause ;
- put two checkers on the 4-square and two checkers on the 1-square.



Look at many of the students' answers before asking someone to say the number aloud.

S: 150.

T: What calculations did you do to decide that the number is 150?

Let several students tell which calculations they did ; a few possibilities are given here.

S: 2 x 40 = 80 and 80 + 20 = 100.
100 + 20 = 120; 120 + 20 = 140.
140 + 10 = 150.

S: The checkers on the 40-square and on the 10-square together make 100. 100 + 40 + 10 = 150.

S: The number on the tens' board is 140 because 2 x 70 = 140, and the number on the ones' board is 10 because 2 x 5 = 10.
140 + 10 = 150.

T: Look at the Minicomputer. What number is $\frac{1}{2} \times 150$?

S: 75.

T: How can you tell that $\frac{1}{2} \times 150 = 75$ just by looking at these checkers on the Minicomputer?

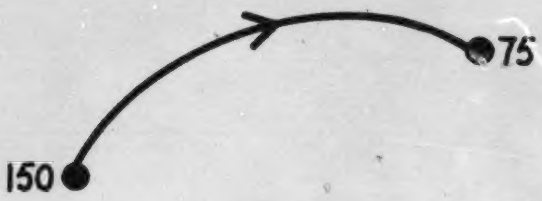
S: The number on the Minicomputer is 150. The checkers are in pairs, so I just looked at one checker in each pair.

Record this number story on the board.

$$\frac{1}{2} \times 150 = 75$$

T: I would like to show this in an arrow picture.

Draw this arrow picture on the board.



T: What could the red arrow be for?

S: $\frac{1}{2} \times$.

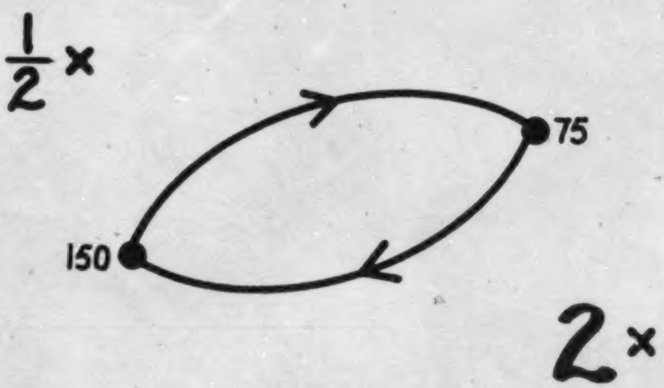
NOTE: The arrow could also be for -75 or "I am more than you" or several other relations, but in this case we are only interested in $\frac{1}{2}\times$.

Write "$\frac{1}{2}\times$" in red near the arrow picture.

T (tracing the red arrow): $\frac{1}{2} \times 150 = 75$. If I drew an arrow here (trace an arrow from 75 to 150), what could it be for?

S: 2×.

Draw a blue arrow from 75 to 150 and write "2×" in blue near the arrow picture.



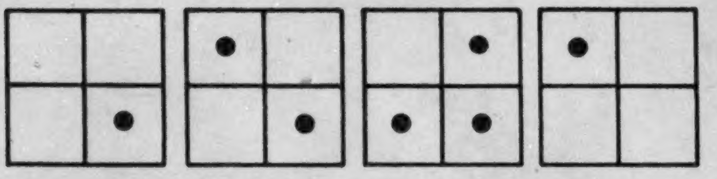
T: What number story is told by this blue arrow?

S: "$2 \times 75 = 150$".

Record this number story on the board.

$$\frac{1}{2} \times 150 = 75$$
$$2 \times 75 = 150$$

Erase the board and then put this configuration on the Minicomputer.



T: What number is on the Minicomputer? Write it on your paper.

Look at several students' answers. Ask a student to write this number (1,978) on the board and to read it aloud.

S: One thousand, nine hundred, seventy-eight.

T: I would like to know which number is $\frac{1}{2} \times 1,978$. Is $\frac{1}{2} \times 1,978$ more than 500? How do you know?

S: Yes, because $\frac{1}{2} \times 1,000 = 500$ and 1,978 is more than 1,000.

T: Is $\frac{1}{2} \times 1,978$ more than 1,000? How do you know?

S: No, because $\frac{1}{2} \times 2,000 = 1,000$ and 1,978 is less than 2,000.

T: So we know that $\frac{1}{2} \times 1,978$ is between 500 and 1,000.

If your students are able to give good explanations, ask if $\frac{1}{2} \times 1,978$ is more than 600, more than 700, more than 800, or even more than 900.

T: What number is $\frac{1}{2} \times 1,978$?

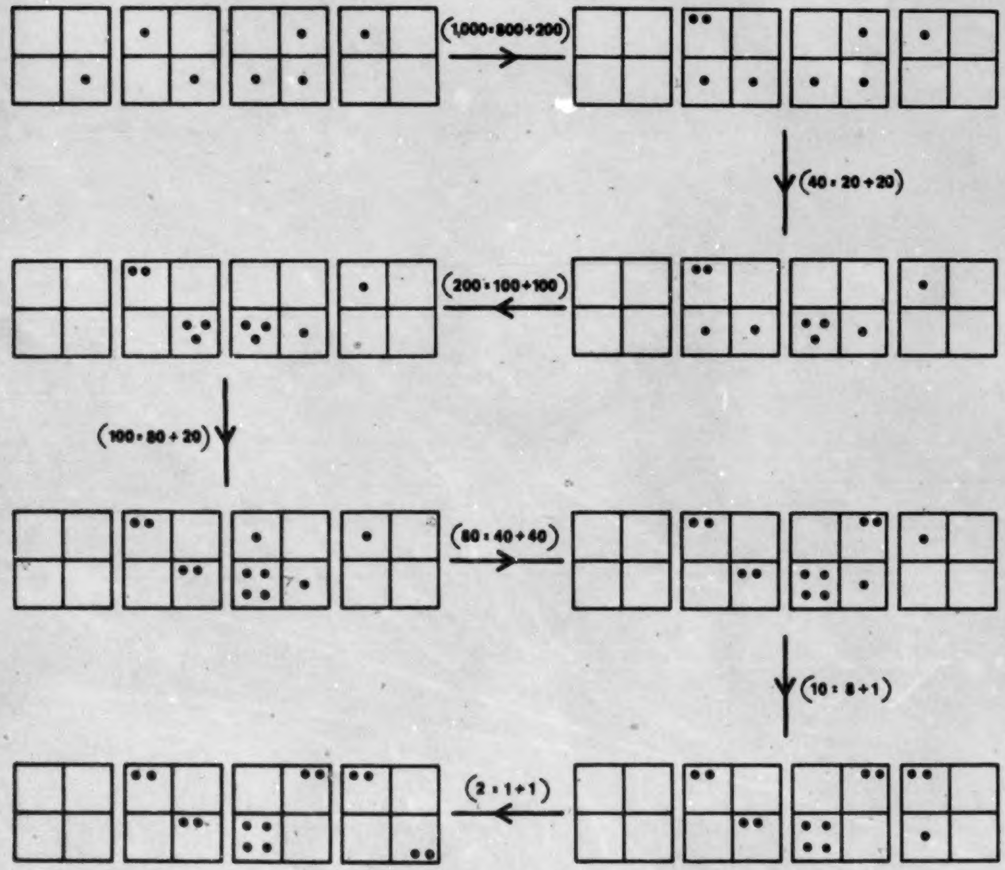
Record some students' guesses on the board, but only record those between 500 and 1,000. If, for example, a student guesses 1,434, simply remind the class that $\frac{1}{2} \times 1,978$ is less than 1,000.

N 11

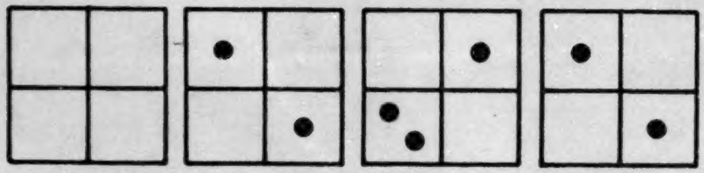
T: How can we find out exactly what number $\frac{1}{2} \times 1{,}978$ is?

S: We need to make backward plays until all the checkers are in pairs (the two checkers in any pair being on the same square).

Invite students to make plays. Any initial backward play made will result in a pair of checkers on one of the squares. Continue this activity until all the checkers are in pairs. Discourage any student who wants to make a play with checkers which are already paired. One possible sequence of plays is shown in the next illustration.



When all the checkers are paired, ask a student to remove some checkers so that one-half of the number will be on the Minicomputer. This student should remove one checker from each pair of checkers. Perhaps you will have this configuration on the Minicomputer.



T: What could we do to make this number easier to read?

S: Make some (forward) plays.

Ask for a volunteer to make plays until the standard configuration for 989 is obtained.

T: What number is $\frac{1}{2} \times 1{,}978$?

S: 989.

Write "$\frac{1}{2} \times 1{,}978 = 989$" to one side of the Minicomputer.

T: How could we check that 989 is $\frac{1}{2} \times 1{,}978$?

S: We could calculate "989 + 989."

T: If $\frac{1}{2} \times 1{,}978 = 989$, then 2×989 should be 1,978. Calculate "989 + 989" on your papers.

N 11

Write this addition problem on the board.

$$\begin{array}{r} 989 \\ +\,989 \\ \hline \end{array}$$

Allow the students some time to solve this problem on their own and then ask for a volunteer to solve the problem at the board and to explain each step. Conclude that 2 × 989 = 1,978 and $\frac{1}{2}$ × 1,978 = 989.

With the students decide which guess was closest to 989.

N12 The Number Line Game

CAPSULE LESSON SUMMARY

Locate numbers on a number line where the marks are for the even numbers. In order to subtract 24 from 52 do a series of calculations.

$$\begin{array}{r} 52 \\ \underline{-10} \\ 42 \\ \underline{-10} \\ 32 \\ \underline{-\ 4} \\ 28 \end{array}$$

Show the corresponding jumps on the number line. Do several similar problems by subtracting tens one at a time and then by subtracting the appropriate number of ones. Play the Number Line Game.

MATERIALS

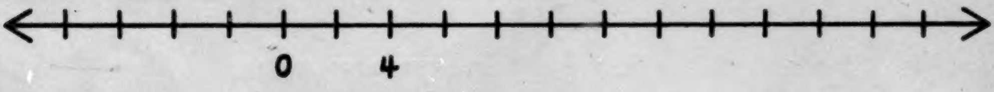
Teacher: Colored chalk

Student: None

DESCRIPTION OF LESSON

Exercise 1: Introduction to the Number Line

Draw a line on the board and graduate it using intervals about six centimeters in length. Locate 0 at one of the marks and locate the number 4 two marks to the right.



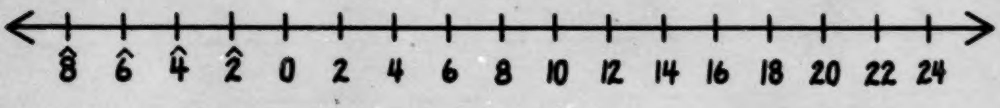
T: Why did I put arrows in my picture?

S: To show that the number line goes on and on in both directions.

N12

T: I've shown you where 0 and 4 are on this number line. Do you know where any other numbers belong on this line?

Invite students to locate other numbers on this number line. Let each of them label several marks so that the activity does not drag. Continue until numbers are associated with each of the marks.



Point to the number line.

T: What do all these numbers have in common?

S: They are all even numbers.

S: They are all multiples of 2.

T: Where is 9 on this number line?

S: 9 is between 8 and 10.

T: Where is $\hat{1}$ on this number line?

S: Between $\hat{2}$ and 0.

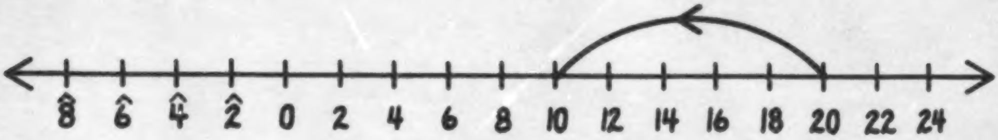
T: Name another odd number which is between $\hat{8}$ and 24.

Suppose a student suggests 15. Invite someone to show where 15 is on this number line.

T: What is the largest odd number between $\hat{8}$ and 24?

S: 23.

Ask a student to show where 23 is on this number line. (Halfway between 22 and 24) Draw an arrow from 20 to 10 on the number line.


$\hat{8}$ $\hat{6}$ $\hat{4}$ $\hat{2}$ 0 2 4 6 8 10 12 14 16 18 20 22 24

T: What could this red arrow be for?

S: -10.

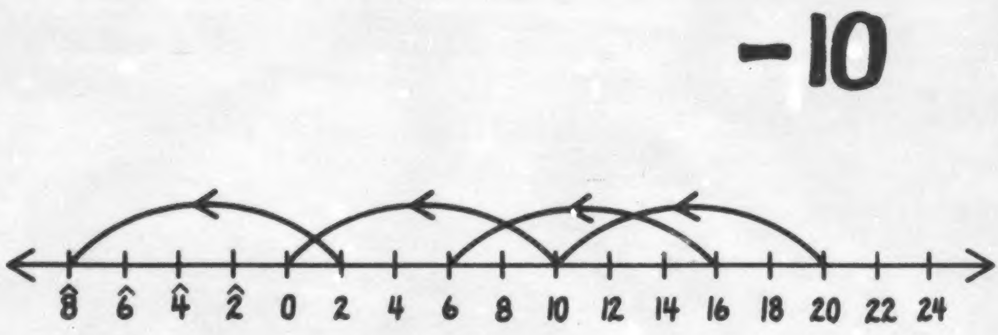
T: Right! 20 - 10 = 10.

NOTE: The red arrow could also be for several other relations including $\frac{1}{2}$x and "I am more than you". Accept these suggestions as possibilities, but tell the class that this red arrow is for -10.

Write "-10" in red on the board near the number line.

T: Where could we draw another -10 arrow?

Invite students to trace other -10 arrows. Draw a few of these arrows yourself, but do not try to draw all of them as the picture will become too complex. Perhaps your number line will look similar to this one.


-10
$\hat{8}$ $\hat{6}$ $\hat{4}$ $\hat{2}$ 0 2 4 6 8 10 12 14 16 18 20 22 24

N12

T: Suppose we start at 40 and make a -10 jump. Where would we land?

S: At 30.

T: If we start at 44 and make a -10 jump, . . . ?

S: At 34.

Continue this activity with the following or similar starting numbers which are appropriate for the abilities of your students.

62 (52)
80 (70)
78 (68)

16 (6)
116 (106)
106 (96)

0 ($\widehat{10}$)
5 ($\hat{5}$)
6 ($\hat{4}$)

Refer to the number line if the students have difficulty calculating "5 - 10" or "6 - 10".

T: If we start at 30 and make two -10 jumps, where would we land?

S: At 10, 30 - 10 = 20 and 20 - 10 = 10.

T: And, if we start at 35 and make two -10 jumps, . . . ?

S: At 15.

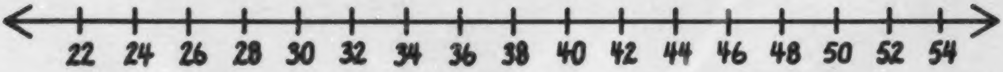
Continue this activity with the following starting numbers or others appropriate for the abilities of your students. Occasionally ask students to explain their answers.

42 (22)
70 (50)
100 (80)

89 (69)
66 (46)
166 (146)

Exercise 2: Subtraction Problems

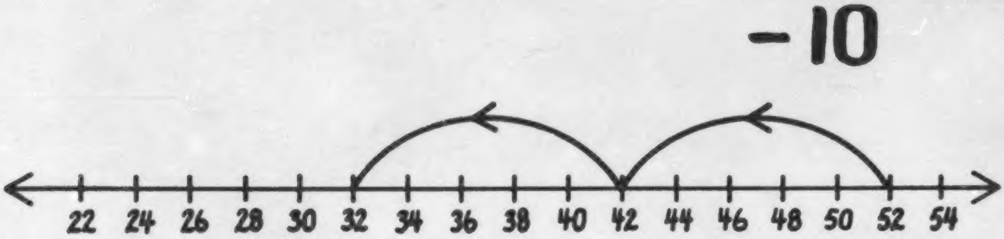
Draw this number line on the board.


22
24
26
28
30
32
34
36
38
40
42
44
46
48
50
52
54

T: If I start at 52 and make two -10 jumps, which numbers will I meet?

S: 42 and 32.

Start at 52 and draw two consecutive -10 arrows.


-10
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50
52
54

T (tracing the red arrows): We start at 52 and subtract 10; 52 - 10 = 42. Then we subtract 10 again; 42 - 10 = 32.

What number is 52 - 20?

S: 32.

The illustrations on the right show what is to be written on the board during the following dialogue.

T: If we subtract 10 . . .

$$\begin{array}{r} 52 \\ -10 \\ \hline 42 \end{array}$$

and subtract 10 again . . .

$$\begin{array}{r} 52 \\ -10 \\ \hline 42 \\ -10 \\ \hline 32 \end{array}$$

it is the same as if we subtract 20.

$$\begin{array}{r} 52 \\ -10 \\ \hline 42 \\ -10 \\ \hline 32 \end{array} \qquad \begin{array}{r} 52 \\ -20 \\ \hline 32 \end{array}$$

T: 52 - 20 = 32, so what number is 52 - 24?

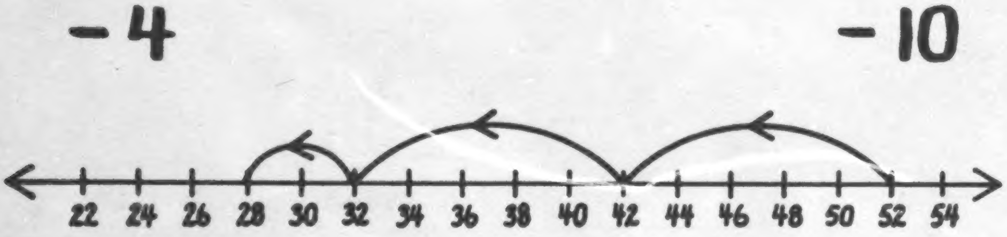
S: 28.

T: How did you do the calculation?

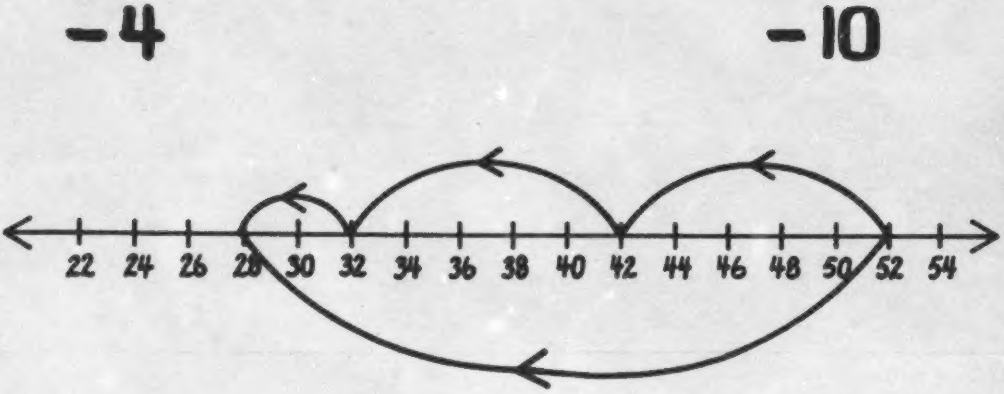
S: I just counted backward 4 from 32.

S: 52 - 20 = 32 and 32 - 4 = 28.

Draw a blue arrow from 32 to 28 on the number line. Write "- 4" in blue near the arrow picture.



Draw a green arrow from 52 to 28.



T: What could the green arrow be for?

S: -24.

Write "-24" in green near the arrow picture. Trace the appropriate arrows as you say:

T: If we subtract 10, then subtract 10 again, and then subtract 4, it is the same as if we subtract 24.

Erase the arrows and the subtraction problems. Write this problem on the board.

$$\begin{array}{r} 40 \\ -12 \\ \hline \end{array}$$

T: Could anyone suggest an easy way to solve this problem?

S: Subtract 10 and then subtract 2.

T: What number is 40 - 10?

S: 30.

$$\begin{array}{r} 40 \\ -12 \\ \hline \end{array} \qquad \begin{array}{r} 40 \\ -10 \\ \hline 30 \end{array}$$

T: Now we need to subtract 2 more.
What number is 30 - 2?

S: 28.

$$\begin{array}{r} 40 \\ -12 \\ \hline \end{array} \qquad \begin{array}{r} 40 \\ -10 \\ \hline 30 \\ -\ \ 2 \\ \hline 28 \end{array}$$

T: Subtracting 10 and then subtracting 2 is the same as subtracting 12.
What number is 40 - 12?

S: 28.

$$\begin{array}{r} 40 \\ -12 \\ \hline 28 \end{array} \qquad \begin{array}{r} 40 \\ -10 \\ \hline 30 \\ -\ \ 2 \\ \hline 28 \end{array}$$

If your students suggest making jumps on the number line to find the answer, follow this suggestion and then calculate the answer as indicated in the previous dialogue. Do not suggest using the number line yourself.

Erase the board and write this problem.

$$\begin{array}{r} 73 \\ -\ 26 \\ \hline \end{array}$$

T: When we wanted to subtract 24, we subtracted 10 twice and then subtracted 4. In this problem we want to subtract 26. What do you suggest?

S: Subtract 10 twice and then subtract 6.

Indicate the calculations on the board.

$$\begin{array}{r} 73 \\ -\ 26 \\ \hline \end{array} \qquad \begin{array}{r} 73 \\ -\ 10 \\ \hline 63 \\ -\ 10 \\ \hline 53 \\ -\ \ 6 \\ \hline 47 \end{array}$$

T: What number is 73 - 26?

S: 47.

N 12

Complete the subtraction problem.

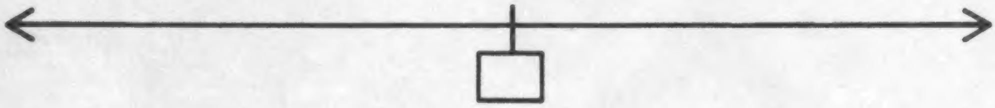
$$\begin{array}{r} 73 \\ -26 \\ \hline 47 \end{array} \qquad \begin{array}{r} 73 \\ -10 \\ \hline 63 \\ -10 \\ \hline 53 \\ -\ 6 \\ \hline 47 \end{array}$$

Exercise 3: The Number Line Game

The Number Line Game is one which may be played frequently throughout the year whenever you finish a lesson early or have some extra time. Each game takes approximately five to ten minutes. Several variations of this game are given at the end of the lesson. Use these variations when your class is ready for them.

T: Today we are going to play the Number Line Game. I'm thinking of a number between 0 and 200. I'll show you where it is on the number line; my number is hidden in the box.

Draw a line on the board and position a box for the secret number near the center of your picture.



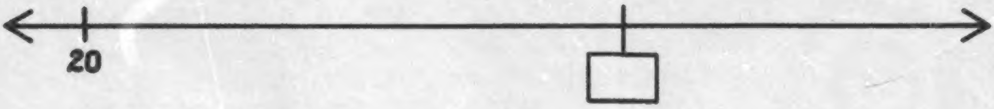
T: Guess what my secret number is.

If someone suggests a number which is not between 0 and 200, do not write anything. Just remind the class that the secret number is between 0 and 200 and ask someone else to guess.

This is a description of a possible game. Suppose the secret number is 114.

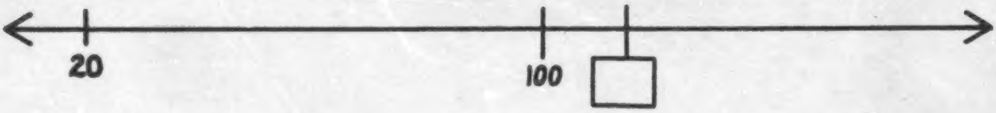
S: 20.

T: I'll show you where 20 is on this number line. It might not be in exactly the right place, but I'll try to make it as close as possible.



T: 20 is less than my secret number.

S: 100.

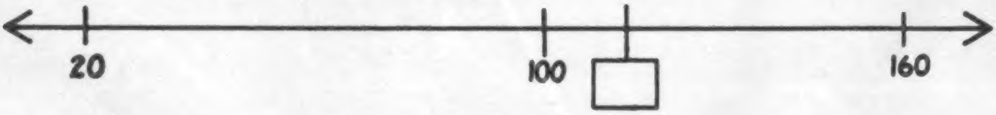


T: 100 is less than my number.

S: 200.

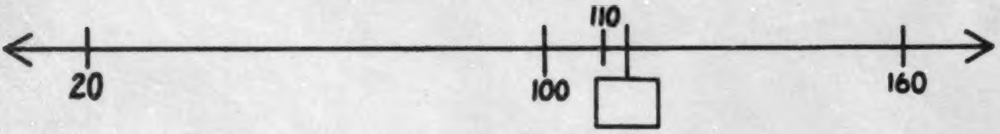
T: My number is between 0 and 200. 200 cannot be my number.

S: 160.



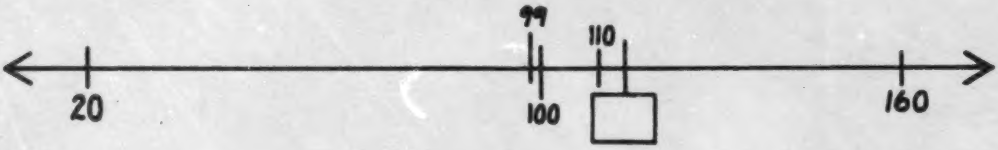
T: 160 is more than my secret number.

S: 110.


110
20
100
160

T: 110 is less than my secret number.

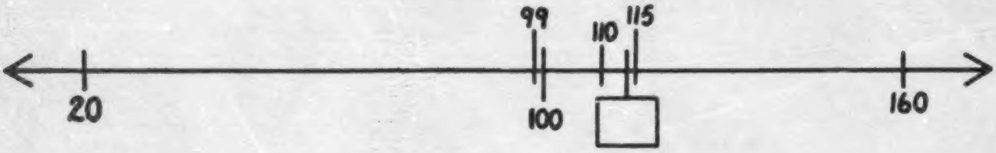
S: 99.


99
110
20
100
160

T: 99 is less than my secret number.

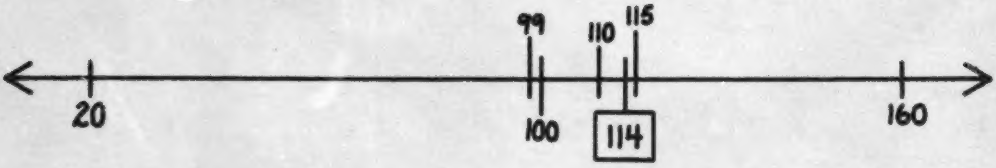
NOTE: Although a guess may give no new information such as 99 in this sample game, do not criticize or comment except to say that it is more or less than your number.

S: 115.


99
110
115
20
100
160

T: 115 is more than my number.

S: 114.



T: You have discovered my secret number!

Play the game a few more times. You may wish to let the student who correctly guessed your number choose the secret number for the next game and whisper it to you. In this case, ask the student to keep the number a secret and not to play in that game.

VARIATIONS OF THE NUMBER LINE GAME

Variation 1

As your class becomes familiar with the Number Line Game, you can expand the range of possible numbers. Some possibilities are:

- numbers between 0 and 500;
- numbers between 500 and 1,000;
- numbers between $\widehat{50}$ and 50;
- numbers between $\widehat{100}$ and 100.

Variation 2

As the students become familiar with numbers such as $2\frac{1}{2}$, occasionally you might choose a non-integer as your secret number. It is best to limit the range of possible numbers to those between 0 and 10 when you use this variation.

Variation 3

When you are using whole numbers only for your range of possible numbers, ask a student to choose a secret number and to judge the guesses of the other students. The student who chooses the secret number tells the class whether a guess is more or less than the secret number. You label a mark on the number line for each guess and, if necessary, help the student to judge. Remind student judges not to give additional clues.

Variation 4

Divide the class into two teams. The teams alternate turns and the members of each team take turns guessing what the secret number is.

N13 Minicomputer Tug of War #1

CAPSULE LESSON SUMMARY

Play Minicomputer Tug of War. Given the answer to an addition calculation and one of the addends, find the other addend.

MATERIALS

Teacher: Minicomputer set; colored chalk

Student: Paper; pencil

DESCRIPTION OF LESSON

Exercise 1: Minicomputer Tug of War

Play several games of Minicomputer Tug of War as described in Lesson W4 FESTIVAL OF PROBLEMS #1 (Lesson 1). Allow about 15 minutes for Exercise 2.

Exercise 2: Addition Algorithm

T: Today I have some puzzles for you. I am going to write an addition problem on the board but one of the numbers will be missing. I want you to copy the addition problem and find the missing number.

Write this problem on the board.

$$\begin{array}{r} 235 \\ + \\ \hline 377 \end{array}$$

T: 235 plus the mystery number is 377. Find the missing number.

Let the students work individually. After a short period of time, ask a student with the correct answer to complete the addition problem on the board.

$$\begin{array}{r} 235 \\ +142 \\ \hline 377 \end{array}$$

N 13

Check the addition collectively to show that 235 + 142 = 377. Continue this activity with this problem.

$$\begin{array}{r} 504 \\ + \\ \hline 568 \end{array}$$

After a short time for individual work, ask a student with the correct answer to complete the addition problem on the board.

$$\begin{array}{r} 504 \\ +\ 64 \\ \hline 568 \end{array}$$

Check the addition collectively to show that 504 + 64 = 568. Continue with this problem.

$$\begin{array}{r} 328 \\ + \\ \hline 692 \end{array}$$

Some of your students might think that the missing number is 376 or 374. If so, calculate "328 + 376" ("328 + 374") and conclude that 376 (374) is not the missing number.

Ask a student with the correct answer to complete the addition problem on the board.

$$\begin{array}{r} 328 \\ +364 \\ \hline 692 \end{array}$$

Do the addition collectively to show that 364 is correct. Emphasize that 8 + 4 = 12, so there is 1 more ten to add in the tens' column. Continue with this problem.

$$\begin{array}{r} 1{,}059 \\ \\ \hline 2{,}281 \end{array}$$

(Answer: 1,222)

Put the following problems on the board and ask the students to copy them and to find the missing numbers. Answers are in the boxes.

$$\begin{array}{r} 43 \\ +\boxed{35} \\ \hline 78 \end{array} \qquad \begin{array}{r} 246 \\ +\boxed{41} \\ \hline 287 \end{array} \qquad \begin{array}{r} 58 \\ +\boxed{36} \\ \hline 94 \end{array} \qquad \begin{array}{r} 425 \\ +\boxed{116} \\ \hline 541 \end{array}$$

If some students finish quickly, you may wish to challenge them with more difficult problems. Answers are in the boxes.

$$\begin{array}{r} 1{,}294 \\ +\boxed{555} \\ \hline 1{,}849 \end{array} \qquad \begin{array}{r} 790 \\ +\boxed{15} \\ \hline 805 \end{array} \qquad \begin{array}{r} 366 \\ +\boxed{107} \\ \hline 473 \end{array} \qquad \begin{array}{r} 828 \\ +\boxed{74} \\ \hline 902 \end{array}$$

N 14

N14 Roads #3

CAPSULE LESSON SUMMARY

Build roads between pairs of numbers using +10 and +1 arrows and as few as possible in each road. After several experiences, describe shortest roads without actually building them.

MATERIALS

Teacher: Colored chalk

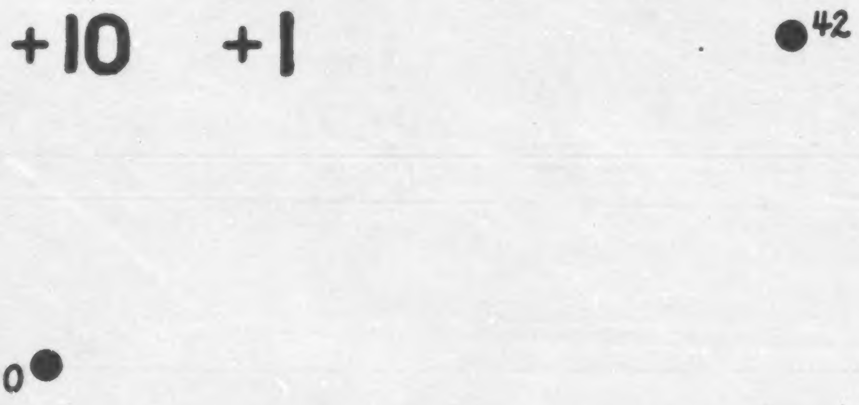
Student: Paper; colored pencils; pencil

DESCRIPTION OF LESSON

Exercise 1

Ask students to count by tens beginning at 0, at 5, at 1, at 6, and at 8. Emphasize that when you count by tens the final digit remains unchanged.

Write this information on the board.



T: Let's build a road from 0 to 42 using only +10 and +1 arrows. How could we build a shortest road?

S: Use as many +10 arrows as we can.

T: How many +10 arrows and how many +1 arrows would we need for a shortest road?

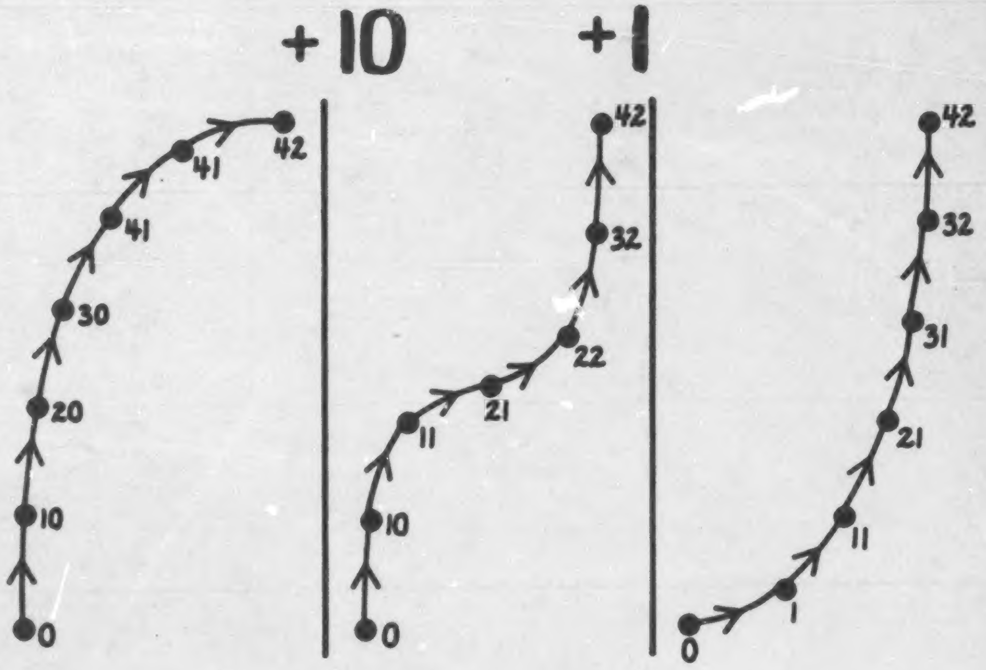
S: Four +10 arrows and two +1 arrows.

T: But how do you know that would give us a road which ends at 42?

S: Because 42 is 4 tens and 2 ones.

Point to "42" on the board and repeat that 42 is 4 tens and 2 ones.

Ask the students to build a red-blue road from 0 to 42 on their papers and to use as few arrows as possible. If necessary, remind them to copy the "key" on their papers to indicate what each color arrow represents. Ask students who finish quickly either to build a different road from 0 to 42 using the same number of arrows or to put their arrow roads on the board. Try to get pictures on the board in which the order of the blue and the red arrows vary. Here are three of the fifteen possible arrow roads.



N 14

Count the +10 arrows and the +1 arrows in the roads on the board and confirm that each has four blue and two red arrows. Conclude that in this situation the order in which the arrows occur does not affect the ending number.

T: If you start a 0 and build a road using five +10 arrows and four +1 arrows, where would the road end?

S: At 54.

T: If you start at 0 and build a road using two +10 arrows and seven +1 arrows, where would the road end?

S: At 27.

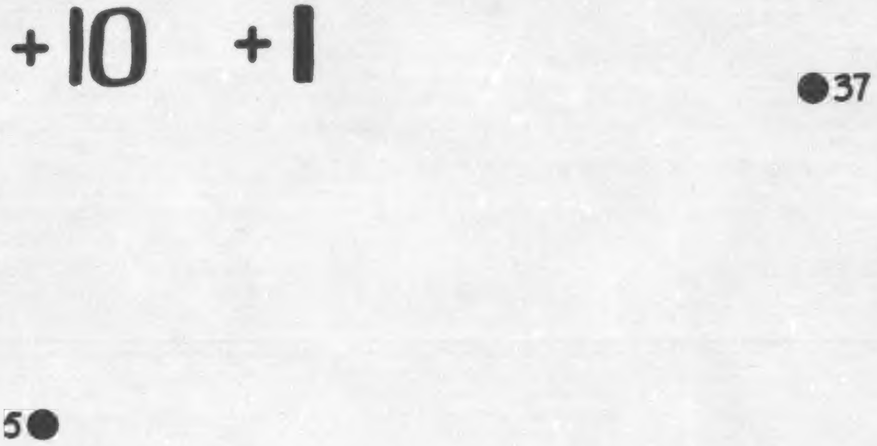
Continue this activity with the following or similar problems.

Starting Number	Number of +10 Arrows	Number of +1 Arrows	Ending Number
0	8	9	89
0	6	10	70
0	0	15	15
0	4	20	60
0	10	0	100
0	15	0	150
0	15	3	153

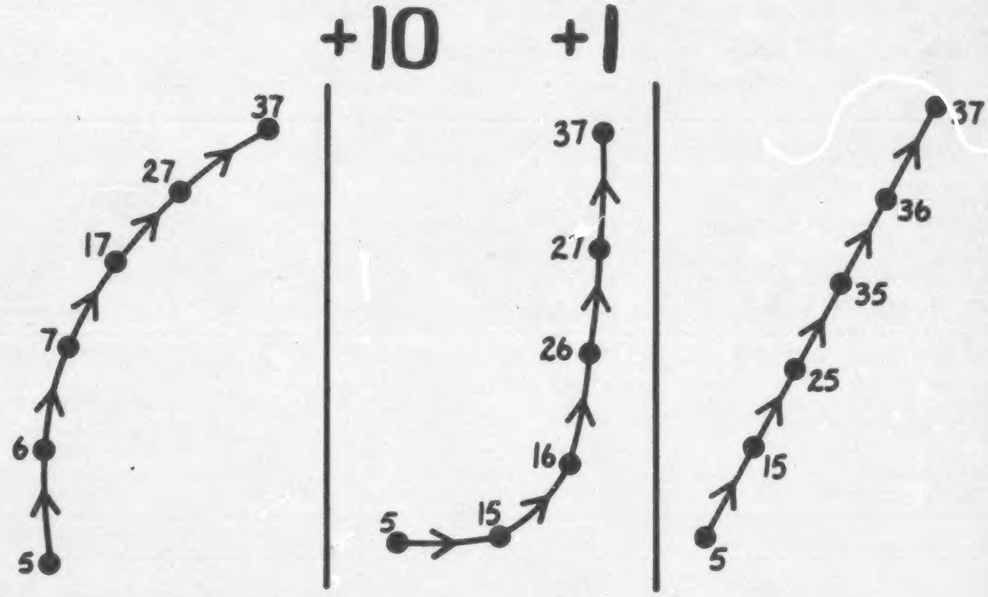
Erase the board before going on to Exercise 2.

Exercise 2

Write the following information on the board. Ask the students to build a red-blue road from 5 to 37 using as few arrows as possible. Remind them to indicate on their papers what each color arrow represents.



Walk around the room and look at the students' work. Invite several students with different shortest roads to put them on the board. These are three of the ten possible roads.


+10
+1
37
27
17
7
6
5
37
27
26
16
15
5
37
36
35
25
15
5

Ask the students to count the +10 and the +1 arrows in the roads on the board and on their papers. Conclude that a shortest road has three +10 arrows and two +1 arrows.

T: If we start at 0 instead of 5 and build a road with three +10 arrows and two +1 arrows (and no others), where would the road end?

S: At 32.

T: If we start at 10 . . . ?

S: At 42.

T: If we start at 12 . . . ?

S: At 44.

T: If we start at 100 . . . ?

S: At 132.

Erase the board before going on to Exercise 3.

Exercise 3

Write this information on the board.

+10 +1

T: What is the smallest number of arrows we would need to build a blue-red road from 8 to 49?

S: Five arrows; four +10 arrows and one +1 arrow.

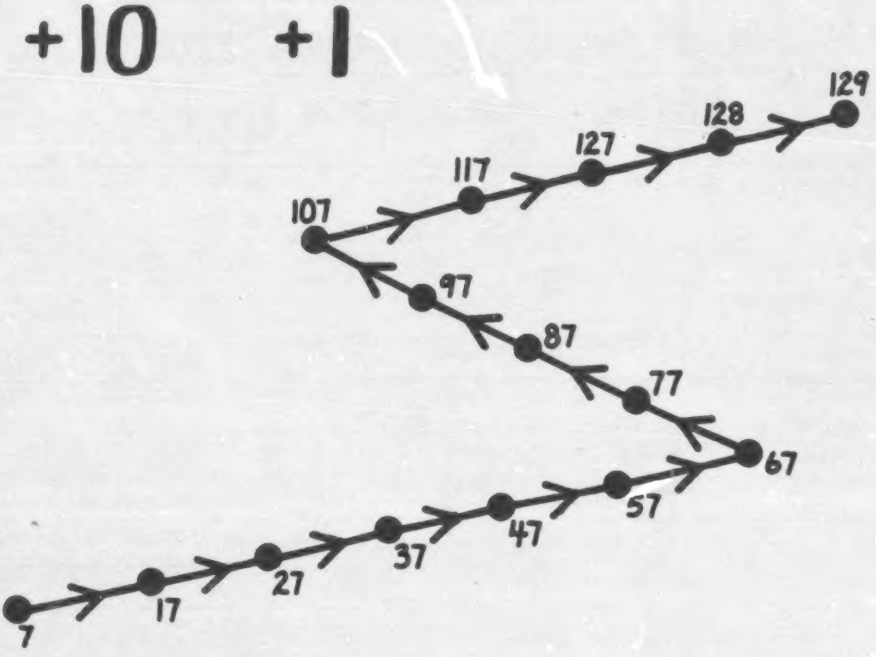
T: What is the smallest number of arrows we would need to build a blue-red road from 4 to 67?

S: Nine arrows; six +10 arrows and three +1 arrows.

T: What is the smallest number of arrows we would need to build a blue-red road from 9 to 21?

S: Three arrows, one +10 arrow and two +1 arrows.

If there is time remaining for the lesson, ask your students to build a blue-red road from 7 to 129 using as few arrows as possible. One possible shortest road is shown here.



N 15

N15 Subtraction on the Minicomputer

CAPSULE LESSON SUMMARY

Explore the effect of moving various checkers in a configuration on the Minicomputer — after a move, is the number on the Minicomputer more than, less than, or the same as before the move? Use the Minicomputer to decide how much money Tracy's mother will have left out of $100 if she buys a bicycle which costs $69. Also decide how much more money she needs to buy a pool table which costs $253 if she presently has $186.

MATERIALS

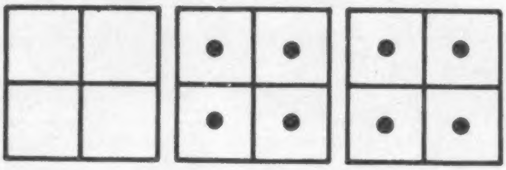
Teacher: Minicomputer set

Student: Paper; pencil; Minicomputer set

DESCRIPTION OF LESSON

Exercise 1: Transforming a Number

Put this configuration on the Minicomputer.



T: What number is this? Write it on your paper.

Look at many of the students' answers. Invite several students to explain to the class what calculations they did to get 165. Record the corresponding number stories on the board as they are suggested. Write "=165" to one side of the Minicomputer.

T: We know that the number on the Minicomputer is 165. Now let's compare this number to some other numbers.

Write these words on the board.

more
same
less

T: I am going to move, remove, or add some checkers. Each time tell me if the number on the Minicomputer is more than, the same as, or less than 165.

Move a checker from the 4-square to the 8-square. Point to each of the three words in turn as you say:

T: If you think this number is more than 165, raise your hand.
If you think this number is also 165, raise your hand.
If you think this number is less than 165, raise your hand.

Repeat the move very obviously if many students do not know that the number now on the Minicomputer is more than 165.

T: How much more is this number than the number we had before?
How do you know?

S: It is 4 more, because you moved the checker from the 4-square to the 8-square.

T: What number is on the Minicomputer now?

S: 169.

T: 165 + 4 = 169.

Erase the "165" and replace it with "169".

Continue this activity with the following or similar moves.

- Make a "40 = 20 + 20" play. (Same)
- Move a checker from the 1-square to the 4-square. (3 more)
- Move a checker from the 8-square to the 2-square. (6 less)
- Make an "8 + 2 = 10" play. (Same)
- Move a checker from the 20-square to the 40-square. (20 more)
- Move a checker from the 10-square to the 8-square. (2 less)
- Move a checker from the 20-square to the 1-square. (19 less)
- Make a "20 + 80 = 100" play. (Same)

Exercise 2: Subtraction Problem

Tell the following or a similar story about one of the students in your class.

T: Tracy's mother decides to buy a bicycle for Tracy. She puts $100 in her purse and goes shopping for a bicycle. What bills could she have in her purse if she has exactly $100 in cash?

Encourage the students to suggest many possibilities before you continue the lesson. A sample dialogue is given here.

S: She could have all $10 bills.

T: How many $10 bills . . . ?

S: Ten.

T: 10 x 10 = 100, so ten $10 bills is $100.

S: She could have two $50 bills.

T: 2 x 50 = 100, so two $50 bills is $100.

S: She could have eight $10 bills and twenty $1 bills.

T: 8 x 10 = 80 and 80 + 20 = 100, so eight $10 bills and twenty $1 bills is $100.

T: Tracy's mother finds a bicycle she likes for $69. Does Tracy's mother have enough money with her so that she can buy it?

S: Yes.

Write this inequality on the board.

$$69 < 100$$

T: 69 is less than 100. What calculation could we do to find out how much money Tracy's mother will have left after she buys the bicycle?

S: "100 - 69".

Write "100 - 69 = ☐ " on the board and point to it as you ask:

T: Is there another way we could say this?

S: "69 + the number in the box = 100".

If necessary, suggest this number story yourself. Write "69 + ☐ = 100" on the board.

$$100 - 69 = \square$$
$$69 + \square = 100$$

N15

T: How much money will Tracy's mother have left after she buys the bicycle?

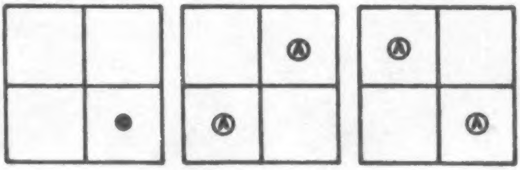
Encourage several students to predict how much money Tracy's mother will have left. Ask students who say there will be $31 left to explain how they calculated the answer.

T: How can we calculate "100 - 69" on the Minicomputer?

S: Put 100 on the Minicomputer and then take off 69.

S: Put 100 on the Minicomputer with regular checkers and then put $\widehat{69}$ on the Minicomputer with negative checkers.

If no one suggests calculating "100 + $\widehat{69}$", suggest this yourself. Ask students to put 100 and $\widehat{69}$ on the Minicomputer.



S: We need to make some backward plays.

Invite students to make plays on the Minicomputer until the standard configuration for 31 is obtained. Remind the students they they are trying to get pairs of checkers, one positive and one negative on the same square. Emphasize that such pairs can be removed from the Minicomputer because a number plus its opposite equals 0; for example, $80 + \widehat{80} = 0$. Conclude that
$100 - 69 = 31$.

T: Tracy's mother buys the bicycle and has $31 left over. How could the salesperson give her $31 in change?

Encourage the students to suggest several possibilities.

Erase the board and remove the checkers from the Minicomputer.

T: Another day Tracy's mother has $186 with her and she wants to buy a pool table which cost $253.

Does she have enough money?

S: No.

Write this inequality on the board.

$$253 > 186$$

T: 253 is more than 186. Tracy's mother does not have enough money to buy the pool table. How could we find out how much more money Tracy's mother needs?

S: We need to find a number so 186 + that number = 253.

Write "186 + ☐ = 253" on the board and point to it as you ask:

T: Is there another way we could say this?

S: 253 - 186 = the number in the box.

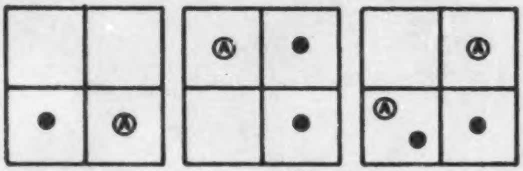
If necessary, suggest these number stories yourself. Write "253 - 186 = ☐" on the board.

$$186 + \square = 253$$
$$253 - 186 = \square$$

N 15

T: How much more money does Tracy's mother need?

Record the students' guesses on the board. Someone very likely will suggest calculating "253 + $\widehat{186}$" on the Minicomputer. Ask students to put 253 and $\widehat{186}$ on the Minicomputer.



Invite students to make plays on the Minicomputer until the standard configuration for 67 is obtained. Conclude that Tracy's mother needs $67 more and that 243 - 186 = 67. Write the vertical form of this number story on the board.

$$\begin{array}{r} 253 \\ -186 \\ \hline 67 \end{array}$$

Allow the students to determine which guess was the closest to 67.

Write these problems on the board and suggest that your students use their individual Minicomputers to solve them. The stars indicate levels of difficulty.

*	**	***	****
56 + $\widehat{10}$	75 + $\widehat{11}$	248 + $\widehat{4}$	153 + $\widehat{8}$
56 + $\widehat{4}$	75 + $\widehat{14}$	248 + $\widehat{20}$	153 + $\widehat{80}$
56 + $\widehat{40}$	75 + $\widehat{21}$	248 + $\widehat{100}$	153 + $\widehat{89}$

N16 The Functions 2x and 1/2x

CAPSULE LESSON SUMMARY

Relate the functions 2x and $\frac{1}{2}$x through the use of a simple arrow diagram: the arrow for one is the return arrow for the other. Use return arrows to label the dots in an arrow picture which has 2x and -1 arrows. Notice where the largest and the smallest numbers are in the picture.

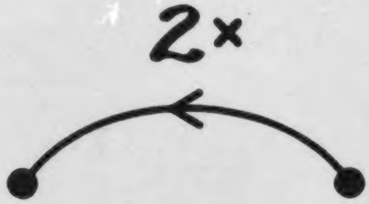
MATERIALS

Teacher: Colored chalk
Student: Paper; pencil; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Draw this arrow picture on the board and use it to do a mental arithmetic exercise.



T:	What is this red arrow for?
S:	2x.

Point to the starting dot of the red arrow.

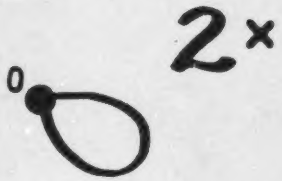
T:	If this dot were for 4, what would the ending dot be for?
S:	8.

Repeat this activity assigning other numbers to the starting dot. Occasionally ask students to tell which calculations they did to find the ending number. These are some possible choices; corresponding ending numbers are given in parentheses.

13	(26)	100	(200)	24	(48)
23	(46)	3	(6)	25	(50)
8	(16)	103	(206)	26	(52)
5	(10)	30	(60)	126	(252)
0	(0)	130	(260)	$\hat{1}$	($\hat{2}$)
9	(18)	134	(268)	$\widehat{15}$	($\widehat{30}$)

Comments on the Preceding Problems

- Since 2 x 0 = 0, the arrow picture on the board is not appropriate because there would be two dots for 0. Ask the students how to show 2 x 0 = 0 in an arrow picture. When a 2 x loop is suggested, draw it on the board.



- Take whispers for "2 x 134". Ask a student to write the corresponding addition problem on the board. Write the vertical and the horizontal forms of the multiplication number story on the board and tell the class that both number stories are read, "two times 134 is equal to 268."

$$\begin{array}{r}134\\+134\\\hline 268\end{array} \qquad 2\times 134 = 268 \qquad \begin{array}{r}134\\\times\ 2\\\hline 268\end{array}$$

Point to the ending dot.

T: If this number were 12, what would the starting dot be for?

S: 6.

T: How do you know?

Any of the following calculations are acceptable explanations.

- $6 + 6 = 12$
- $\frac{1}{2} \times 12 = 6$
- $2 \times 6 = 12$

Repeat this activity assigning other numbers to the ending dot. Frequently ask students to tell which calculations they did to find the starting number. These are some possible choices; corresponding starting numbers are shown in parentheses.

18 (9)	600 (300)	20 (10)
100 (50)	6,000 (3,000)	28 (14)

T: What number is $\frac{1}{2} \times 20$?

S: 10.

Write the number story on the board.

$$\frac{1}{2} \times 20 = 10$$

T: What number is $\frac{1}{2} \times 8$?

S: 4.

N 16

Write "$\frac{1}{2} \times 8 = 4$" below the first number story.

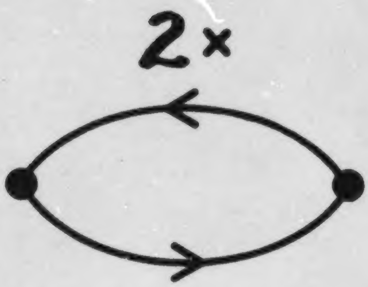
$$\frac{1}{2} \times 20 = 10$$
$$\frac{1}{2} \times 8 = 4$$

T: $\frac{1}{2} \times 20 = 10$ and $\frac{1}{2} \times 8 = 4$, so what number is $\frac{1}{2} \times 28$?

S: 14.

$$\begin{array}{c} \frac{1}{2} \times 20 = 10 \\ \frac{1}{2} \times 8 = 4 \\ \hline \frac{1}{2} \times 28 = 14 \end{array}$$

Draw the return arrow in blue.



T: The red arrow is for 2x. What could the blue arrow be for?

S: $\frac{1}{2}$x.

Write "$\frac{1}{2}$x" in blue near the blue arrow.

Point to the starting dot of the blue arrow.

T: If this dot were for 60, what number would the other dot be for?

S: 30.

Repeat this activity assigning other starting numbers to the blue arrow. Occasionally ask students to tell which calculations they did to find the ending number of the blue arrow. These are possible choices; corresponding ending numbers of the blue arrow are shown in parentheses.

60	(30)	114	(57)	5	$(2\frac{1}{2})$
64	(32)	40	(20)	8	(4)
200	(100)	50	(25)	9	$(4\frac{1}{2})$
210	(105)	58	(29)	3	$(1\frac{1}{2})$
100	(50)	4	(2)	10	(5)
106	(53)	6	(3)	11	$(5\frac{1}{2})$

Comments on the Preceding Problems

- Take whispers for "$\frac{1}{2} \times 114$". Ask students to tell which calculations they did to find the answer and record the corresponding number stories on the board. Perhaps these calculations will be suggested.

$$\begin{array}{l} \frac{1}{2} \times 100 = 50 \\ \frac{1}{2} \times 14 = 7 \\ \hline \frac{1}{2} \times 114 = 57 \end{array} \qquad \begin{array}{r} 50 \\ +\ 7 \\ \hline 57 \end{array}$$

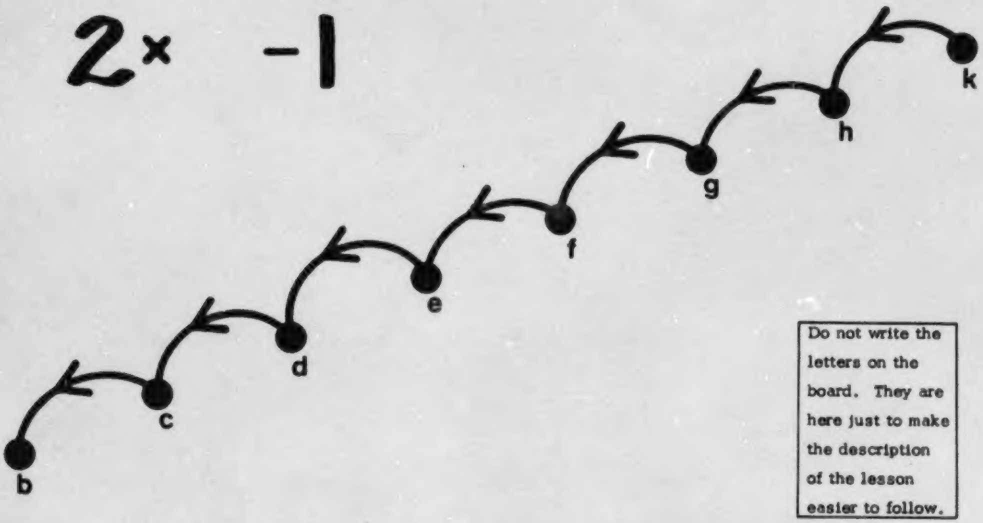
- Take whispers for "$\frac{1}{2} \times 58$". Ask students to tell which calculations they did to find the answer and record the corresponding number stories on the board.

- For at least one of the calculations which result in non-integers, ask a student to write the corresponding number story on the board. Give help if it is needed.

$$\frac{1}{2} \times 11 = 5\frac{1}{2}$$

Exercise 2

Distribute unlined paper and colored pencils. Draw this arrow picture on the board and ask the students to copy it very carefully onto their papers.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Quickly check the students' drawings to make sure the color, the order, and the direction of the arrows are correct.

T: Where is the largest number in this arrow picture? How do you know?

Let several students answer. Commend any student who gives a good explanation but do not indicate where the largest number is in the arrow picture.

T: Where is the smallest number in this arrow picture? How do you know?

Let several students answer, but do not indicate yourself where the smallest number is in the arrow picture.

NOTE: If all the numbers in this arrow picture are more than 1, the largest number is at c and the smallest is at k. This is for your information only.

Label b "115".

T: If this number (at b) is 115, what are these other numbers? Label all the dots in your arrow picture.

Allow the students to work independently for a few minutes. Look at many of their papers since several students might have difficulty labeling the dots. Label b and c collectively. A possible dialogue is given here.

Perhaps several students have incorrectly labeled c "114". Label c "114" in the arrow picture on the board.

T: Many of you have labeled this dot "114".

Trace the blue arrow from 114 to 115.

T: This blue arrow is for -1. Is 114 - 1 = 115? (No) This number can't be 114. What should it be? How do you know?

S: 116, because 115 + 1 = 116 (or 116 - 1 = 115).

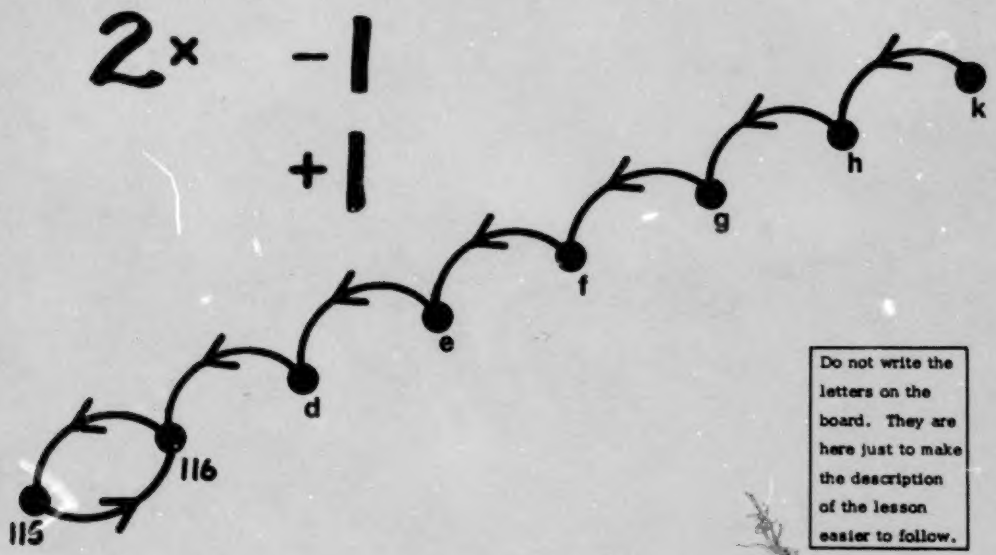
Erase "114" and replace it with "116". Trace an arrow from 115 to 116.

T: If we draw an arrow from 115 to 116, what could this arrow be for? (+1)

The blue arrow is for - 1, so the return arrow is for + 1.

N 16

Draw the return arrow in orange.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Trace the arrow from d to 116.

T: This red arrow goes from this dot (d) to 116. But we need to go from 116 to this dot (d). We need to draw a return arrow.

Draw an arrow from 116 to d in green.

T: What could the green arrow be for?

S: $\frac{1}{2}$x.

Write "$\frac{1}{2}$x" in green near the arrow picture.

T: We need to calculate "$\frac{1}{2} \times 116$". How do you suggest we do this calculation?

S: $\frac{1}{2} \times 100 = 50$ and $\frac{1}{2} \times 16 = 8$, so $\frac{1}{2} \times 116 = 58$.

Record these number stories on the board.

$$\begin{array}{r}
\frac{1}{2} \times 100 = 50 \\
\frac{1}{2} \times 16 = 8 \\
\hline
\frac{1}{2} \times 116 = 58
\end{array}$$

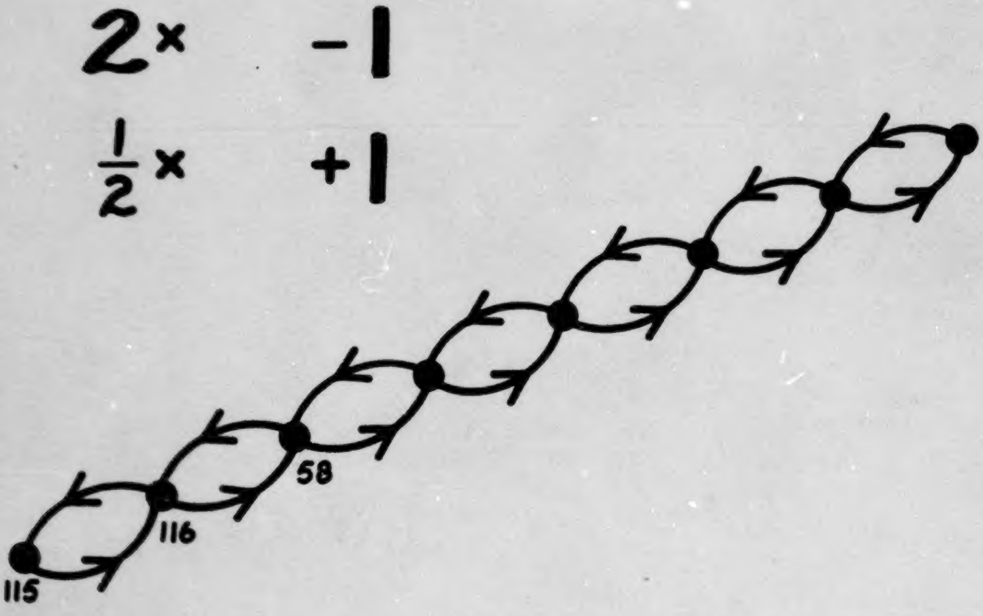
Label d "58".

T: Where can we draw another $\frac{1}{2}$x arrow?

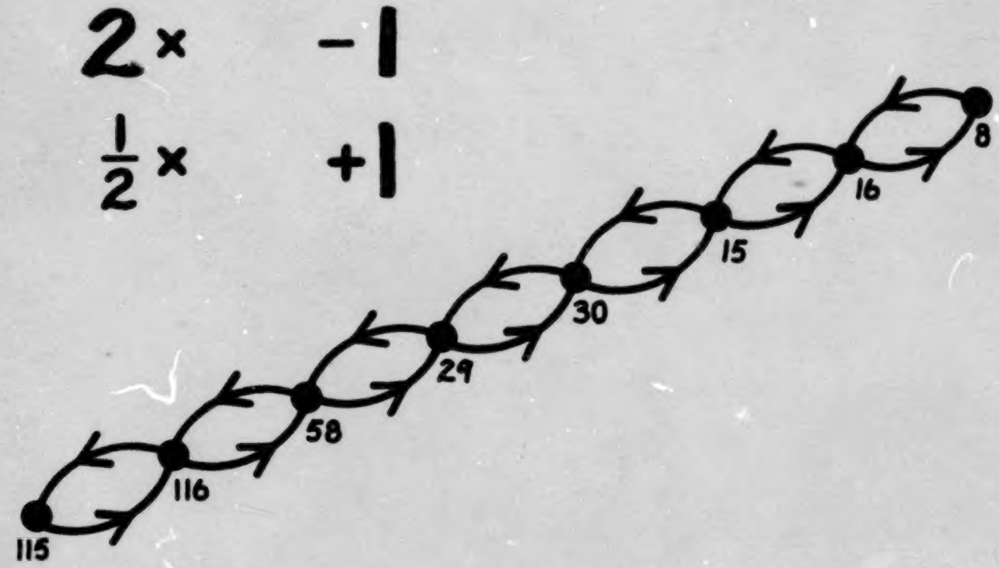
Continue until all the $\frac{1}{2}$x arrows are drawn.

T: Where can we draw another +1 arrow?

Continue until all the +1 arrows are drawn.



Allow a few minutes for students to label dots in their arrow pictures ; help those who are having difficulty. When a few students have labeled all the dots, invite them to label the dots on the board.



Notice where the largest and the smallest numbers are in the picture.

NOTE : If you think the calculations required to label the dots in this arrow picture are too difficult for your class, you could start with 51 at b instead of 115.

N17 Road Map of Nevada

CAPSULE LESSON SUMMARY

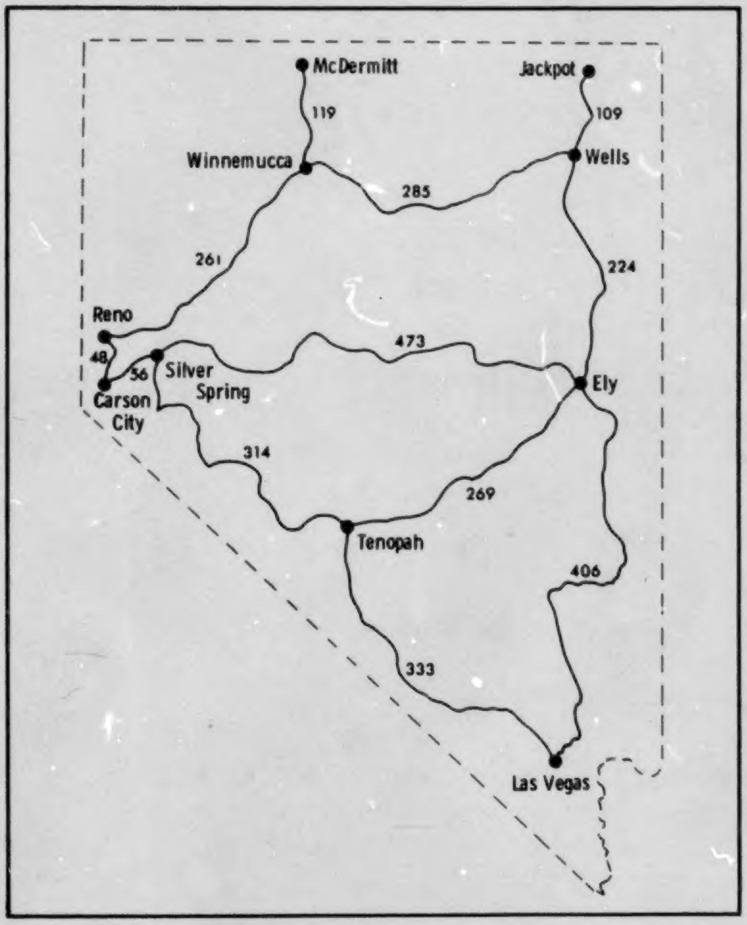
Find shortest routes between cities in Nevada. Find which of two cities is closer to a third. (Optional) Plan a trip starting at a particular city which is approximately 800 km long.

MATERIALS

Teacher: World of Numbers Poster #1 for UPG III; tape
Student: Worksheet N17; paper; pencil

DESCRIPTION OF LESSON

Distribute copies of Worksheet N17 and display World of Numbers Poster #1.


McDermitt
Jackpot
119
109
Winnemucca
Wells
285
261
224
Reno
473
48
56
Silver Spring
Ely
Carson City
314
269
Tenopah
406
333
Las Vegas

N17

NOTE: If you and your students are in Nevada, adjust the following dialogue so that it is appropriate.

T: This is a map of one of the states of the United States of America. What is the name of this state? Do you know any of these cities?

You may need to tell your class that this state is Nevada. Encourage the students to share what they know about Nevada and the cities indicated on this map. Tell the students that the numbers show how far it is in kilometers from one city to another. Trace the road from Las Vegas to Tenopah.

T: How far is it from Las Vegas to Tenopah according to this map? (333 kilometers)
How far is Reno from Winnemucca? (261 kilometers)
How many kilometers long is the shortest route from Las Vegas to Ely? (406 kilometers)
How far is it from Carson City to Silver Spring? (56 kilometers)

Problem 1: Wells to McDermitt

Abbreviate a statement of the problem on the board.

Wells to McDermitt?

T: What is the shortest route from Wells to McDermitt?

S: Go from Wells to Winnemucca and then on to McDermitt.

Ask a student to trace this route on the poster; then record it on the board.

Wells to McDermitt?

Wells to Winnemucca to McDermitt is ____ km

T: How long is this route from Wells to McDermitt? Write your answer on a piece of paper.

Look at many of the students' answers and help students who are having difficulty. Ask a student to answer aloud and complete the sentence on the board.

Wells to McDermitt?
Wells to Winnemucca to McDermitt is 404 km

T: How did you calculate the distance from Wells to McDermitt?

S: I added "285 + 119".

Write this addition problem on the board.

$$\begin{array}{r} 285 \\ +119 \\ \hline \end{array}$$

Ask for a volunteer to solve this problem at the board and to explain each step.

T: Is this the shortest route from Wells to McDermitt?
How do you know?

Let students give explanations, but do not expect well-formed answers involving calculations.

Erase the board before going on to Problem 2.

Problem 2: Silver Spring to Wells

Abbreviate a statement of the problem on the board.

Silver Spring to Wells ?

T: What is the shortest route from Silver Spring to Wells?

N 17

Some students might believe that the shortest route from Silver Spring to Wells is through Ely. Ask a student to trace this route on the poster; then record it on the board.

Silver Spring to Wells ?
Silver Spring to Ely to Wells is ___ km

T: How long is this route from Silver Spring to Wells? Write your answer on a piece of paper.

Look at many of the students' papers before asking a student to answer aloud. Complete the sentence on the board.

Silver Spring to Wells ?
Silver Spring to Ely to Wells is 697 km

T: How did you calculate this distance?

S: I added "473 + 224".

Write this addition problem on the board and ask a student to solve it.

$$\begin{array}{r} 473 \\ +\,224 \\ \hline \end{array}$$

T: Are you sure this is the shortest route from Silver Spring to Wells?

If necessary, tell your class that there is a shorter route from Silver Spring to Wells on this map and encourage them to find it. When a student suggests that the route through Carson City, Reno, and Winnemucca is shorter, ask a student to trace that route. Then record it on the board.

Silver Spring to Wells ?

Silver Spring to Ely to Wells is 697 km

Silver Spring to Carson City to Reno
to Winnemucca to Wells is ___ km

T: How long is this route from Silver Spring to Wells? Write your answer on your paper.

Look at many of the students' papers and help students who are having difficulty.

T: How did you calculate the length of this route from Silver Spring to Wells?

S: I added "56 + 48 + 261 + 285".

Write this addition problem on the board and emphasize that all the hundreds' digits should be in the same column, all the tens' digits in another column, and all the ones' digits in a third column. Ask for a volunteer to solve this problem at the board and to explain each step.

$$\begin{array}{r} {}^{2\,2}\\ 56\\ 48\\ 261\\ +285\\ \hline 650 \end{array}$$

Complete the sentence on the board and conclude that the shortest distance from Silver Spring to Wells is 650 km.

Silver Spring to Wells ?
Silver Spring to Ely to Wells is 697 km
Silver Spring to Carson City to Reno
to Winnemucca to Wells is 650 km

T: How much longer is the route from Silver Spring to Wells which goes through Ely than the route which goes through Reno?

Encourage the students who answer to explain their calculations. If necessary, suggest one or both of these calculations. Answers are in the boxes.

$$\begin{array}{r} 650 \\ +\boxed{47} \\ \hline 697 \end{array} \qquad \begin{array}{r} 697 \\ -650 \\ \hline \boxed{47} \end{array}$$

Conclude that the route through Ely is 47 km longer than the route which goes through Reno.

Erase the board before going on to Problem 3.

Problem 3: Las Vegas to Wells and Las Vegas to Silver Spring

T: Which city is closer to Las Vegas — Wells or Silver Spring?

Abbreviate your question on the board as follows.

Las Vegas to Wells ?

Las Vegas to Silver Spring ?

The students are likely to tell you that Silver Spring is closer to Las Vegas than Wells, but ask them to calculate the distances so they are sure. Allow a few minutes for independent work; then ask the students to describe the shortest routes from Las Vegas to Silver Spring and to Wells, and to tell you the length of each of these routes. Record this information on the board.

Las Vegas to Wells ?

Las Vegas to Ely to Wells is 630 km

Las Vegas to Silver Spring ?

Las Vegas to Tenopah to Silver Spring is 647 km

T: Which distance is shorter?

S: 630 km is less than 647 km.

$$630 < 647$$

T: Is Silver Spring or Wells closer to Las Vegas?

S: Wells.

T: How much farther is it from Las Vegas to Silver Spring than it is from Las Vegas to Wells?

Encourage the students who answer to explain their calculations. If necessary, suggest one or both of these calculations. Answers are in the boxes.

$$\begin{array}{r} 630 \\ +\boxed{17} \\ \hline 647 \end{array} \qquad \begin{array}{r} 647 \\ -630 \\ \hline \boxed{17} \end{array}$$

Conclude that Las Vegas is 17 km farther from Silver Spring than from Wells.

Erase the board before going on to Problem 4.

Problem 4 (Optional): Plan a Trip Starting at Ely

T: I want everyone to plan a trip which starts at Ely and which is approximately 800 km long. Try to make the length of your trip as close to 800 km as possible. You may end your trip in any of these cities.

After allowing the students to work independently for a few minutes, record one of their trips on the board. Try to choose a trip which is at least 900 km long for the collective discussion. An example is given here.

T: Gary has a trip from Ely to Las Vegas to Tenopah and back to Ely.

Record the trip on the board.

Ely to Las Vegas to Tenopah to Ely is ____ km.

T: Did anyone else plan this trip?
How long is this trip.

S: 1,008 km.

Ely to Las Vegas to Tenopah to Ely is 1,008 km.

T: Is this trip more or less than 800 km long?

S: More.

T: How much longer than 800 km is it?

S: 208 km.

T: Is there another trip we could take starting at Ely which is closer to 800 km long?

Perhaps a student will suggest this trip.

Ely to Silver Spring to Tenopah is 787 km.

T: Is this trip more or less than 800 km long?

S: Less.

T: How much less . . . ?

S: 13 km.

If necessary suggest one or both of these calculations.

$$\begin{array}{r} 787 \\ + \boxed{13} \\ \hline 800 \end{array} \qquad \begin{array}{r} 800 \\ - 787 \\ \hline \boxed{13} \end{array}$$

Conclude that 787 is closer to 800 than 1,008 is to 800.

Continue encouraging the students to find a trip closer to 800 km long. Another trip with a length close to 800 km is from Ely to Las Vegas and back to Ely. If none of your students discover this trip, you could give them a few clues; for example,

T: I am thinking of a trip which has a length even closer to 800 km than 787 km. This trip begins and ends in Ely and visits only one other city. Can you discover which trip I am thinking of?

N 17

Encourage the students to find this trip on their own. When a student suggests the trip from Ely to Las Vegas and back to Ely, record this trip and its length on the board.

Ely to Las Vegas to Ely is 812 km.

T: How much more is 812 than 800?

S: 12.

T: Which number is closer to 800; 787 or 812?

S: 812.

T: Why?

S: 812 is 12 more than 800 and 787 is 13 less than 800.

Your students might decide that the trip from Ely to Las Vegas and back to Ely is the trip which starts at Ely with the length closest to 800 km. There are other trips which have a length closer to 800 km than this one, but most of them involve traveling between Carson City and Silver Spring several times. Commend any student who finds a trip which is even closer to 800 km than 812 km, but do not insist that your students find such a trip.

N18 +4 Arrow Roads #1

CAPSULE LESSON SUMMARY

Use the hand-calculator to find multiples of 4. In a string picture, locate some numbers which are multiples of 4 and some which are not. Display multiples of 4 in an arrow picture by drawing part of a +4 arrow road with the number 0 on it, and by indicating that the road continues in both directions. Compare the +4 arrow road with the number 1 on it to the + 4 arrow road with the number 0 on it. Draw a + 4 arrow road with numbers on it which are on neither of those two roads.

MATERIALS

Teacher: Colored chalk
Student: Hand-calculator[*]; paper; colored pencils; pencil

DESCRIPTION OF LESSON

Ask two or three students to count by fours from 0 to 20. Then ask several students to count by fours from 0 to 40.

Distribute as many hand-calculators as are available. It is desirable that each student have a hand-calculator.

T: Try to find a way to make your hand-calculator count by fours.

Allow a few minutes for experimentation; then encourage several students to share their methods with the class. Someone might suggest pressing $\boxed{+}\boxed{4}\boxed{=}\boxed{=}\boxed{=}$ . . . ; if necessary, suggest this yourself. Ask all the students to try that method on their hand-calculators.

T: What kind of numbers do we get when we count by fours from 0?

S: Even numbers.

---

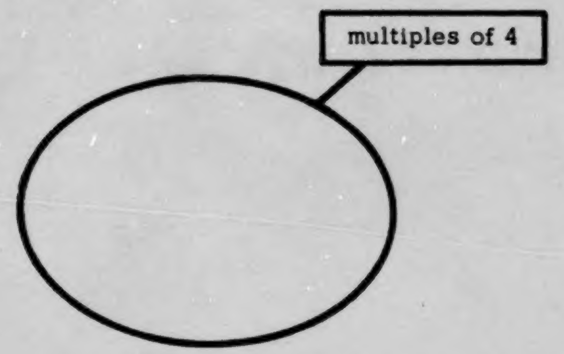
[*] See the Note on the Use of Hand-Calculators in the General Introduction.

N18

S: Multiples of 4.

T: If we start at 0 and press $\boxed{+}\boxed{4}\boxed{=}\boxed{=}\boxed{=}$ . . . , all the numbers which will appear on the display are multiples of four.

Draw this string picture on the board.



T: Give me some numbers which belong inside this string.

Let students suggest many multiples of 4 and record them in the string picture. If the students are not suggesting a wide range of numbers, ask for a multiple of 4 which is more (or less) than a particular number such as 100; 0; or 1,000. Explain why a number is (or is not) a multiple of 4 whenever the explanation would be fairly simple and concise. A sample dialogue is given here.

S: 100 belongs inside the string.

T: That's correct; 100 is a multiple of 4.

Record "100" inside the string.

S: 104.

T: 104 is a multiple of 4 because 100 is a multiple of 4 and
100 + 4 = 104.

Record "104" inside the string.

S: 90.

T: We know 100 is a multiple of 4.
100 - 4 = . . . ? (96)
96 - 4 = . . . ? (92)
92 - 4 = . . . ? (88)
We skipped 90, so 90 is not a multiple of 4.

Record "90" outside the string.

S: 412.

T: 400 is a multiple of 4 and 12 is a multiple of 4, so 412 is a multiple of 4.

Record "412" inside the string.

NOTE: Since 100 is a multiple of 4, the last two digits of a whole number indicate whether or not it is a multiple of 4; for example, 1,372 is a multiple of 4 because 72 is a multiple of 4, and 494 is not a multiple of 4 because 94 is not a multiple of 4. This is for your information only.

Continue this activity until many students have participated.

T: What are some numbers which are not multiples of 4?
How do you know?

As numbers are suggested, record them in their appropriate place in the string picture. Students might explain that a number is not a multiple of 4 because it is 1, 2, or 3 more or less than a particular number appearing on the hand-calculator. Encourage the students to suggest small and large numbers which are not multiples of 4.

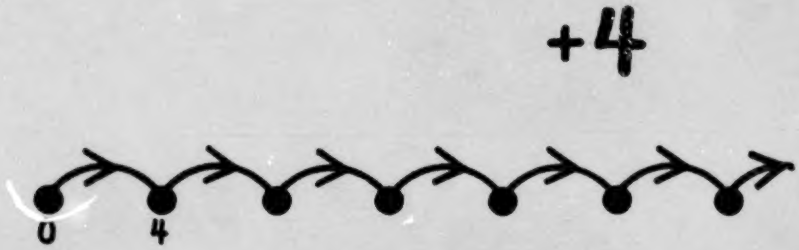
N18

T: How could we include all the multiples of 4 in an arrow picture?

S: Draw a +4 arrow road with 0 (4) on it.

Draw a +4 arrow picture on the board. After drawing several arrows, ask if you've drawn enough +4 arrows. Students should tell you that the road goes on and on.

T: I will show that this arrow road continues by drawing an arrow without an ending dot.

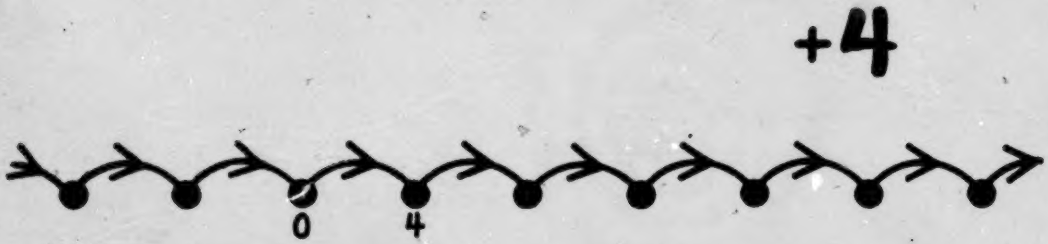


T: Are there any numbers smaller than 0 which are multiples of 4?

S: Yes ; $\hat{4}$, $\hat{8}$, $\widehat{12}$, $\widehat{16}$, ...

Extend the arrow road a few dots to the left.

T: I will show that there are more multiples of 4 which are less than 0 by drawing an arrow without a starting dot.



Invite a student to label the dots. Remind the students that even though you have only drawn a few dots on the board, all the numbers which are multiples of 4 are on this arrow road.

T: Are there any whole numbers which are not on this arrow road? (Yes) What are some of those numbers? How do you know?

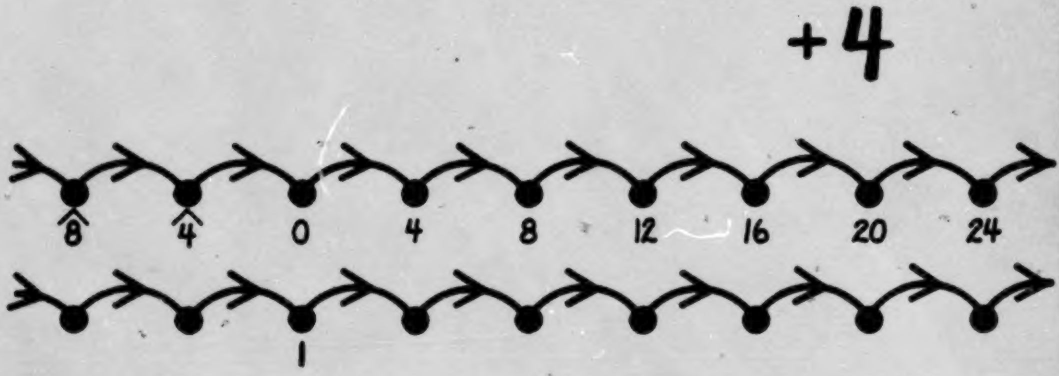
Let students suggest many numbers which are not multiples of 4. If a student indicates that a certain number is skipped on the arrow road on the board, you should indicate where it is skipped. For example, 18 is skipped because 18 is between 16 and 20.

T: Is 1 on this arrow road?

S: No.

T: 1 is on another +4 arrow road.

Draw a second arrow road directly below the first one. Carefully align the dots of the two arrow roads and label the dot for 1 directly below the dot for 0.



Remind the students that this arrow road also extends in both directions.

N 18

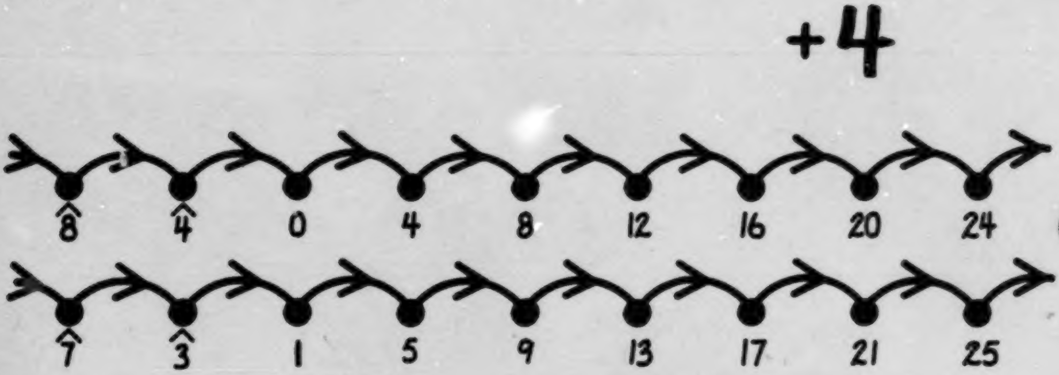
T: What are some of the numbers which are on the second arrow road? How do you know?

S: 9 is on the second arrow road because 1 + 4 = 5 and 5 + 4 = 9.

S: 13 is on the second arrow road because 12 is a multiple of 4 and 12 + 1 = 13.

S: 20 is on the first arrow road, so 20 + 1 is on the second arrow road.

Invite a student to label the dots in the second arrow road.



T: Do you notice any patterns in this arrow picture?

There are several patterns which your students might observe; for example,

- all the numbers in the first arrow road are even;
- all the numbers in the second arrow road are odd;
- each number in the second arrow road is 1 more than the number directly above it;
- the last digits of the whole numbers in each road form a repeating sequence. In the first road the repeating part of the sequence is: 0, 4, 8, 2, 6. In the second road the repeating part of the sequence is: 1, 5, 9, 3, 7.

T: Would it be possible for us to build another +4 arrow road which would have numbers from neither of these arrow roads on it? (Yes) Which numbers could be on this new arrow road? How do you know?

A possible dialogue is given here.

S: 2 could be on the new +4 arrow road because it is skipped by both of the other arrow roads.

T: The first arrow road jumps from 0 to 4 and the second arrow road jumps from 1 to 5, so 2 isn't on either of these arrow roads.

Perhaps a student incorrectly will assume that a number not shown in the picture is not on either road; for example,

S: 36 could be on the new +4 arrow road because 36 isn't on the board.

T: 36 isn't on the board, but are you sure it isn't on one of these arrow roads? Remember that these arrow roads extend in both directions and I have only drawn a small part of them on the board.

S: 36 is a multiple of 4, so it is on the first arrow road.

Continue this activity until several numbers which are not on either of the two arrow roads on the board have been suggested. Any number which is 1 or 2 less than a multiple of 4 would not be on either of these arrow roads. Ask the students to choose one of those numbers and to draw (on their papers) a +4 arrow road with that number on it. Students who finish quickly should be encouraged to draw another + 4 arrow road, different from the two roads on the board and from the road they just drew.

N 19

N19 3 x and 1/3 x on the Minicomputer

CAPSULE LESSON SUMMARY

Calculate "3 x 175" and "$\frac{1}{3}$ x 378" on the Minicomputer. Do similar calculations as an individual activity.

MATERIALS

Teacher : Minicomputer set

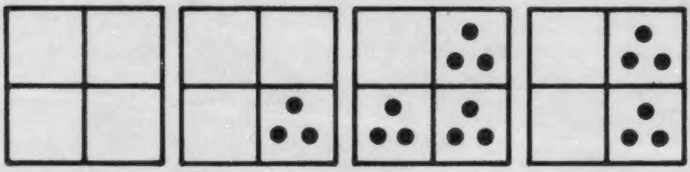
Student : Minicomputer set ; paper ; pencil

DESCRIPTION OF LESSON

Begin the lesson with some mental arithmetic activities using the following or similar calculations which are appropriate for the abilities of your students.

3 x 10	(30)	3 x 20	(60)	3 x 25	(75)
$\frac{1}{3}$ x 30	(10)	$\frac{1}{3}$ x 60	(20)	$\frac{1}{3}$ x 75	(25)
3 x 11	(33)	3 x 30	(90)	3 x 15	(45)
$\frac{1}{3}$ x 33	(11)	$\frac{1}{3}$ x 90	(30)	$\frac{1}{3}$ x 45	(15)
3 x 13	(39)	3 x 21	(63)	3 x 16	(48)
$\frac{1}{3}$ x 39	(13)	$\frac{1}{3}$ x 63	(21)	$\frac{1}{3}$ x 48	(16)

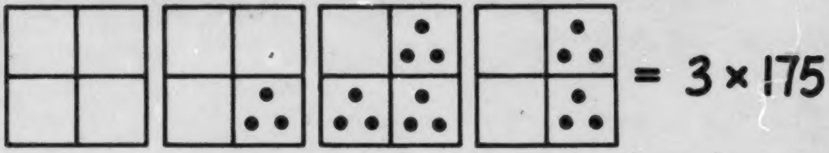
Put this configuration on the Minicomputer.



T : What calculation are we doing on the Minicomputer?

S : "3 x 175". You put 175 on the Minicomputer three times.

Write "= 3 x 175" to one side of the Minicomputer.



T: What calculation are we doing on the hundreds' board?

S: "3 x 100".

T: What number is 3 x 100?

S: 300.

Write "3 x 100 = 300" on the board.

T: What calculation are we doing on the tens' board?

S: "3 x 70".

T: What number is 3 x 70?

S: 210.

Write "3 x 70 = 210" on the board below "3 x 100 = 300".

T: What calculation are we doing on the ones' board?

S: "3 x 5".

T: What number is 3 x 5?

S: 15.

N 19

Write "3 × 5 = 15" on the board.

$$\begin{array}{r}3 \times 100 = 300\\ 3 \times 70 = 210\\ 3 \times 5 = 15\end{array}$$

T: How could we calculate "3 × 175"?

S: We could do the addition problem "175 + 175 + 175".

Record this problem on the board, but do not calculate the answer.

$$\begin{array}{r}175\\ 175\\ +\,175\\ \hline\end{array}$$

S: We could make plays on the Minicomputer.

S: We could add "300 + 210 + 15".

T: Why could we calculate "3 × 175" by adding 300, 210, and 15?

S: Because 3 × 100 = 300, 3 × 70 = 210, and 3 × 5 = 15.

If no one suggests calculating "300 + 210 + 15", suggest this yourself and write the addition problem on the board.

$$\begin{array}{r}175\\ 175\\ +\,175\\ \hline\end{array} \qquad \begin{array}{r}300\\ 210\\ +\,15\\ \hline\end{array}$$

Also record the problems corresponding to other methods of calculating "3 x 175" which your students suggest. Let your class choose a method to calculate "3 x 175" and then check the answer by calculating "3 x 175" again using a second method. For example, your class might choose to calculate "3 x 175" by solving this addition problem . . .

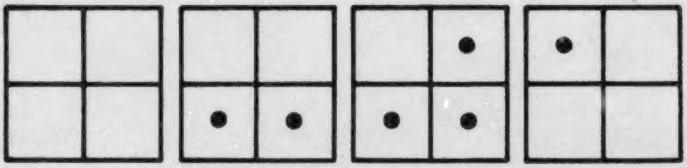
$$\begin{array}{r} 175 \\ 175 \\ +\ 175 \\ \hline \end{array}$$

and then check the calculation by solving another addition problem.

$$\begin{array}{r} 300 \\ 210 \\ +\ 15 \\ \hline \end{array}$$

If some students insist on making plays on the Minicomputer, call on one of them to do this while the rest of the class is solving the second addition problem. Conclude that 3 x 175 = 525. Erase the board and remove the checkers from the Minicomputer.

Put 378 on the Minicomputer.



T: What number is this?

S: 378.

Write "$\frac{1}{3}$ x 378 = ?" on the board.

T: Is $\frac{1}{3}$ × 378 more than 50?

S: Yes, because 3 × 50 = 150.

T: Is $\frac{1}{3}$ × 378 more than 100?

S: Yes, because 3 × 100 = 300 and 378 is more than 300.

T: Is $\frac{1}{3}$ × 378 more than 200?

S: No, because 3 × 200 = 600 and 600 is more than 378.

T: We know that $\frac{1}{3}$ × 378 is between 100 and 200.

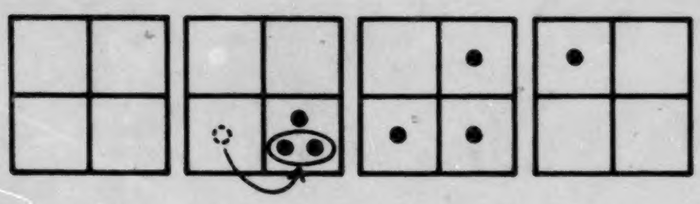
Allow the students to guess the answer. Record on the board only guesses which are between 100 and 200.

T: How can we calculate "$\frac{1}{3}$ × 378"?

S: We can make backward plays until all the checkers are in groups of three checkers.

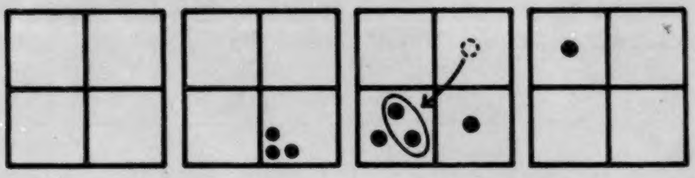
T: Who can make a play which will result in three checkers on the same square?

There are several plays which can be made which will result in three checkers on the same square of the Minicomputer. A possible sequence of plays and dialogue are given here.



S: 200 = 100 + 100.

T: Now we have a group of three checkers on the 100-square. I'll push these three checkers together so we will remember not to break up this group of three checkers. Who can make another play which will put three checkers on the same square?

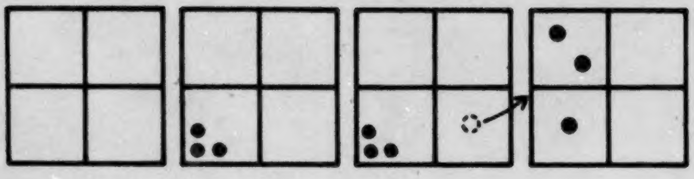


S: 40 = 20 + 20.

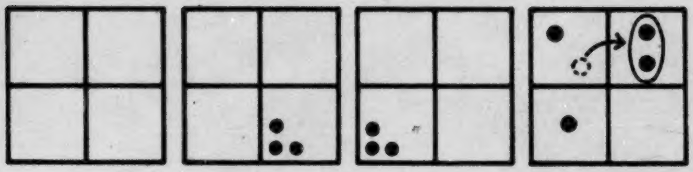
T: We have another group of three checkers on the 20-square. I'll push them together because we don't want to make any more plays with these checkers.

It is no longer possible to create a group of three checkers on the same square with only one standard move.

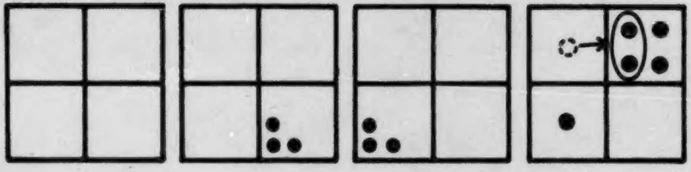
T: There are only two checkers left which aren't in groups of three checkers on the same square. Who can make a play which will help us form another group of three checkers?



S: 10 = 8 + 2.

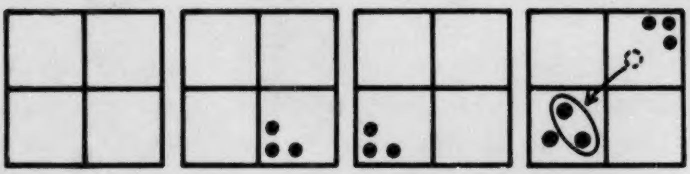


S: 8 = 4 + 4.



S: 8 = 4 + 4.

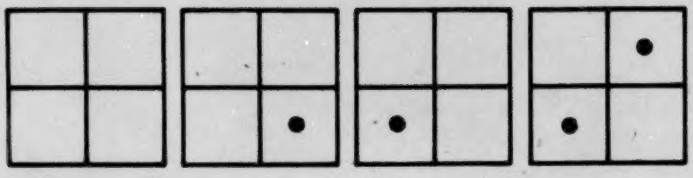
T: We now have a group of three checkers on the 4-square. I'll push this group of three checkers together. Who can make a play which will give us another group of three checkers?



S: 4 = 2 + 2.

T: Now all the checkers are in groups of three. Who can remove some checkers so that one-third of this number will be on the Minicomputer?

The volunteer should remove two checkers from each group of three checkers on the Minicomputer. If necessary, make plays until the standard configuration for 126 is obtained.



Complete the number story on the board.

$$\frac{1}{3} \times 378 = 126$$

Let the students determine which guess was closest to 126.

Record some multiplication problems on the board and ask the students to do the calculations using whichever methods they prefer. Encourage the students to use their individual Minicomputers. Problems of varying levels of difficulty are suggested below.

*	**	***
2 x 74	3 x 164	4 x 99
3 x 36	4 x 256	3 x 468
4 x 27	3 x 93	5 x 582
3 x 153	4 x 187	4 x 484
$\frac{1}{3} \times 96$	$\frac{1}{3} \times 156$	$\frac{1}{3} \times 528$

N 20

N20 + 4 Arrow Roads #2

CAPSULE LESSON SUMMARY

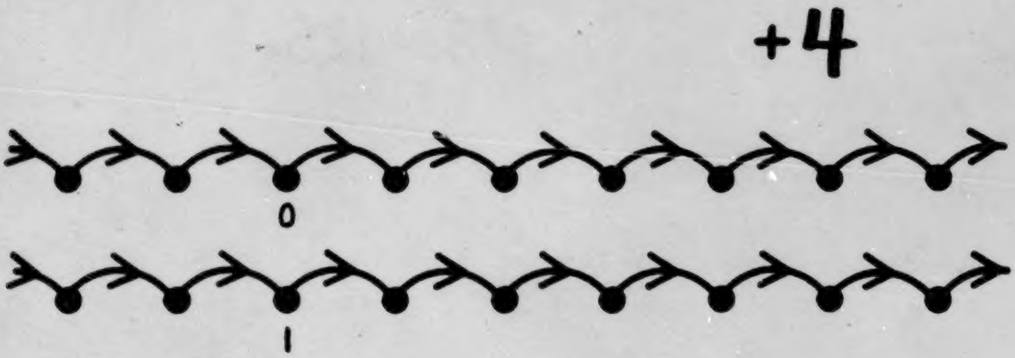
Draw a + 4 arrow road with the number 0 on it and one with the number 1 on it. Notice that 2 is not on either road and build a third road with the number 2 on it. Notice that 3 is on none of those roads and build a fourth road with the number 3 on it. Ask if there are any (whole) numbers which are not on any of the roads.

MATERIALS

Teacher: Colored chalk
Student: Paper; pencil; colored pencils

DESCRIPTION OF LESSON

Draw this arrow picture on the upper part of the board.



T: What do you remember about these + 4 arrow roads from our discussion last week?

Encourage the students to comment. They should recall at least the following information:

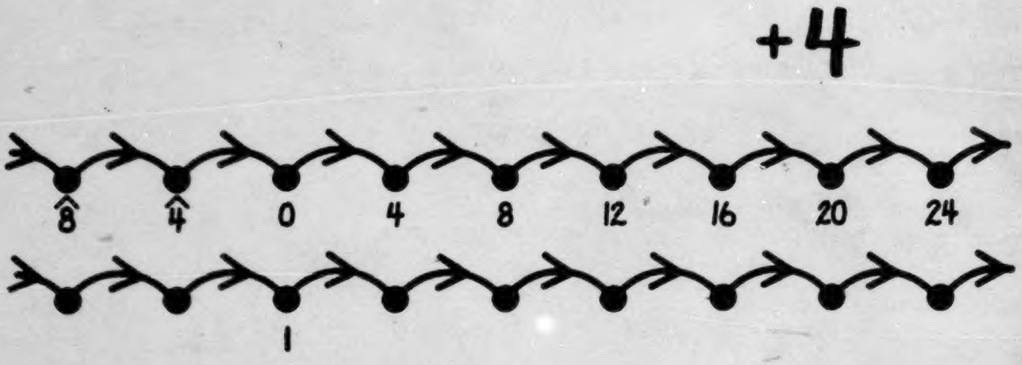
- the unfinished arrows indicate that the arrow roads continue forever in both directions;
- the numbers in the first arrow road are the multiples of 4;

- all the numbers in the second arrow road are not multiples of 4;
- all the numbers in the first arrow road are even;
- all the numbers in the second arrow road are odd.

Ask the students to suggest many numbers which are on the first arrow road. Encourage the students to suggest a variety of numbers including positive and negative, large and small multiples of 4.

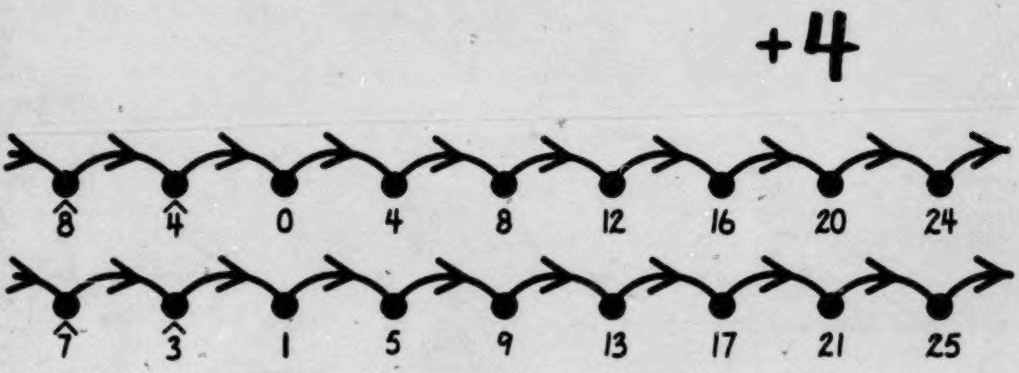
NOTE: Since 100 is a multiple of 4, the last two digits of a whole number indicate whether or not it is a multiple of 4; for example, 956 is a multiple of 4 because 56 is a multiple of 4, and 20,846 is not a multiple of 4 because 46 is not a multiple of 4. This is for your information only.

Invite a student to label the dots in the first arrow road.



Ask the students to suggest numbers which are on the second arrow road including positive and negative, large and small numbers. All the numbers in the second arrow road are 1 more than a multiple of 4. For example, $\hat{7}$ ($\hat{8} + 1$), 33 (32 + 1), and 101 (100 + 1) are all on the second arrow road.

Invite a student to label the dots in the second arrow road.


+4
$\hat{8}$ $\hat{4}$ 0 4 8 12 16 20 24
$\hat{7}$ $\hat{3}$ 1 5 9 13 17 21 25

Emphasize that each number in the second arrow road is 1 more than the number directly above it. If necessary, use the number line to help your students see that $\hat{7}$ is 1 more than $\hat{8}$ and that $\hat{3}$ is 1 more than $\hat{4}$.

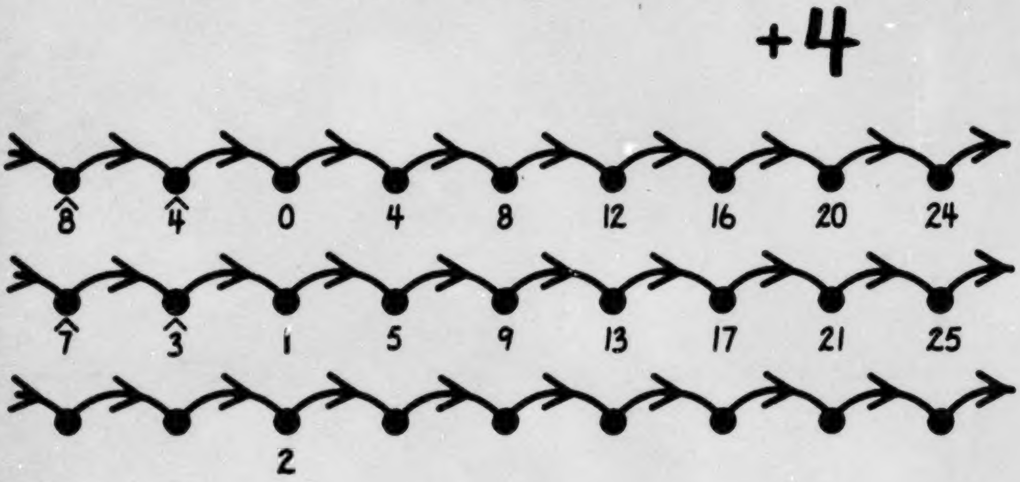
T: Which whole numbers are not on either of these arrow roads? How do you know?

Encourage the class to find several numbers and to explain why each number is not on either of the +4 arrow roads on the board.

T: Would 2 be on either of these arrow roads?

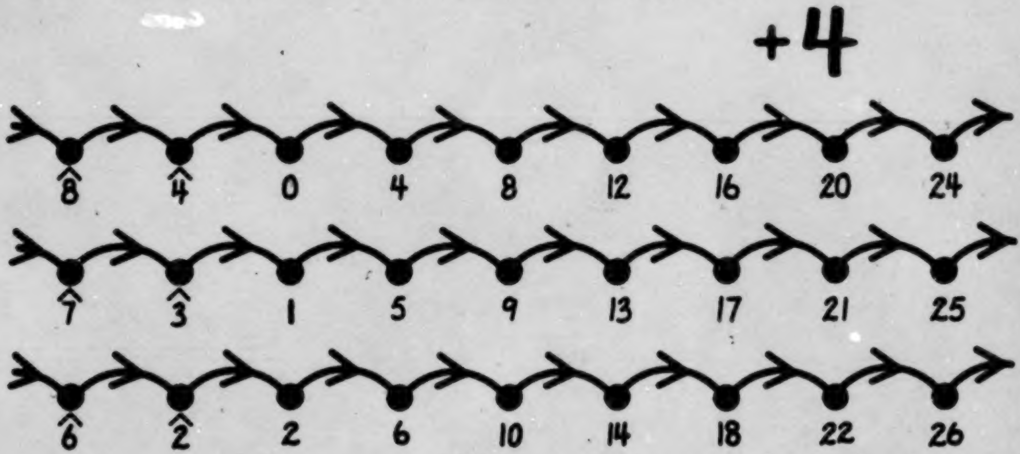
S: No.

T: I'll draw a +4 arrow road with the number 2 on it.


+4
$\hat{8}$ $\hat{4}$ 0 4 8 12 16 20 24
$\hat{7}$ $\hat{3}$ 1 5 9 13 17 21 25
2

Ask the students to suggest many numbers which are on the third arrow road. All the numbers on the third arrow road are 2 more than a multiple of 4. For example, 18 (16 + 2), 50 (48 + 2), and 822 (820 + 2) are on the third arrow road.

Invite a student to label the dots in the third arrow road.



T: What patterns do you see in this arrow picture?

There are several patterns the students could mention, but emphasize the pattern that each number is 1 more than the number directly above it.

T: Are there any whole numbers which are skipped by all three of these +4 arrow roads? (Yes)
What are they? How do you know they are skipped?

If necessary, focus the class' attention on the numbers between $\hat{8}$ and 26 and ask which numbers are skipped. If a student cannot give a good reason why a number would not be on any of the arrow roads on the board, do not offer any information and do not draw any conclusion.

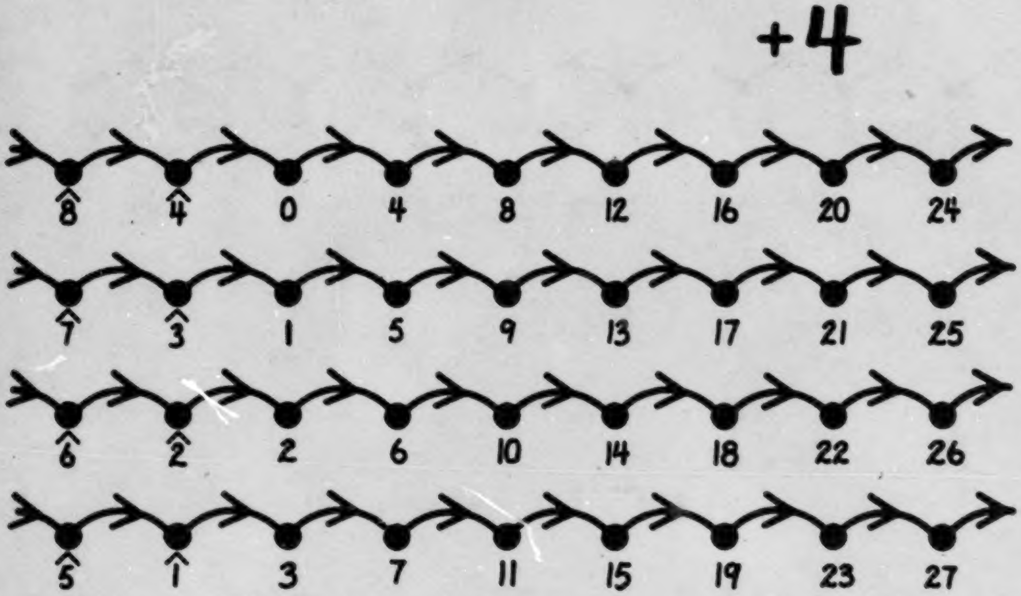
NOTE: All the integers which are 1 less (3 more) than a multiple of 4 are not on one of these +4 arrow roads. For example; 23 (24 - 1), $\hat{9}$ ($\hat{8}$ - 1), and 999 (1,000 - 1) are skipped on all three roads.

T: Would 3 be on one of these arrow roads?

S: No, because the first arrow road jumps from 0 to 4, the second arrow road jumps from 1 to 5, and the third arrow road jumps from 2 to 6.

T: 3 isn't on any of these roads, so I'll draw another +4 arrow road with the number 3 on it.

Draw a fourth arrow road below the third arrow road and align the dots. Put the dot for 3 directly below the dot for 2. Ask the students to suggest many numbers which are on the fourth arrow road before asking a student to label the dots.


+4
$\hat{8}$ $\hat{4}$ 0 4 8 12 16 20 24
$\hat{7}$ $\hat{3}$ 1 5 9 13 17 21 25
$\hat{6}$ $\hat{2}$ 2 6 10 14 18 22 26
$\hat{5}$ $\hat{1}$ 3 7 11 15 19 23 27

T: What patterns do you see?

There are several patterns which your students might observe; for example,

- all the numbers in the first and third arrow roads are even;
- all the numbers in the second and fourth arrow roads are odd;

- each number is 1 more than the number directly above it;
- all the whole numbers between $\hat{8}$ and 27 are represented by dots on the board;
- the last digits of the whole numbers in each arrow road form a repeating sequence. In the first and third arrow roads the repeated part of the sequence is: 0, 4, 8, 2, 6. In the second and fourth arrow roads the repeated part of the sequence is: 1, 5, 9, 3, 7.

T: Are there any whole numbers which are skipped by all four of these +4 arrow roads? (No)

Someone might claim that there is a number which is not on any of these +4 arrow roads. Ask the rest of the class if they agree or disagree and why. Remind the class that each of the arrow roads continues forever in both directions. A possible dialogue is given here.

S: 9,236 isn't on any of these +4 arrow roads.

T: How do you know? Remember that these "unfinished" arrows tell us that these arrow roads continue forever.

S: If 9,236 is on one of these +4 arrow roads, it would be on either the first one or the third one because 9,236 is an even number.

S: We could find out which arrow road 9,236 is on by starting at 8,000 and counting from 8,000 to 9,236. 8,000 is on the first arrow road because it is a multiple of 4; 8,001 is on the second road; 8,002 is on the third road; 8,003 is on the fourth road; 8,004 is on the first road; . . . .

T: It is difficult for us to decide which of these arrow roads 9,236 would be on because it is so large, but can we be sure it is on one of them?

S: If we had our calculators, we could start at 8,000 and press $\boxed{+}\boxed{4}\boxed{=}\boxed{=}\boxed{=}$ . . . to see if 9,236 would be on the first arrow road.

Most of your students should be confident that all of the whole numbers are on one of the + 4 arrow roads.

T: On your paper draw a + 5 arrow picture similar to the + 4 arrow picture on the board. Try to include each of the whole numbers in one of your + 5 arrow roads.

Just before the end of the lesson, after the students have been working independently for several minutes, ask the following questions.

T: How many different + 4 arrow roads did we need to draw to include all the whole numbers in our picture? (Four)
How many different + 5 arrow roads do we need to draw if we want to include all the whole numbers in our + 5 arrow picture? (Five)

N21 Multiplication and Subtraction Problems

CAPSULE LESSON SUMMARY

Set up a multiplication calculation on the Minicomputer and decide which number it is by doing mental calculations. Add a positive and a negative number on the Minicomputer and ask for the corresponding subtraction problem.

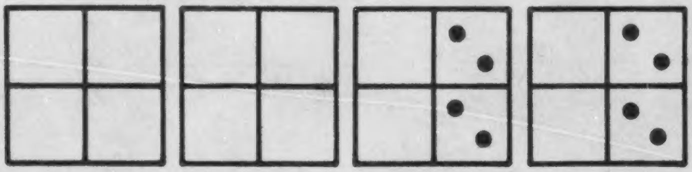
MATERIALS

Teacher : Minicomputer set
Student : Paper; pencil; Worksheets N21*, **, ***, and ****

DESCRIPTION OF LESSON

Exercise 1 : Multiplication on the Minicomputer

Put this configuration on the Minicomputer.



T : What number is this? Write it on a piece of paper.

Ask several students to explain how they determined that the number on the Minicomputer is 110. If no one suggests that the number on the Minicomputer is 2 × 55, mention this yourself. Write "2 × 55 = 110" on the board.

T : What number is on the tens' board?

S : 100; 2 × 50 = 100.

T : What number is on the ones' board?

S : 10; 2 × 5 = 10.

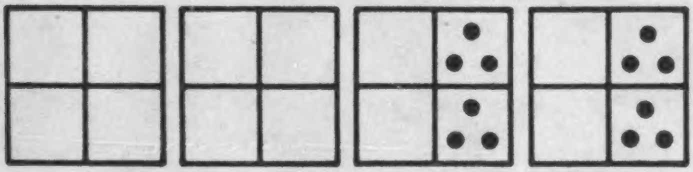
N 21

Record these number stories on the board.

$$
\begin{aligned}
2 \times 55 &= 110\\
2 \times 50 &= 100\\
2 \times 5 &= 10
\end{aligned}
$$

T: 2 x 55 = 110.

Put the standard configuration for 55 on the Minicomputer three times.



T: What number is this? Write it on your paper.

Ask several students to explain how they determined that the number on the Minicomputer is 165. If necessary, suggest that this number is 3 x 55. Write "3 x 55 = 165" on the board.

T: What number is 3 x 50?

S: 150.

T: What number is 3 x 5?

S: 15.

Record these number stories on the board.

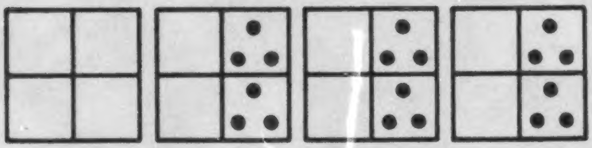
$$3 \times 55 = 165$$
$$3 \times 50 = 150$$
$$3 \times 5 = 15$$

T: What number is 150 + 15?

S: 165.

T: 3 x 55 = 165.

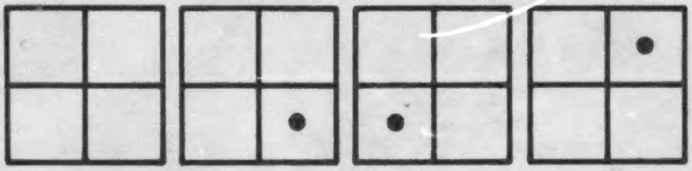
Continue this activity with this configuration.



= $3 \times 555 = 1{,}665$

Exercise 2

Put 124 on the Minicomputer.

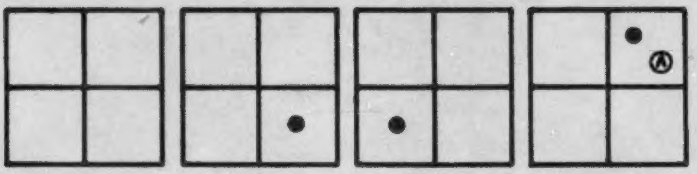


T: What number is this?

S: 124.

N 21

Put a negative checker on the 4-square.



T: I put a negative checker on the Minicomputer. What calculation am I doing?

S: "$124 + \hat{4}$".

T: What subtraction problem has the same answer?

S: "124 - 4".

Write both number stories on the board.

$$124 + \hat{4} = ?$$

$$124 - 4 = ?$$

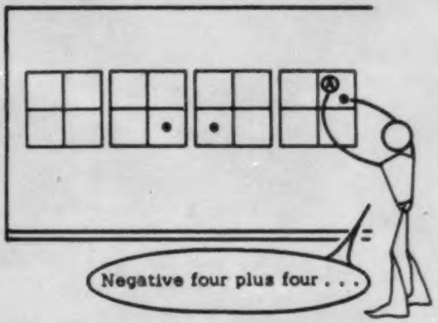
T: What number is this? Write it on your paper.

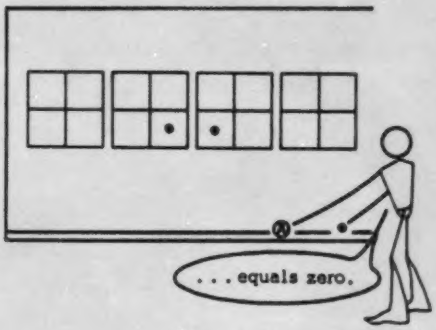
Look at several of the students' papers.

T: Many of you wrote "120". Why is 120 correct?

S: Because the two checkers on the 4-square can be taken off.

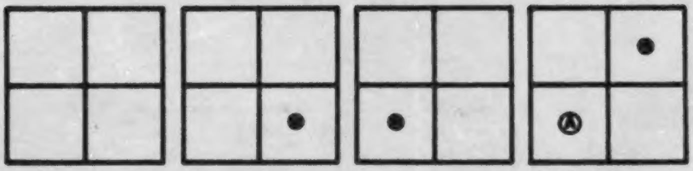
Make the play yourself.


Negative four plus four . . .


. . . equals zero.

Complete the number stories on the board.

Put this configuration on the Minicomputer.



T: What calculation are we doing on the Minicomputer?

S: "$124 + \hat{2}$".

T: What subtraction problem has the same answer?

S: "124 - 2".

Write both number stories on the board.

$$124 + \hat{2} = ?$$

$$124 - 2 = ?$$

N 21

T: What number is this? Write it on your paper.

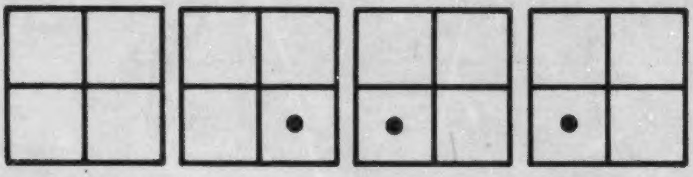
Look at several of the students' papers.

T: Many of you have written "122" on your papers. How do you know that this number is 122?

S: 124 - 2 = 122.

S: I thought about making a backward play with the checker on the 4-square.

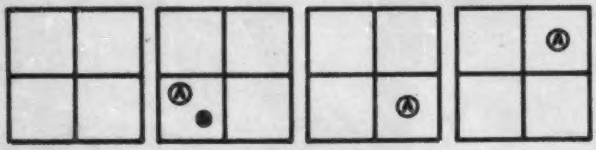
Invite a student to make the backward play on the Minicomputer. Then the student should remove a regular checker and a negative checker from the 2-square.



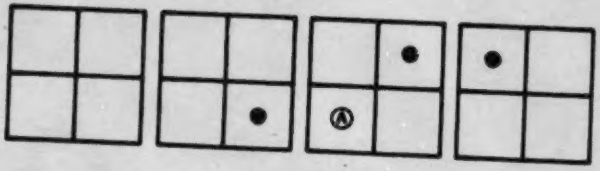
$= 122$

Complete the number stories on the board.

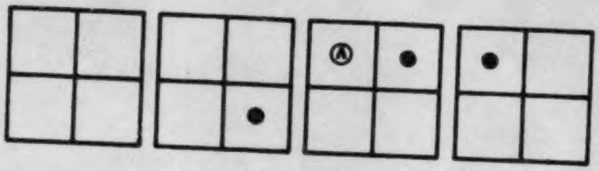
Repeat this activity with the following configurations.



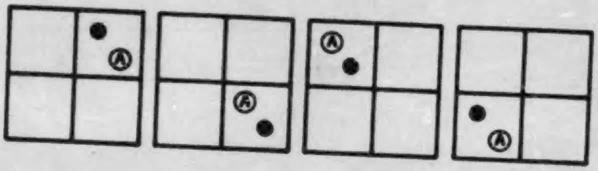
$= \widehat{214} + 200 = \widehat{14}$



$= 148 + \widehat{20} = 128$



$= 148 + \widehat{80} = 68$

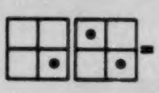


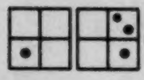
$= \widehat{4{,}182} + 4{,}182 = 0$

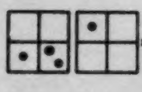
Worksheets N21*, **, ***, and **** are available for individual work.

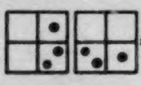
Name ______________________ **N 21 ***

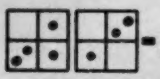
What number is on the Minicomputer?

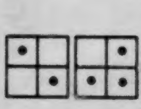
 - $\hat{19}$

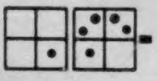
 - $\hat{21}$

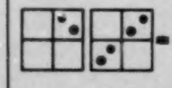
 - 28

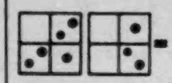
 - $\hat{41}$

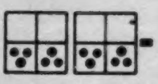
 - 52

 - 97

 - 12

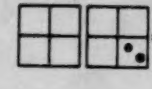
 - 0

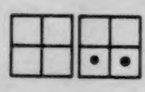
 - $\hat{14}$

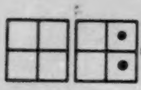
 - 33

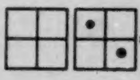
Name ______________________ **N 21 ****

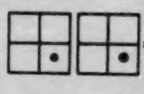
What number is on the Minicomputer?

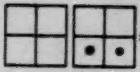
 - 0

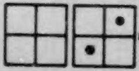
 - 1

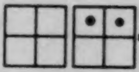
 - 3

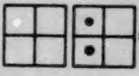
 - 7

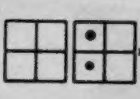
 - 9

 - $\hat{1}$

 - 2

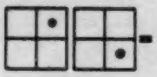
 - $\hat{4}$

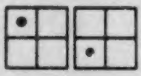
 - 6

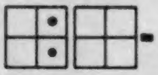
 - $\hat{6}$

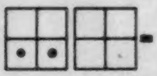
Name ______________________ **N 21 *****

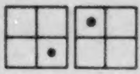
What number is on the Minicomputer?

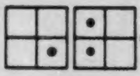
 - 39

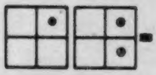
 - $\hat{78}$

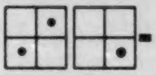
 - $\hat{30}$

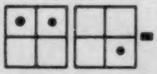
 - 10

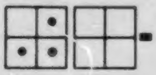
 - 2

 - 0

 - 35

 - 59

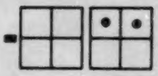
 - $\hat{39}$

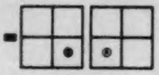
 - 50

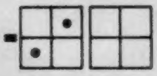
Name ______________________ **N 21 ******

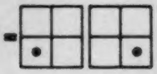
Put these numbers on the Minicomputer.

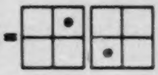
Use exactly one regular checker and one negative checker for each number.

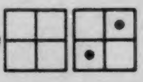
4 - 

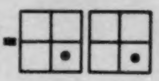
8 - 

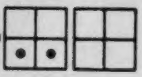
20 - 

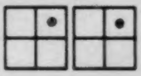
19 - 

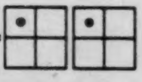
38 - 

$\hat{2}$ - 

$\hat{9}$ - 

$\hat{10}$ - 

$\hat{36}$ - 

$\hat{72}$ - 

N22 Subtraction and Composition #1

CAPSULE LESSON SUMMARY

Locate numbers on a number line where the marks are for even numbers. Make +10 jumps on the number line. Find the composite of several +10 jumps. Solve a subtraction problem by building an arrow road with arrows that compose to give the appropriate subtraction function. Use as many - 10 arrows as possible.

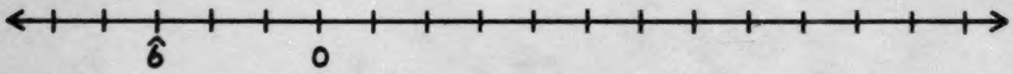
MATERIALS

Teacher: Colored chalk
Student: Paper; pencil; Worksheets N22*, **, ***, and ****

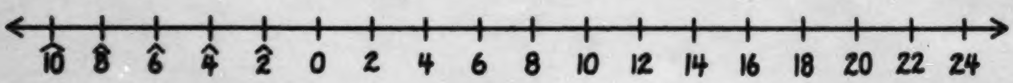
DESCRIPTION OF LESSON

Exercise 1: Mental Arithmetic

Draw a line on the board and graduate it into intervals about six centimeters in length. Locate 0 at one of the marks and locate $\hat{6}$ three marks to the left.



Invite students one at a time to associate numbers with each of the marks on this number line.


$\hat{10}$ $\hat{8}$ $\hat{6}$ $\hat{4}$ $\hat{2}$ 0 2 4 6 8 10 12 14 16 18 20 22 24

Point to the number line.

T: What do all these numbers have in common?

S: They are all multiples of 2.

S: They are all even numbers.

T: Where are some odd numbers on this number line?

Invite a student to point to where some odd numbers are on this number line. The student should point halfway between consecutive pairs of even numbers. Emphasize that between consecutive even numbers there is an odd number. Ask students to locate several specific odd numbers between $\widehat{10}$ and 24 on this number line.

Write "- 10" in red near the number line and invite students to trace - 10 arrows. As each arrow is traced, record the appropriate number story on the board.

When you have about ten number stories written on the board, ask the students if they see a pattern. Emphasize that the last digit of a number does not change when you subtract 10 except for numbers between 0 and 10.

T: Suppose we start at 15 and make a - 10 jump. Where would we land?

S: At 5.

T: If we start at 30 and make a - 10 jump, . . . ?

S: At 20.

Continue this activity with the following or similar starting numbers which are appropriate for the abilities of your students.

31 (21)
50 (40)
52 (42)
20 (10)
25 (15)
35 (25)
45 (35)

10 (0)
100 (90)
106 (96)
200 (190)
300 (290)
1,000 (990)
1,001 (991)

19 (9)
76 (66)
43 (33)
69 (59)
5 ($\hat{5}$)
9 ($\hat{1}$)
2 ($\hat{8}$)

Refer to the number line if the students have difficulty calculating "5 - 10", "9 - 10", or "2 - 10".

T: If we start at 24 and make a -10 jump, where would we land?

S: At 14.

T: 24 - 10 = 14.

Write this calculation on the board.

$$\begin{array}{r} 24 \\ -10 \\ \hline 14 \end{array}$$

T: If we start at 24 and make two -10 jumps, where would we land?

S: At 4.

T: 24 - 10 = 14 and 14 - 10 = 4.

$$\begin{array}{r} 24 \\ -10 \\ \hline 14 \\ -10 \\ \hline 4 \end{array}$$

T: Instead of subtracting 10 twice, what number could we subtract?

S: 20; 24 - 20 = 4.

$$\begin{array}{r} 24 \\ -10 \\ \hline 14 \\ -10 \\ \hline 4 \end{array} \qquad \begin{array}{r} 24 \\ -20 \\ \hline 4 \end{array}$$

T: I would like to calculate "61 - 20". How many times do I need to subtract 10?

S: Twice.

T: Yes, subtracting 10 twice gives the same result as subtracting 20. What number is 61 - 20? Write it on your paper.

Look at several of the students' papers and then do the calculation on the board. The following illustrations on the right show what is to be written on the board during the dialogue.

T: If we subtract 10 ...

$$\begin{array}{r} 61 \\ -10 \\ \hline 51 \end{array}$$

then subtract 10 again ...

$$\begin{array}{r} 61 \\ -10 \\ \hline 51 \\ -10 \\ \hline 41 \end{array}$$

it is the same as if we subtract 20.

$$\begin{array}{r} 61 \\ -10 \\ \hline 51 \\ -10 \\ \hline 41 \end{array} \qquad \begin{array}{r} 61 \\ -20 \\ \hline 41 \end{array}$$

N 22

T: I would like to calculate "104 - 30". How many times do I need to subtract 10?

S: Three times.

Write "104 - 30 = ?" on the board.

T: Before we do this calculation, what can you tell me about the last digit of the answer?

S: The last digit is "4", because we are subtracting tens.

T: Is 104 - 30 more or less than 100?

S: Less.

T: . . . more or less than 50?

S: More.

T: We know that 104 - 30 is between 100 and 50 and that the last digit of this number is 4. What number is 104 - 30? Write it on your paper.

Look at the students' papers and then solve the problem at the board.

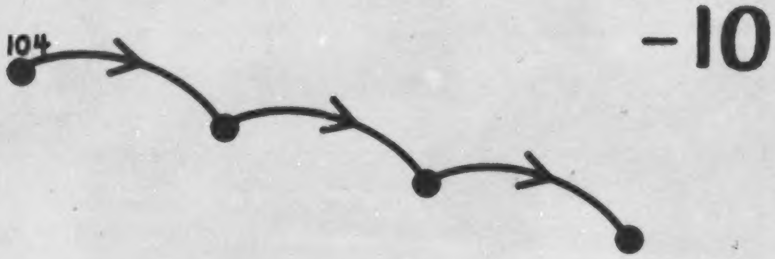
$$\begin{array}{r} 104 \\ -10 \\ \hline 94 \\ -10 \\ \hline 84 \\ -10 \\ \hline 74 \end{array} \qquad \begin{array}{r} 104 \\ -30 \\ \hline 74 \end{array}$$

N 22

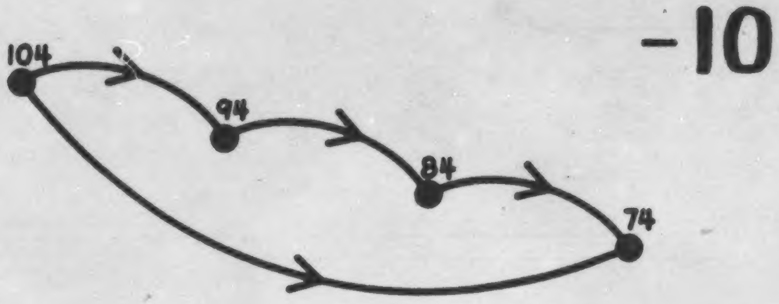
T: We can show this calculation with a -10 arrow picture. How many -10 arrows will we need to draw?

S: Three.

Draw this arrow picture on the board.



Invite a student to label the dots. Then draw a blue arrow from 104 to 74.



T: What could this blue arrow be for?

S: -30.

Write "-30" in blue on the board near the arrow picture. Erase the labels for the dots and point to the starting dot.

T: If this number were 50, what number would the ending dot be for?

S: 20.

T: If this number were 41, . . . ?

S: 11.

Continue this activity with the following or similar numbers which are appropriate for the abilities of your students.

79 (49)   95 (65)

112 (82)   268 (238)

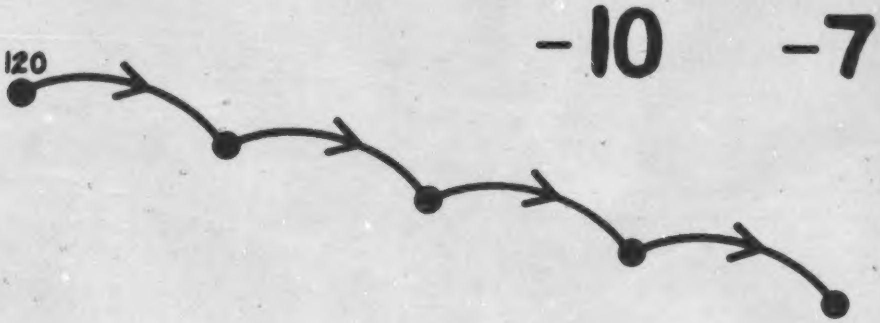
Erase the blue arrow and "- 30" (blue). Then write this calculation on the board.

$$\begin{array}{r} 120 \\ \underline{-\ 37} \end{array}$$

T: Before we wanted to subtract 30, so we subtracted 10 three times. In this problem we want to subtract 37. What do you suggest?

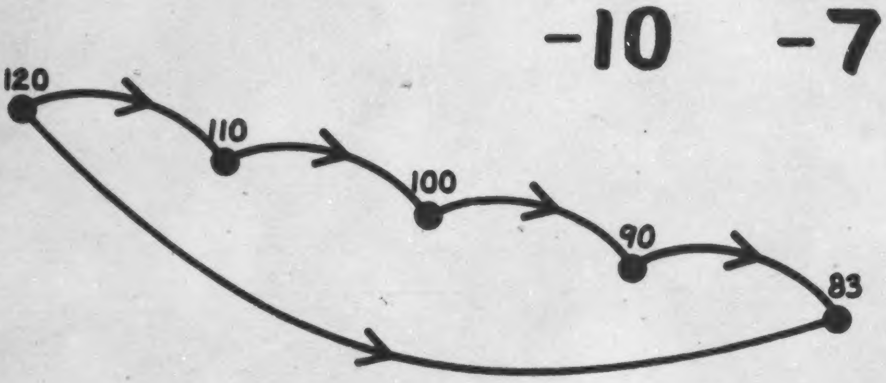
S: Subtract 10 three times and then subtract 7.

Extend the arrow picture on the board by adding a - 7 arrow. Label the starting dot "120".



N 22

Invite students to label the dots in this arrow picture. Draw a blue arrow from 120 to 83.



T: What could this blue arrow be for?

S: -37.

Write "-37" in blue near the arrow picture.

T: Subtracting 10 three times and then subtracting 7 is the same as subtracting 37.

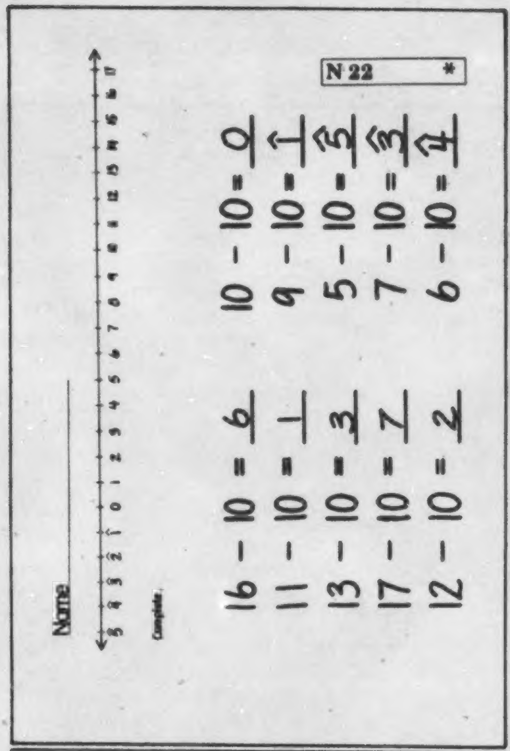
T: What number is 120 - 37?

S: 83.

Record the answer to the subtraction problem.

$$\begin{array}{r} 120 \\ -\ 37 \\ \hline 83 \end{array}$$

Worksheets N22*, **, ***, and **** are available for individual work.



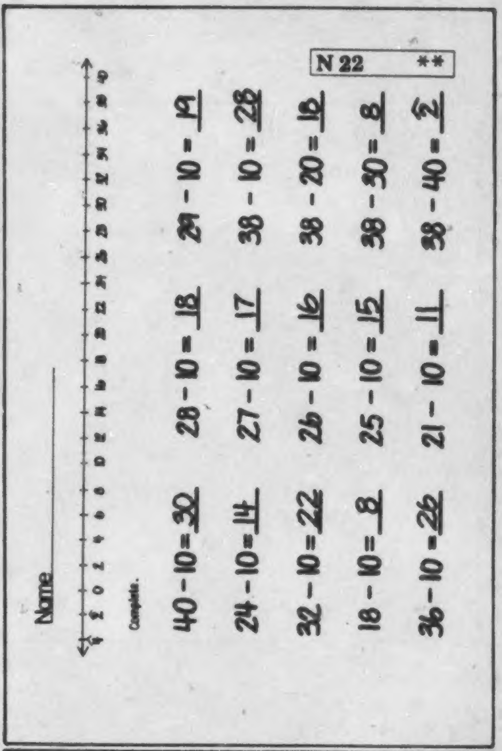
Name

N 22 *

Complete.

16 − 10 = 6
11 − 10 = 1
13 − 10 = 3
17 − 10 = 7
12 − 10 = 2

10 − 10 = 0
9 − 10 = $\hat{1}$
5 − 10 = $\hat{5}$
7 − 10 = $\hat{3}$
6 − 10 = $\hat{4}$



Name

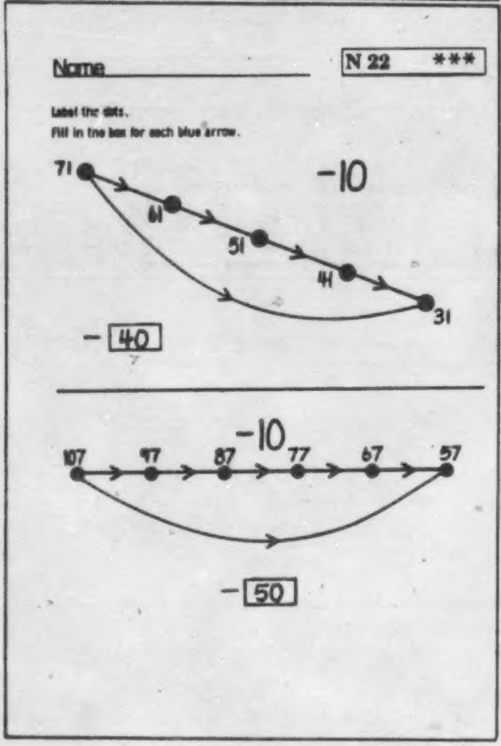
N 22 **

Complete.

40 − 10 = 30
24 − 10 = 14
32 − 10 = 22
18 − 10 = 8
36 − 10 = 26

28 − 10 = 18
27 − 10 = 17
26 − 10 = 16
25 − 10 = 15
21 − 10 = 11

29 − 10 = 19
38 − 10 = 28
38 − 20 = 18
38 − 30 = 8
38 − 40 = $\hat{2}$

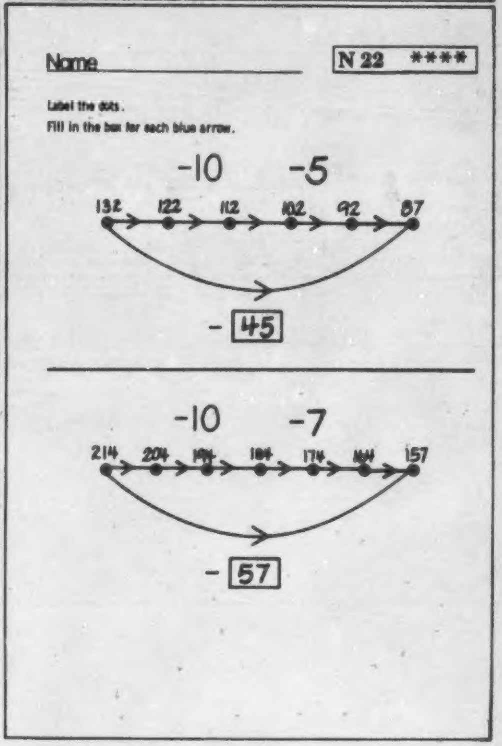


Name

N 22 ***

Label the dots.
Fill in the box for each blue arrow.

71
61
51
41
31
−10
− 40
107
97
87
−10
77
67
57
− 50



Name

N 22 ****

Label the dots.
Fill in the box for each blue arrow.

−10
−5
132
122
112
102
92
87
− 45
−10
−7
214
204
194
184
174
164
157
− 57

N23 10 x on the Minicomputer

CAPSULE LESSON SUMMARY

Add a positive number and a negative number on the Minicomputer and ask for the corresponding subtraction problem. Make plays which replace ten checkers on a square with one checker on the square of the same color on the next board to the left. Introduce ⑩-checkers (a ⑩-checker on a square names the same number as ten regular checkers on the same square). Make the same play replacing a ⑩-checker on a board with a regular checker on the next board to the left.

MATERIALS

Teacher: Minicomputer set; ⑩-checkers (see the note which follows)

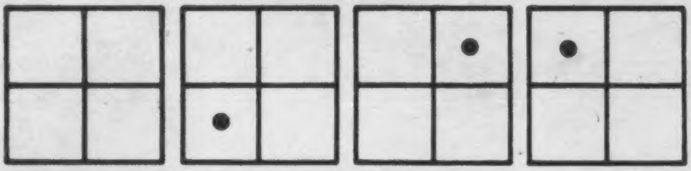
Student: Worksheets N23*, **, ***, and ****; paper; pencil

NOTE: Before the lesson begins, magnetize each of the ⑩-checkers by sticking a small piece of magnetic material to the back.

DESCRIPTION OF LESSON

Exercise 1

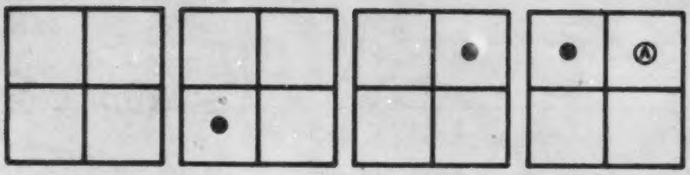
Put 248 on the Minicomputer.



T: What number is on the Minicomputer?

S: 248.

Put a negative checker on the 4-square.



T: I put a negative checker on the Minicomputer. What calculation am I doing?

S: "248 + $\hat{4}$".

T: What subtraction problem has the same answer?

S: "248 - 4".

Write both number stories on the board.

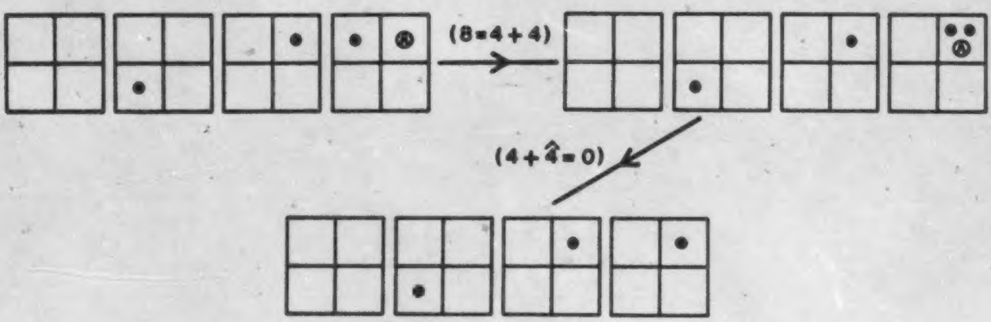
$$248 + \hat{4} = ?$$
$$248 - 4 = ?$$

T: What number is this? Write it on a piece of paper.

Look at several of the students' papers. Ask a student to answer aloud.

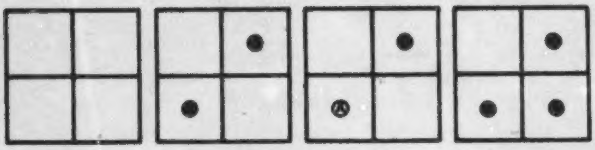
S: 244.

Complete the number stories on the board and invite a student to make some plays to show that the number on the Minicomputer is 244.

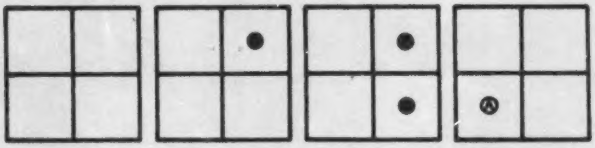


N 23

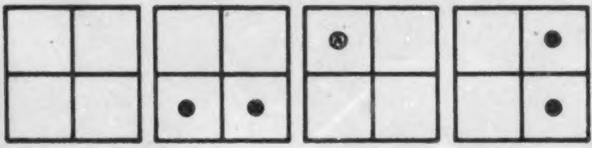
Repeat this activity with these configurations.



$= 647 + \widehat{20} = 627$



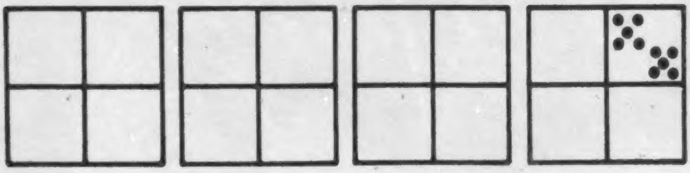
$= 450 + \widehat{2} = 448$



$= 305 + \widehat{80} = 225$

Exercise 2

Put this configuration on the Minicomputer.



T: What multiplication calculation are we doing on the Minicomputer?

S: "10 x 4".

Write the calculation on the board.

10 × 4

NOTE: If a student suggests "4 x 10", remark that although 4 x 10 = 10 x 4, "4 x 10" would be suggested by four checkers on the 10-square of the Minicomputer.

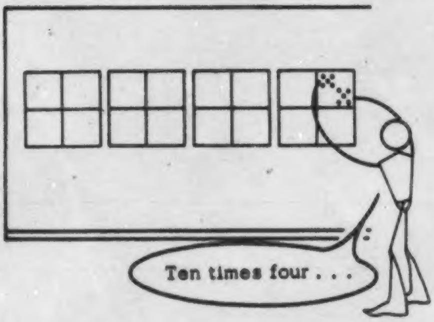
T: What number is this? Write it on your paper.

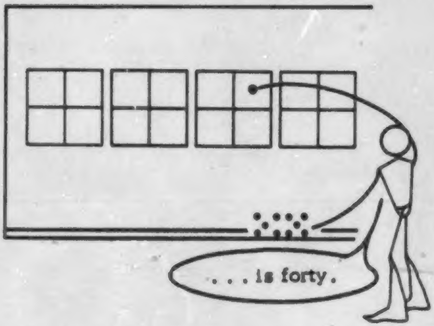
Look at many of the students' papers and then ask a student to answer aloud.

S: 40.

T: 10 x 4 = 40. Ten checkers on this purple square (point to the 4-square) is the same as one checker on this purple square (point to the 40-square).

Make a "10 x 4 = 40" play.





Complete the number story.

$$10 \times 4 = 40$$

Repeat this activity for "10 x 40" and "10 x 400". Write each number story below the previous number story.

$$\begin{aligned} 10 \times \quad 4 &= \quad 40 \\ 10 \times \ \ 40 &= \ \ 400 \\ 10 \times 400 &= 4{,}000 \end{aligned}$$

N 23

T: Do you see a pattern?

Encourage many different responses. If any of your students suggest that when you multiply a number by 10 you simply add a "0" to the right of it, reinforce this idea but do not suggest it yourself.

NOTE: Adding a "0" to the right of the numeral works only when an integer is multiplied by 10. For example: 10 × 0.3 is not 0.30.

Extend the list on the board another step.

10 × 4 = 40

10 × 40 = 400

10 × 400 = 4,000

10 × 4,000 =

T: What number is 10 × 4,000? How do you know?

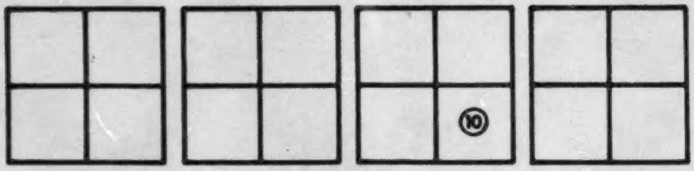
S: 40,000. 40 has one zero, 400 has two zeros, 4,000 has three zeros, and 40,000 has four zeros.

Ask a student to complete the number story on the board. Point out that a comma is inserted between the thousands' place and the hundreds' place and that it helps us to read large numbers. The digits to the left of the comma tells us how many thousands there are.

Erase the board and remove the checkers from the Minicomputer.

T (holding up a ⑩-checker): I am going to use this ⑩-checker to show that there are ten checkers on the same square.

Put a ⑩ -checker on the 10-square of the Minicomputer.

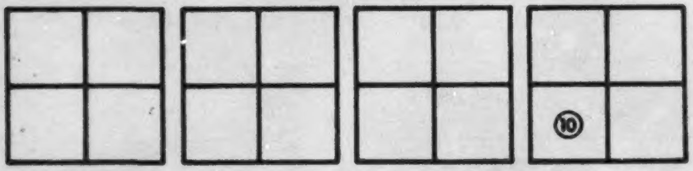


T: What number is this?

S: 100.

T: 10 × 10 = 100.

Move the ⑩ -checker to the 2-square of the Minicomputer.

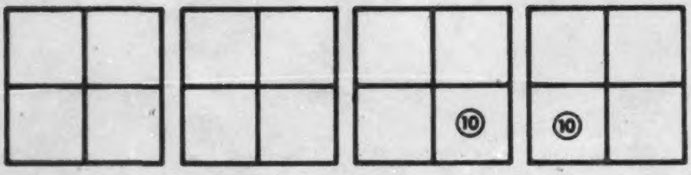


T: What number is this?

S: 20.

T: 10 × 2 = 20.

Put this configuration on the Minicomputer.



T: What calculation do these checkers suggest?

N 23

S: "10 × 12".

Write this calculation on the board.

10 × 12

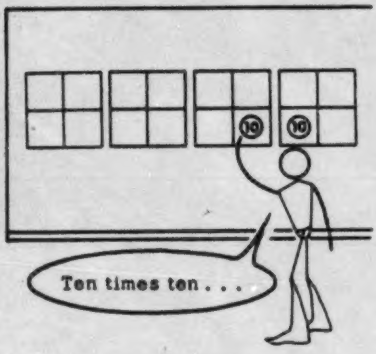
T: What number is this? Write it on your paper.

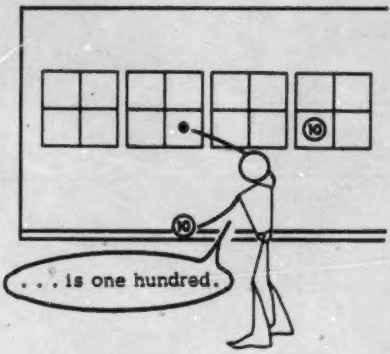
Look at several of the students' papers and then ask a student to answer aloud.

S: 120.

T: 10 × 12 = 120. Ten checkers on this white square (point to the 10-square) is the same as one checker on this white square (point to the 100-square).

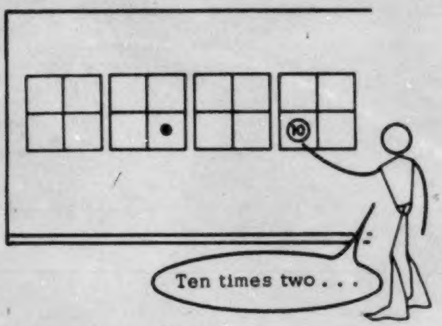
Make a "10 × 10 = 100" play.

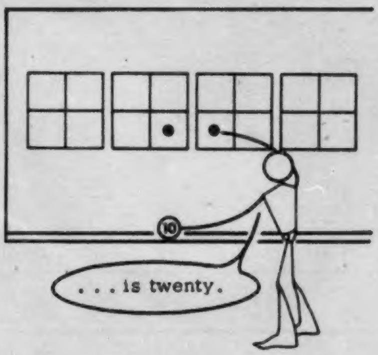




T: Ten checkers on this red square (point to the 2-square) is the same as one checker on this red square (point to the 20-square).

Make a "10 × 2 = 20" play.


10
Ten times two . . .


10
. . . is twenty.

Complete the number story.

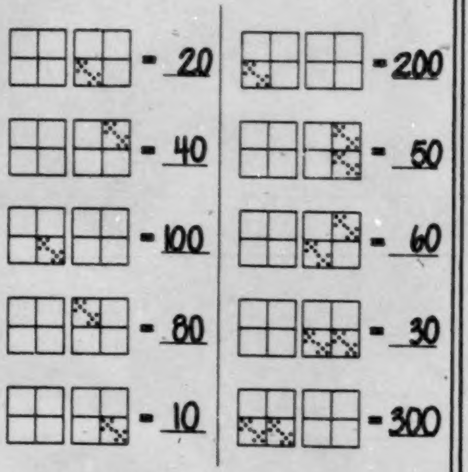
$$10 \times 12 = 120$$

Repeat this activity with "10 x 120", "10 x 15", and "10 x 150".

Worksheets N23*, **, **, and **** are available for independent work. Provide individual Minicomputers to students who wish to use them.

Name ______________ N 23 *

What number is on the Minicomputer?



= 20

= 40

= 100

= 80

= 10

= 200

= 50

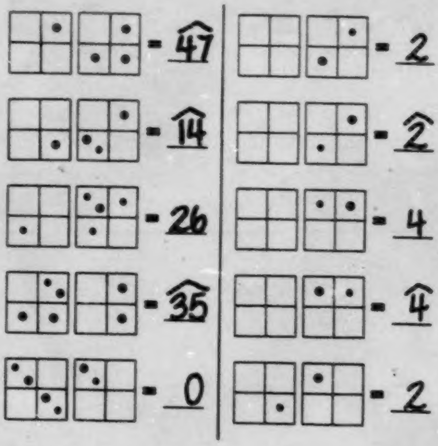
= 60

= 30

= 300

Name ______________ N 23 **

What number is on the Minicomputer?



= $\widehat{47}$

= $\widehat{14}$

= 26

= $\widehat{35}$

= 0

= 2

= $\widehat{2}$

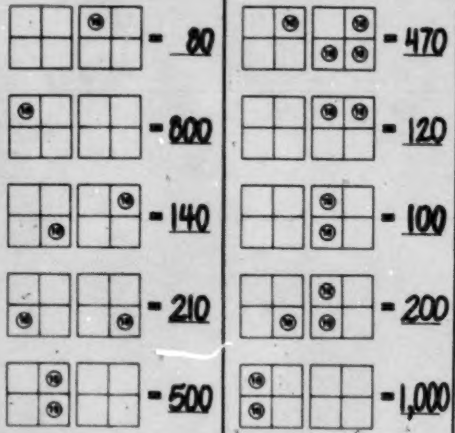
= 4

= $\widehat{4}$

= 2

Name ______________ N 23 ***

What number is on the Minicomputer?



= 80

= 800

= 140

= 210

= 500

= 470

= 120

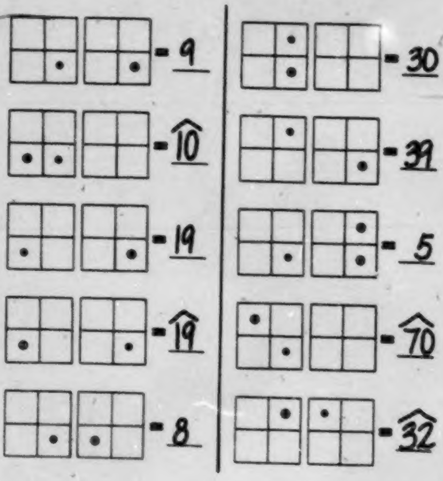
= 100

= 200

= 1,000

Name ______________ N 23 ****

What number is on the Minicomputer?



= 9

= $\widehat{10}$

= 19

= $\widehat{19}$

= 8

= 30

= 39

= 5

= $\widehat{70}$

= $\widehat{32}$

N24 The Functions 3x and 1/3x

CAPSULE LESSON SUMMARY

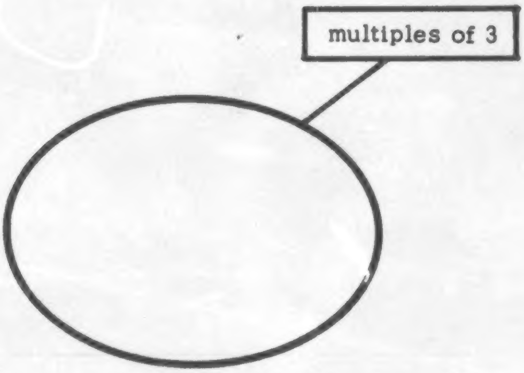
In a string picture locate some numbers which are multiples of 3 and some which are not. Do 3x and $\frac{1}{3}$x calculations to label the dots in a 3x arrow road. One of the calculations is "$\frac{1}{3}$ x 4".

MATERIALS

Teacher: Colored chalk
Student: Paper; pencil; Worksheets N24*, **, ***, and ****; colored pencils

DESCRIPTION OF LESSON

Draw this string picture on the board.



T: What are some numbers which are multiples of 3?

Encourage many students to give you numbers. When a multiple of 3 is given, locate it inside the red string. If the suggested number is not a multiple of 3, locate it outside the red string. Whenever possible, explain to the class why a number is not a multiple of 3. Let the students' suggestions determine the direction of your discussion. A student may suggest that 24 is a multiple of 3.

T: 24 is how many threes?

N 24

If necessary, count by threes from 0 to 24 and ask a student to keep track of how many times you add three. Conclude that 24 is 8 threes. Draw and label a dot for 24 inside the string. Write this number story on the board.

$$8 \times 3 = 24$$

T: 8 x 3 = 24. What number is $\frac{1}{3}$ x 24 (read: one-third of twenty-four)?

S: 8.

Write this number story on the board below the first one.

$$8 \times 3 = 24$$
$$\frac{1}{3} \times 24 = 8$$

T: Are there any numbers less than 24 which are also multiples of 3? Name one.

Perhaps a student will give an incorrect answer; for example,

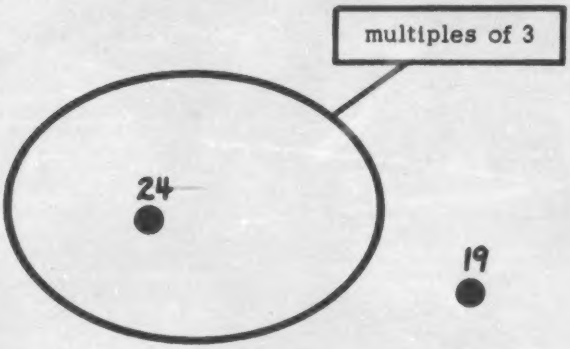
S: 19.

T: Is 19 a multiple of 3? How do you know?

S: 19 can't be a multiple of 3 because 24 - 3 = 21 and 21 - 3 = 18.

S: If we start at 0 and count by threes, we would skip 19.

T: 19 is not a multiple of 3, so I will put it outside of the red string.



T: Are there any numbers less than 19 which are multiples of 3?

Continue this activity until a negative multiple of 3 is suggested; for example, $\hat{6}$.

T: What number is $\frac{1}{3} \times \hat{6}$?

S: $\hat{2}$.

Write these number stories on the board.

$$\frac{1}{3} \times \hat{6} = \hat{2}$$
$$3 \times \square = \hat{6}$$

T: Three times what number is $\hat{6}$?

S: $\hat{2}$.

T: $\hat{2} + \hat{2} + \hat{2} = \hat{6}$, so $3 \times \hat{2} = \hat{6}$.

Write "$\hat{2}$" inside the box and put $\hat{6}$ inside the string.

N 24

T: What are some large numbers which are multiples of 3?

Perhaps a student will give an incorrect answer; for example,

S: 1,000.

T: We need to determine if 1,000 is a multiple of 3. Do you know any large numbers close to 1,000 which are multiples of 3?

If no one suggests 999, ask successively if 9, 99, and 999 are multiples of 3.

T: What number is $\frac{1}{3} \times 999$?

S: 333.

T: We know that 999 is a multiple of 3 because 3 x 333 = 999. What does this information tell us about 1,000?

S: 1,000 can't be a multiple of 3 because it is only 1 more than 999.

Put 1,000 outside the string and 999 inside the string.

Accept another suggestion.

S: 3,021.

T: Is 3,000 a multiple of 3?

S: Yes, because 3 x 1,000 = 3,000.

T: Is 21 a multiple of 3?

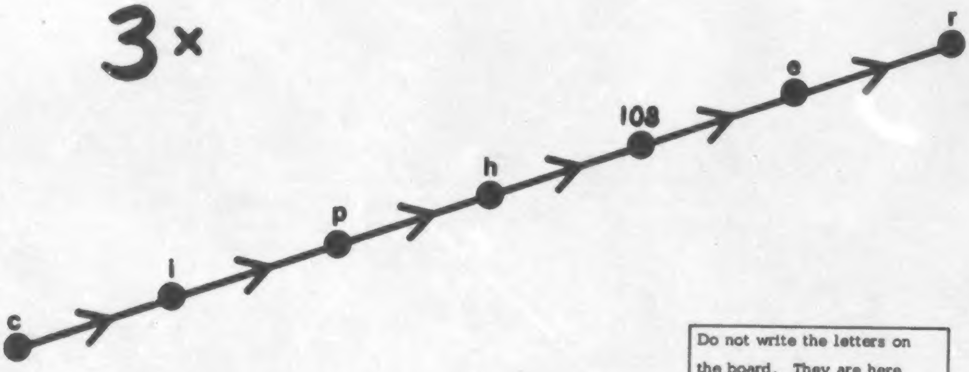
S: Yes, because 3 x 7 = 21.

T: 3,000 is a multiple of 3 and 21 is a multiple of 3, so 3,021 is a multiple of 3.

Continue this activity until many students have participated, but avoid turning it into a drill.

NOTE: You can determine easily if a number is a multiple of 3 by summing its digits. If the sum of the digits is a multiple of 3, then the number is also a multiple of 3. For example; since 7 + 8 = 15 is a multiple of 3, then 78 is also a multiple of 3. We know that 500 is not a multiple of 3 because 5 + 0 + 0 = 5 and 5 is not a multiple of 3. This is for your information only.

Draw this arrow picture on the board and ask the students to copy it on their papers.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: What are the red arrows for?

S: 3 x.

T (pointing to e): How can we determine what number is here?

S: We need to calculate "3 x 108".

T: Each of you calculate "3 x 108" and then label this dot (point to e) in your picture.

N 24

If your students have difficulty getting started, suggest adding "108 + 108 + 108" as a method of calculating "3 x 108". After a few minutes, ask students to share with the class their methods of doing the calculation. When someone explains that they added "108 + 108 + 108", write the addition problem on the board.

$$\begin{array}{r} 108 \\ 108 \\ +\,108 \\ \hline \end{array}$$

T (pointing to the ones' column): What number is 3 x 8?

S: 24.

T: We have 2 tens and 4 ones.

$$\begin{array}{r} 1\overset{2}{0}8 \\ 108 \\ +\,108 \\ \hline 4 \end{array}$$

T (pointing to the tens' column): What number is 3 x 0?

S: 0.

T: 0 + 2 = 2. We have 2 tens.

$$\begin{array}{r} 1\overset{2}{0}8 \\ 108 \\ +\,108 \\ \hline 24 \end{array}$$

T (pointing to the hundreds' column): What number is 3 x 1?

S: 3.

T: We have 3 hundreds.

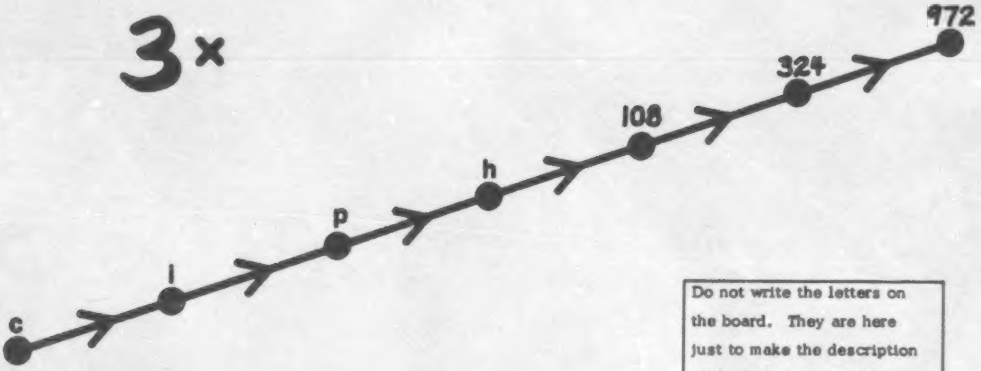
$$\begin{array}{r} {}^{2}\\ 108\\ 108\\ +108\\ \hline 324\end{array}$$

T: 3 x 108 = 324.

Write the corresponding multiplication problem on the board and label e "324".

$$\begin{array}{r} 108\\ \times\ \ 3\\ \hline 324\end{array}$$

Decide which number is at r in the same way. Conclude that 3 x 324 = 972 and label r "972".



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

N 24

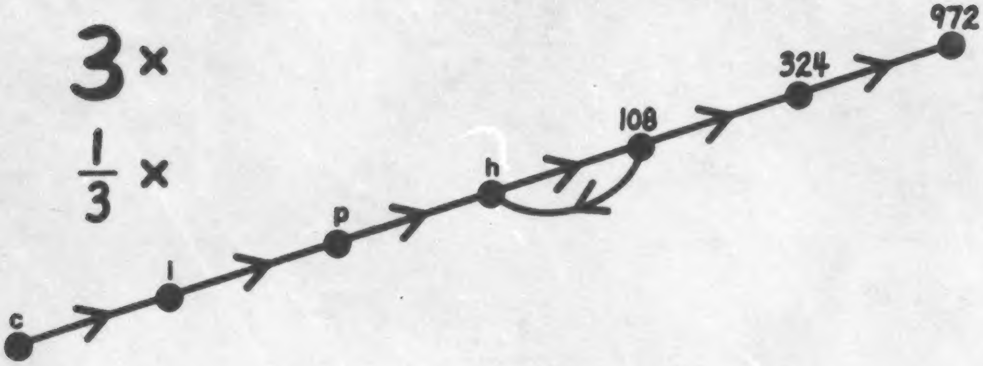
T (pointing to h): How can we determine what number is here?

S: We need to draw a return arrow.

Draw an arrow from 108 to h.

T: What could this arrow be for?

S: $\frac{1}{3}$ x.



T: Is $\frac{1}{3}$ x 108 more than 10?

S: Yes, because 3 x 10 is only 30.

T: Is $\frac{1}{3}$ x 108 more than 30?

S: Yes, because 3 x 30 = 90 and 90 is less than 108.

T: Is $\frac{1}{3}$ x 108 more than 40?

S: No, because 3 x 40 = 120 and that's too much.

T: We know that $\frac{1}{3}$ x 108 is between 30 and 40. Each of you calculate "$\frac{1}{3}$ x 108" and label this dot (point to h) in your picture.

Tell students that to check an answer multiply it by 3 and see if the product is 108. Avoid using the Minicomputer to calculate "$\frac{1}{3}$ x 108" unless none of your students discovers that 3 x 36 = 108. Write the corresponding addition problem on the board and invite a student to solve it. Encourage the students to use multiplication facts when doing these addition problems: for example, in this problem to think "3 x 6 = 18" instead of "6 + 6 = 12 and 12 + 6 = 18". Write the corresponding multiplication problem on the board also.

$$\begin{array}{r} \overset{1}{3}6 \\ 36 \\ +\,36 \\ \hline 108 \end{array} \qquad \begin{array}{r} 36 \\ \times\ 3 \\ \hline 108 \end{array}$$

Conclude that $\frac{1}{3}$ x 108 = 36 and label h "36".

T (pointing to p): Now calculate "$\frac{1}{3}$ x 36" and then label this dot in your picture.

When several students have found that $\frac{1}{3}$ x 36 = 12, write the corresponding addition and multiplication problems on the board. Label p "12".

$$\begin{array}{r} 12 \\ 12 \\ +\,12 \\ \hline 36 \end{array} \qquad \begin{array}{r} 12 \\ \times\ 3 \\ \hline 36 \end{array}$$

Repeat this activity for "$\frac{1}{3}$ x 12" and "$\frac{1}{3}$ x 4". The students quickly should find that $\frac{1}{3}$ x 12 = 4 but might be unable to calculate "$\frac{1}{3}$ x 4".

N 24

T: Is $\frac{1}{3} \times 4$ more than 1?

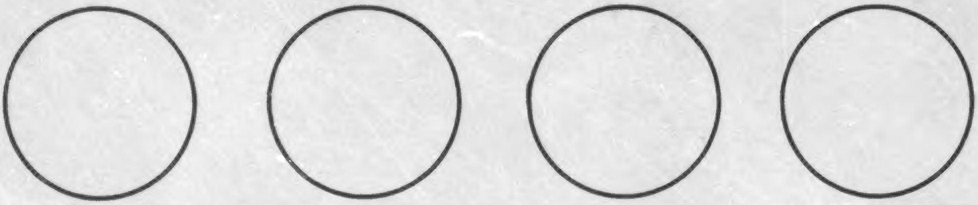
S: Yes, because $3 \times 1 = 3$.

T: Is $\frac{1}{3} \times 4$ more than 2.

S: No, because $3 \times 2 = 6$.

T: We know that $\frac{1}{3} \times 4$ is between 1 and 2.

Draw four circles of the same size on the board.

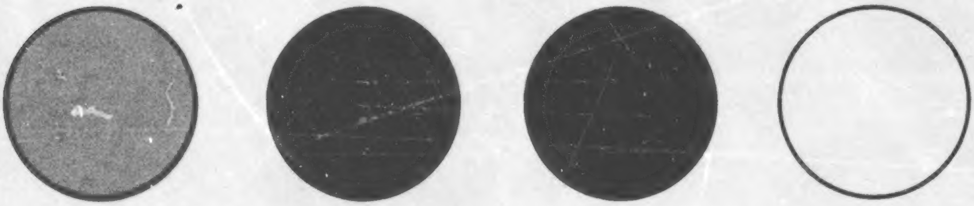


Choose three of your students and tell your class that you want to share four pies fairly between the three students chosen.

T: Will I be able to give each of my friends an entire pie?

S: Yes.

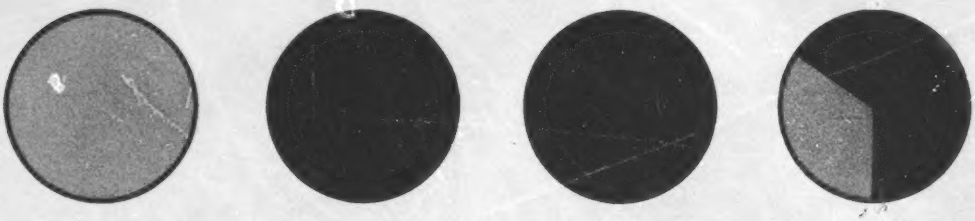
Shade the inside of three of the circles a different color for each.



T: I can give one pie to each of my friends and I will have one pie left. How can I share one pie between my three friends?

S: Cut the last pie into three pieces.

Invite a student to show where the pie should be cut. Be patient! It isn't easy to divide a circle into three pieces the same size. Be prepared to move some of the division lines if students object that the pieces are not the same size. Shade each of the pieces a different color.



T: What part of the pie is each of these pieces (point to the fourth circle)?

S: $\frac{1}{3}$.

T: Each of my friends will receive one whole pie and one-third of a pie. $\frac{1}{3} \times 4 = 1\frac{1}{3}$. (Write "$1\frac{1}{3}$" on the board.) We read this number one and one-third. If we calculate "$3 \times 1\frac{1}{3}$", the answer should be 4.

Write this calculation on the board.

$$\begin{array}{r} 1\frac{1}{3} \\ 1\frac{1}{3} \\ +\,1\frac{1}{3} \\ \hline \end{array}$$

N 24

T (point to the column of "$\frac{1}{3}$"'s): What number is $3 \times \frac{1}{3}$? $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \ldots$?

S: 1.

T: $3 \times \frac{1}{3} = 1$. So we have another one to put in the ones' column.

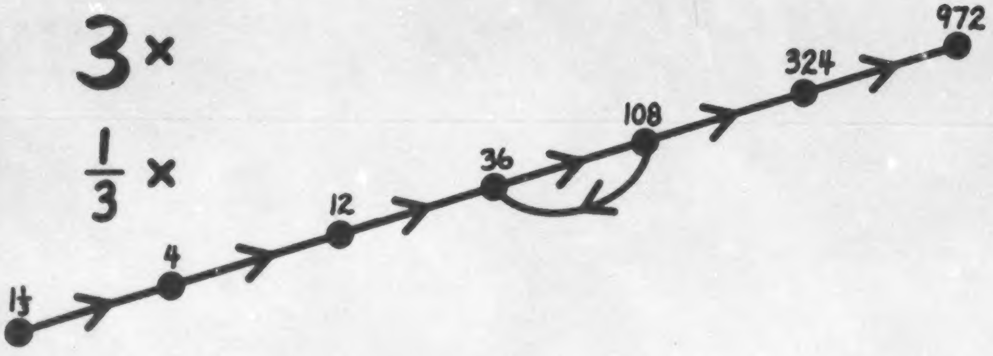
T (point to the ones' column): What number is 4×1?

S: 4.

$$\begin{array}{r} 1\phantom{\tfrac{1}{3}} \\ 1\tfrac{1}{3} \\ 1\tfrac{1}{3} \\ +\,1\tfrac{1}{3} \\ \hline 4\phantom{\tfrac{1}{3}} \end{array}$$

T: $\frac{1}{3} \times 4 = 1\frac{1}{3}$ and $3 \times 1\frac{1}{3} = 4$.

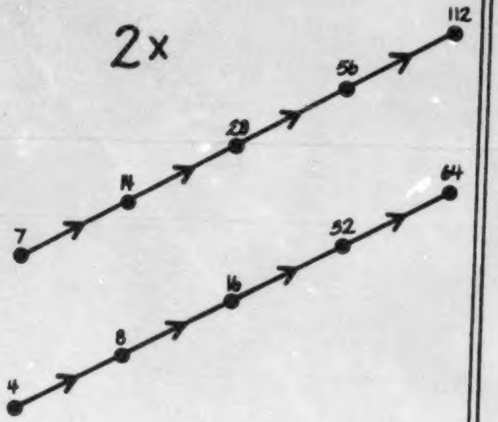
Label c "$1\frac{1}{3}$".



Worksheets N24*, **, ***, and **** are available for individual work.

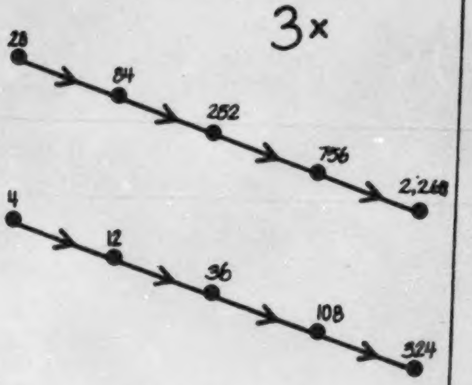
Name ____________________ **N 24 ***

Label the dots.


2x
7
14
28
56
112
4
8
16
32
64

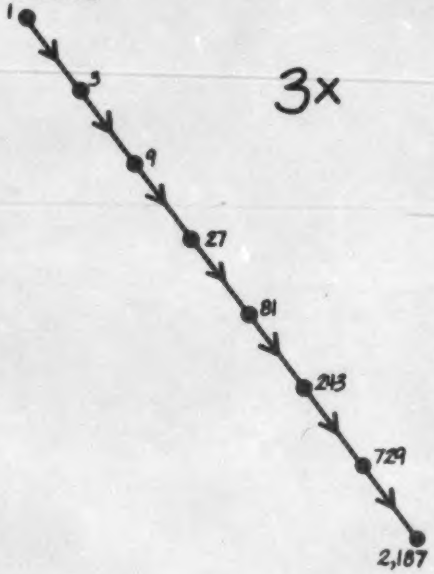
Name ____________________ **N 24 ****

Label the dots.


3x
28
84
252
756
2,268
4
12
36
108
324

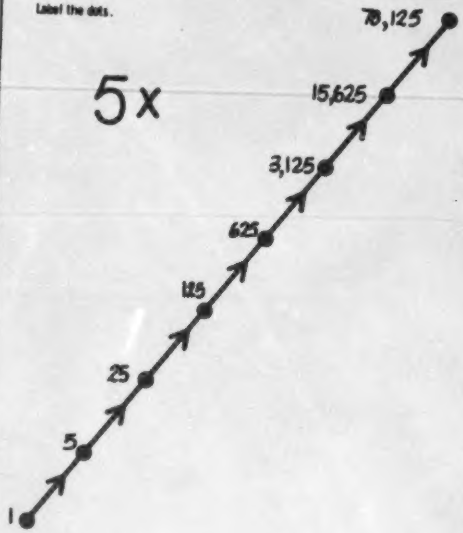
Name ____________________ **N 24 *****

Label the dots.


3x
1
3
9
27
81
243
729
2,187

Name ____________________ **N 24 ******

Label the dots.


5x
1
5
25
125
625
3,125
15,625
78,125

N 25

N25 Introduction to Decimal Numbers #1

CAPSULE LESSON SUMMARY

Through a sequence of situations involving money support a natural extension of the Minicomputer for representing decimal numbers.

MATERIALS

Teacher: Minicomputer set*; colored chalk
Student: None

DESCRIPTION OF LESSON

Begin this lesson with five to ten minutes of mental arithmetic exercises which include halving and doubling numbers.

Choose students from your class to star in these stories. Substitute their names for the names which appear here.

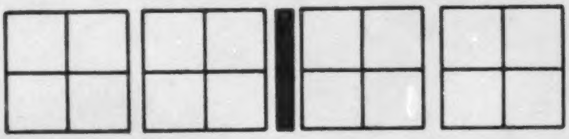
Display three Minicomputer boards.

T: Chuck and Timothy are my friends. I want to share $70 between them. I want to give the same amount of money to each of them. How much money should I give Chuck? How much money should I give Timothy?

Allow several students to make comments. Be sure the class understands that the $70 must be shared equally between the two children.

---

* This lesson requires that at times additional space be left between two of the demonstration Minicomputer boards and that a green bar be drawn in this space.



If your Minicomputers are hanging from permanently attached hooks or are displayed in some other way which fixes their position, you may need to make some adjustment before the lesson begins.

Although many students very likely know that $\frac{1}{2} \times 70 = 35$, the following activity of calculating "$\frac{1}{2} \times 70$" on the Minicomputer reviews the process of halving on the Minicomputer. This review is in preparation for finding one-half of a number on the Minicomputer when the result is a (non-integer) decimal.

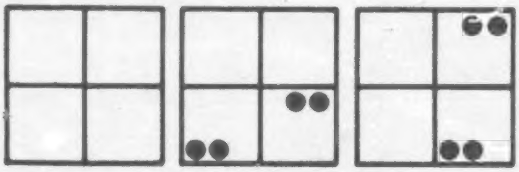
T: Can we use the Minicomputer to find which number is one-half of 70? (Yes)

Ask a student to put 70 on the Minicomputer.

T: How can we find one-half of the number on the Minicomputer?

S: Make some backward plays until all the checkers are in pairs.

Invite students to make plays. Whenever a play results in a pair of checkers on the same square, ask the student who made the play to move those checkers together into a corner of that square. Discourage students from breaking up these pairs. Continue until all the checkers are in pairs.



T: Who can remove some checkers so that one-half of this number will be on the Minicomputer?

A student should remove one checker from each pair. Conclude that $\frac{1}{2} \times 70 = 35$ and write the number story on the board.

$$\frac{1}{2} \times 70 = 35$$

N 25

Clear the Minicomputer and continue with another story.

T : Christy and Laura are also my friends. I want to share $700 between them. How much money should I give Christy? How much money should I give Laura?

Allow several students to make comments. Many students might know that $\frac{1}{2} \times 700 = 350$.

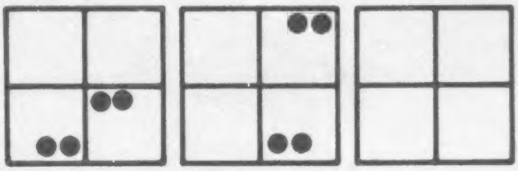
T : Can we use the Minicomputer to find which number is one-half of 700? (Yes)

Ask a student to put 700 on the Minicomputer.

T : What should we do if we want to find one-half of 700 on the Minicomputer?

S : Make backward plays until all the checkers are in pairs.

Let students make plays until all the checkers are in pairs.



T : Who can remove checkers so that one-half of the number will be on the Minicomputer?

The student should remove one checker from each pair. Conclude that $\frac{1}{2} \times 700 = 350$ and write the number story on the board.

$$\frac{1}{2} \times 70 = 35$$

$$\frac{1}{2} \times 700 = 350$$

Clear the Minicomputer and continue with another story.

T: I have two more friends, Jason and Lynn. I want to share $7 between them. How much money should I give Jason? How much money should I give Lynn?

Allow several students to comment. Many students might suggest that each of them be given three and a half dollars.

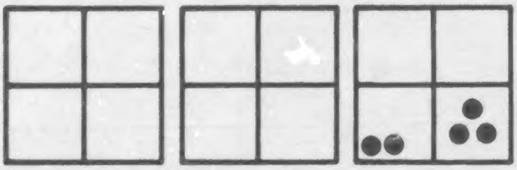
T: How many cents is one-half of a dollar? (Fifty)
Three and one-half dollars is three dollars and fifty cents.

Ask a student to put 7 on the Minicomputer.

T: What should we do if we want to find one-half of 7 on the Minicomputer?

S: Make backward plays until all the checkers are in pairs.

Invite students to make plays until you have this configuration on the Minicomputer.

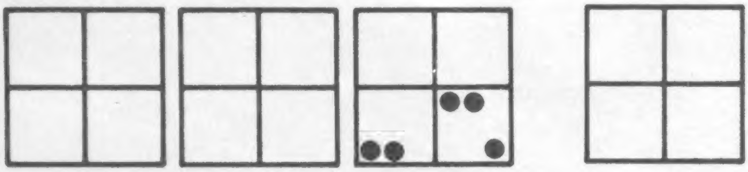


Move two of the checkers on the 1-square into a corner of the square.

T: Now we have two pairs of checkers but we still have one checker without a partner? What should we do now?

N 25

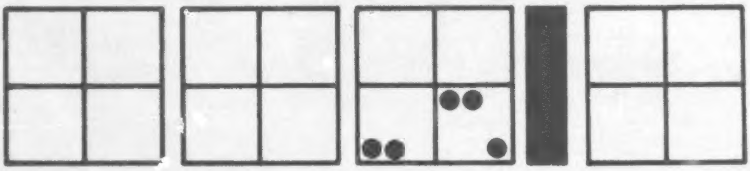
There may be some confusion. Be patient. Encourage the students to express their ideas. In many classes where this question has been asked, at least one student suggested adding another Minicomputer board to the right. If this does not happen, do not worry. Suggest it yourself. Place an extra board to the right of the ones' board, but leave more space than usual.



At least one of your students should object and say that now the number on the Minicomputer is not 7, but rather 70. If no one objects, you should raise the question yourself of whether or not the number on the Minicomputer is still 7.

T: In order not to forget that this number is 7, let's draw a green bar between two of these Minicomputer boards.

Draw a green bar[*] between the Minicomputer boards.



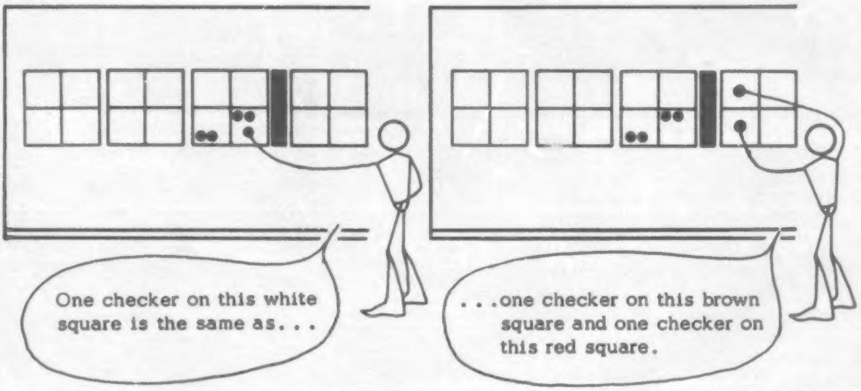
Emphasize that the green bar is there to remind us that the board to the left of the bar is still the ones' board.

T: What should we do now?

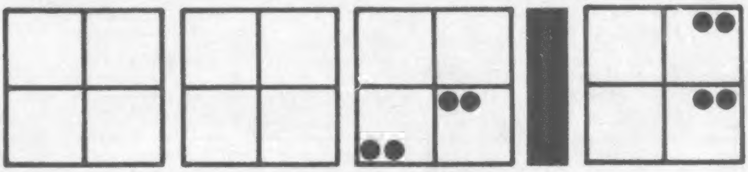
---

[*] The bar should be drawn with green chalk directly on your chalkboard. Do it for this lesson and all subsequent lessons involving the Minicomputer representation of decimals.

Students may be confused and may suggest some incorrect plays. It is natural for the green bar to disturb them. Perhaps you will need to make one play yourself.

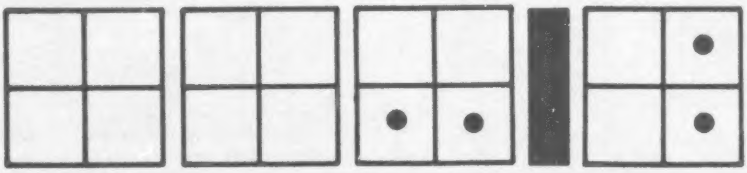

One checker on this white square is the same as. . .
. . .one checker on this brown square and one checker on this red square.

Generally students will make the next two moves without any difficulty.



T: Who can remove some checkers so that one-half of this number will be on the Minicomputer?

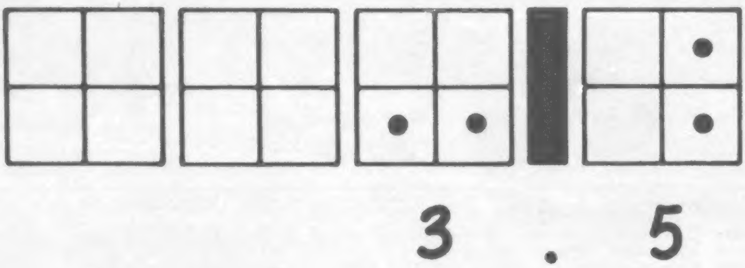
The student who volunteers should remove one checker from each pair.



T: How do you suggest we write this number?

N 25

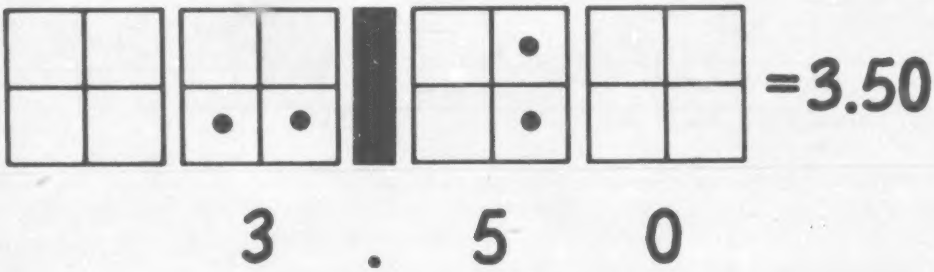
Perhaps a student will suggest using something analogous to the green bar to separate "3" and "5". If no one suggests using a decimal point to give "3.5", suggest this yourself. Many students might be acquainted with the decimal point from seeing prices in stores. Write "3.5" below the Minicomputer.



T: Is this the way people usually write three and one-half dollars? (No)

Without saying anything, remove the Minicomputer board on the far left and place it on the far right. Wait for the reaction of the class.

S: There are no checkers on the new board. We can write "0" below it.



T: 3.5 (read: three point five) is the same as 3.50 (read: three point five zero).

Indicate this equality on the board.

$$3.5 = 3.50$$

T: How do we write three and one-half dollars?

Invite a student to show how to write it on the board.

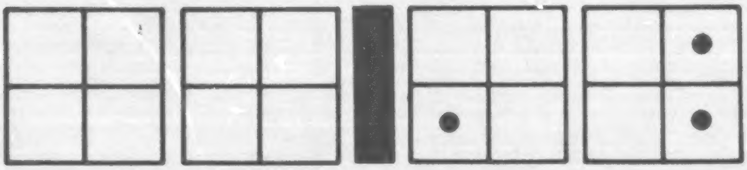
$3.50

T: We know that one-half of $7 is $3.50, so each of my two friends will receive $3.50.

Remove the checkers from the Minicomputer and erase the board.

T: Who can put one quarter on the Minicomputer?
Who can write this number below the Minicomputer?

Be sure the student includes a "0" under the ones' board and a decimal point.



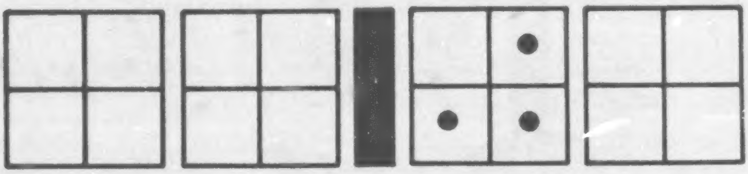
=0.25

0 . 2 5

T: Yes, one quarter is 25 cents. We write "0.25" (read: zero point two five).

Remove the checkers and erase the board.

T: Who can put seven dimes on the Minicomputer?
Who can write this number below the Minicomputer?



=0.70

0 . 7 0

T: Seven dimes is seventy cents and we write "0.70" (read: zero point seven zero). Is there another way to write this number?

If none of the students suggest writing "0.7", remove the pennies' board and repeat the question. If necessary, suggest "0.7" yourself and indicate this equality on the board.

0.70 = 0.7

Point to the board directly left of the green bar (i.e., the ones' board).

T: This board is for dollars.

Point to the board directly right of the green bar.

T: What do you think this board is for?

S: Dimes.

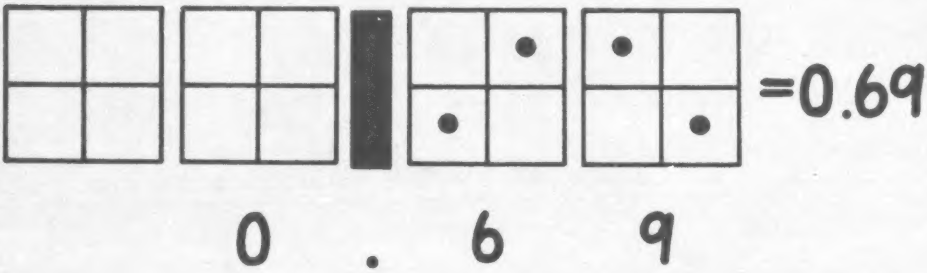
Point to the board to the right of the dimes' board.

T: What do you think this board is for?

S: Pennies.

Remove the checkers and erase the board.

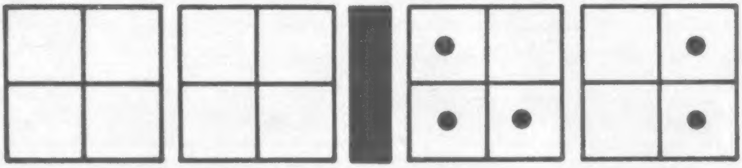
T: Who can put six dimes and nine pennies on the Minicomputer? Who can write this number below the Minicomputer?



T: Six dimes and nine pennies is sixty-nine cents and we write "0.69" (read: zero point six nine).

Remove the checkers and erase the board.

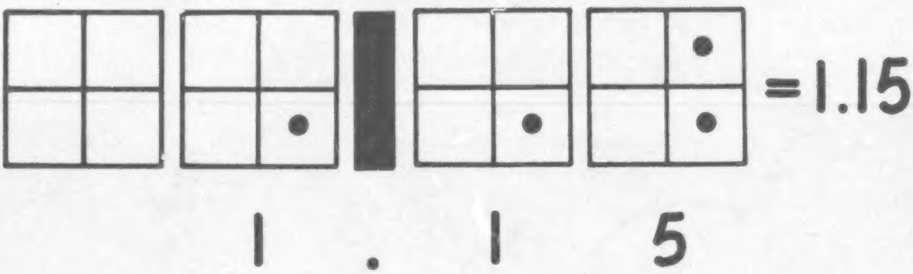
T: Who can put nine dimes on the Minicomputer using blue checkers? Who can put a quarter on the Minicomputer using red checkers?



T: Does anyone see a play we can make?

If students have difficulty verbalizing the play, say "eight dimes and two dimes is one dollar."

T: Who can write this number below the Minicomputer?



T: How much money is nine dimes and one quarter?

S: One dollar and fifteen cents.

T: We write "1.15" (read: one point one five).

N 26

N26 Subtraction and Composition #2

CAPSULE LESSON SUMMARY

Solve subtraction problems by building arrow roads with arrows that compose to give the appropriate subtraction function.

MATERIALS

Teacher: Colored chalk
Student: Colored pencils; paper; pencil

DESCRIPTION OF LESSON

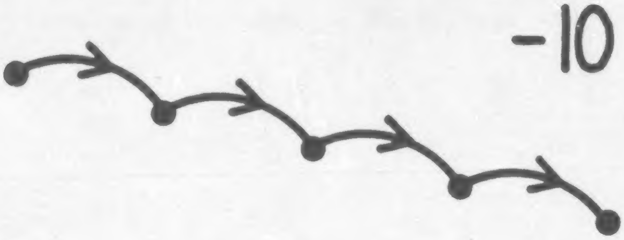
Begin this lesson with several minutes of mental arithmetic activities involving the function -10. Use the following subtraction problems or choose others which are appropriate for the numerical abilities of your students.

20 - 10
21 - 10
60 - 10
62 - 10
30 - 10
35 - 10
38 - 10

100 - 10
102 - 10
109 - 10
200 - 10
205 - 10
1,000 - 10
1,001 - 10

0 - 10
24 - 10
24 - 20
56 - 10
56 - 20
93 - 10
93 - 20

Draw this arrow picture on the board.



T: Where is the smallest number in this arrow picture?

The students should point to the dot on the far right.

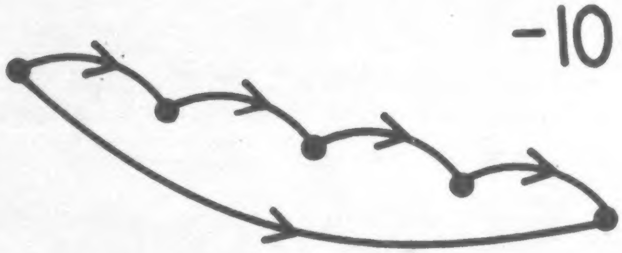
T: How do you know?

S: If you subtract 10 from a number, you get a smaller number.

T: Where is the largest number in this arrow picture?

The students should point to the dot on the far left. Ask a student to explain why that dot is for the largest number.

Draw an arrow from the starting dot to the ending dot.



T: What could this blue arrow be for?

S: -40.

T: How do you know?

S: There are four -10 arrows. 4 × 10 = 40.

Write "-40" in blue on the board near the arrow picture. Label the starting dot 87.

T: If this number were 87, what would the ending number be? Write your answer on a piece of paper.

Ask several students to tell which calculations they did and record them on the board. For example, when a student suggests subtracting 10 four times, record the calculations as follows.

$$\begin{array}{r} 87 \\ -\ 10 \\ \hline 77 \\ -\ 10 \\ \hline 67 \\ -\ 10 \\ \hline 57 \\ -\ 10 \\ \hline 47 \end{array} \qquad \begin{array}{r} 87 \\ -\ 40 \\ \hline 47 \end{array}$$

Conclude that 87 - 40 = 47. Repeat this activity with the following or similar starting numbers.

51	69
113	338

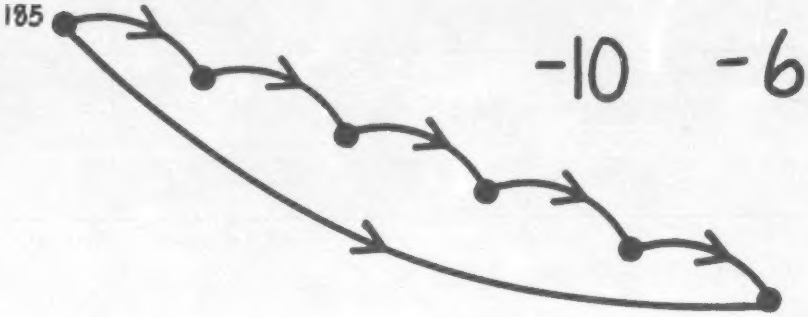
Erase the blue arrow and "- 40" (in blue) and write this problem on the board.

$$\begin{array}{r} 185 \\ -\ 46 \\ \hline \end{array}$$

T: Subtracting 10 four times is the same as subtracting 40. How could we build an arrow road to help us subtract 46 from 185?

Acknowledge any correct suggestions. For example, a student might suggest using nine - 5 arrows and one - 1 arrow. Very likely someone will suggest starting at 185 and building an arrow road with four - 10 arrows and one - 6 arrow. If necessary, suggest this arrow road yourself.

Label the starting dot of the arrow road on the board 185 and add a - 6 arrow. Draw a blue arrow from the starting dot to the ending dot.



T: What could this blue arrow be for?

S: - 46.

Write "- 46" in blue near the arrow picture. Ask the students to calculate "185 - 46" and to write the answer on their papers. Look at many of the students' papers before letting someone answer aloud.

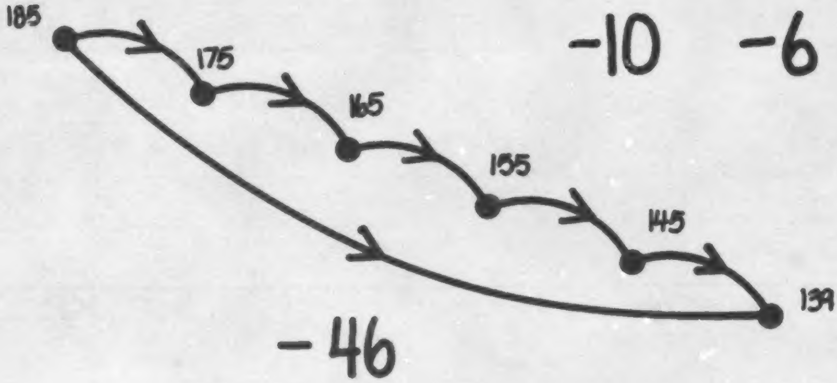
S: 139.

T: Did this arrow picture help any of you to do the calculation? How?

S: The arrow picture shows that to subtract 46 we could subtract 10 four times and then subtract 6.

N 26

Invite a student to label the dots in the arrow picture.



Conclude that 185 - 46 = 139 and record the answer on the board.

$$\begin{array}{r} 185 \\ -\ 46 \\ \hline 139 \end{array}$$

Erase the board and then continue the lesson with this subtraction problem.

$$123 - 35$$

T: How could we build an arrow road to help us solve this problem?

Encourage the students to offer many suggestions. A few of the possible arrow roads are described below. All of the roads start at 123.

- A road with seven - 5 arrows
- A road with three - 10 arrows and one - 5 arrow
- A road with five - 7 arrows

- A road with a -20 arrow, a -10 arrow, and a -5 arrow
- A road with thirty-five -1 arrows

After discussing the problem, let the students build an arrow road to help them calculate "123 - 35". Ask students with different solutions to draw them on the board. Conclude that 123 - 35 = 88 and briefly discuss the various arrow pictures.

Erase the board and repeat this activity with "214 - 121". If necessary, suggest using a -100 arrow yourself.

N 27

N27 Introduction to Decimal Numbers #2

CAPSULE LESSON SUMMARY

Solve addition and subtraction problems which involve decimal numbers and which arise from story situations.

MATERIALS

Teacher: Minicomputer set; colored chalk

Student: Worksheets N27*, **, ***, and ****; pencil

DESCRIPTION OF LESSON

Exercise 1

Tell the following or a similar story to your class. Choose one of your students to star in this story.

T: Anita's grandmother gave her $2 to spend at the school fair. What coins or bills could Anita have if her total amount of spending money for the fair is $2?

Encourage the students to suggest many combinations of coins and bills. A possible dialogue is given here.

S: Anita could have two $1 bills.

S: Anita could have eight quarters.

T: How many cents is a quarter? (25)
What number is 8 × 25? (200)
Eight quarters is 200¢ which is the same as $2.

S: She could have two half-dollars and ten dimes.

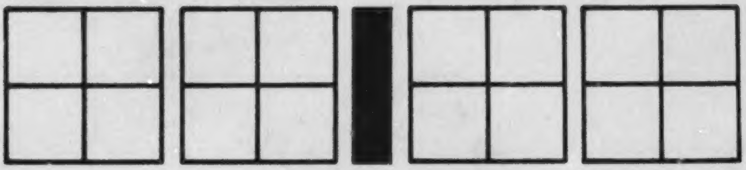
T: How much money is two half-dollars? ($1) 2 × 50 = 100.
How much money is ten dimes? ($1) 10 × 10 = 100.
So two half dollars and ten dimes is $2.

While Anita is at the school fair, she decides to spend 75¢ on the hoop toss and 85¢ on a surprise package.

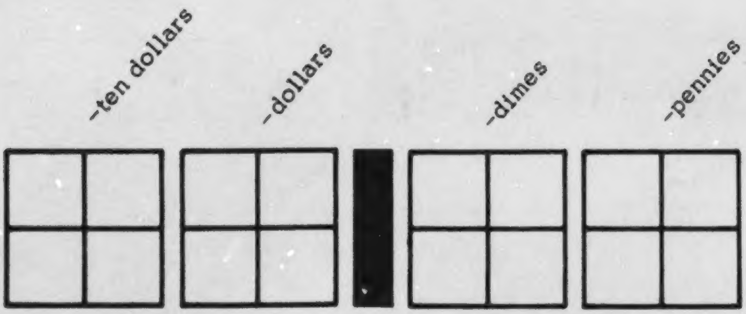
Write "$0.75" and "$0.85" on the board and ask a student to read each of these prices aloud.

T: Does Anita have enough money to spend 75¢ on the hoop toss and 85¢ on a surprise package? (Yes)
How much money would Anita spend? ($1.60)
Let's calculate "$0.75 + $0.85" on the Minicomputer.

Display four Minicomputer boards and draw a green bar between the second and third boards.

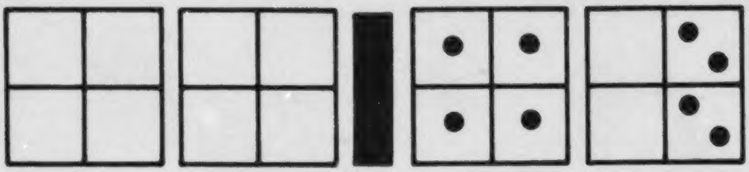


Point to each Minicomputer board and ask what it is for (as indicated in the following illustration).

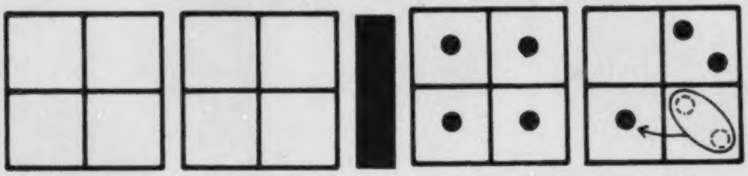


N 27

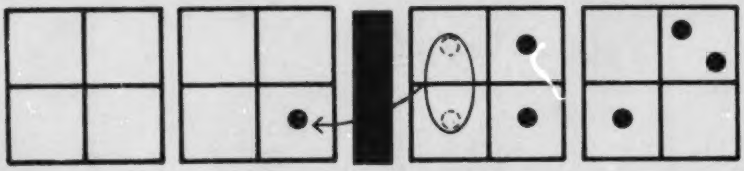
Ask a student to put $0.75 on the Minicomputer and another to put on $0.85.



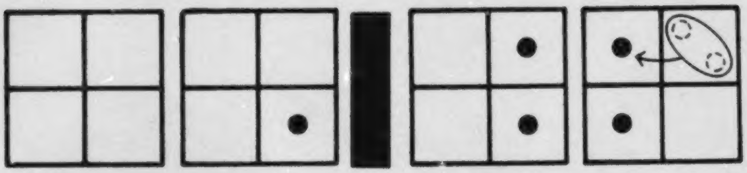
Invite students to make plays. Say the corresponding number stories if the students do not. A possible sequence of plays and dialogue are given here.



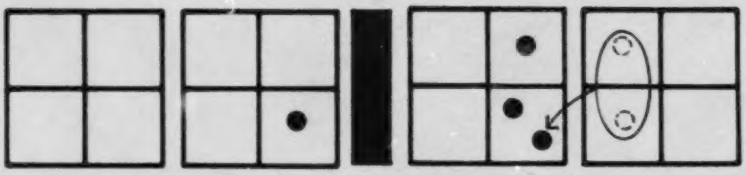
S: One penny plus one penny equals two pennies.



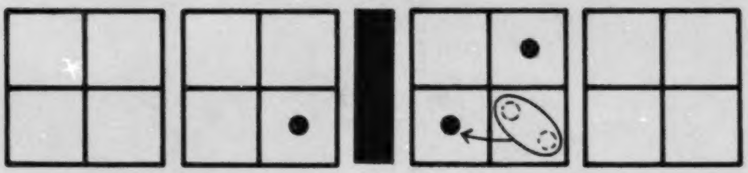
S: Eight dimes plus two dimes equals one dollar.



S: Four pennies plus four pennies equals eight pennies.

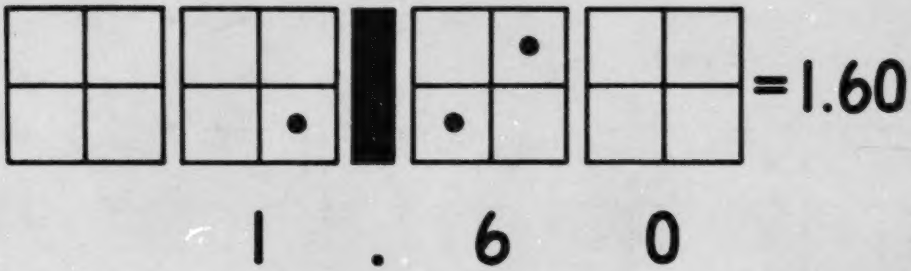


S: Eight pennies plus two pennies equals one dime.



S: One dime plus one dime equals two dimes.

Ask a student to write the number below the Minicomputer.



T: We calculated "\$0.75 + \$0.85" on the Minicomputer and the answer is "\$1.60" as many of you predicted. We could also have calculated the answer another way.

Write the corresponding addition problem on the board and invite a student to solve it. You may need to put in the decimal point yourself.

$$\begin{array}{r} \overset{1}{0}.\overset{1}{7}5 \\ +\,0.85 \\ \hline 1.60 \end{array}$$

N 27

T: Anita took $2 with her to the school fair. After she spends $1.60, how much money will she have left?

S: 40¢.

Write the corresponding subtraction problem on the board.

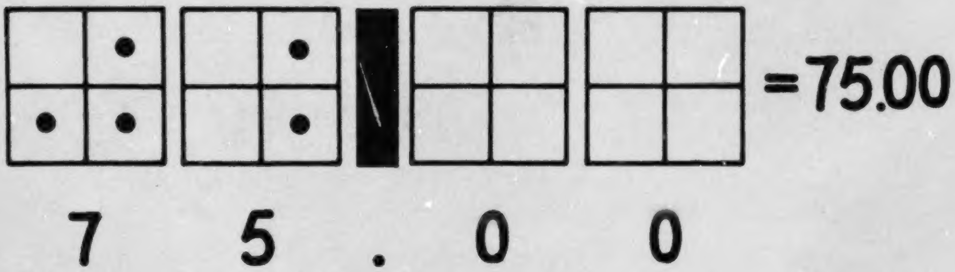
$$\begin{array}{r} 2.00 \\ -\ 1.60 \\ \hline 0.40 \end{array}$$

T: $2 - $1.60 = $0.40.

Erase the board and remove the checkers from the Minicomputer.

Exercise 2

Ask a student to put $75 on the Minicomputer and another to write the number below the Minicomputer.



T: Is there another way to write this number?

When students suggest, "75" and "75.0", write them on the board. If necessary, suggest them yourself.

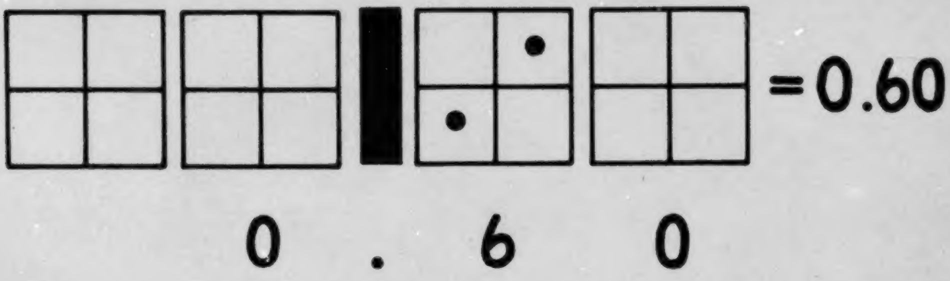
$$75 = 75.0 = 75.00$$

Remove the checkers and erase the board. Ask students to put the following amounts of money on the Minicomputer one at a time and to write the appropriate number below the Minicomputer each time.

$1.06 $8.30

$20.10 $40.09

Ask a student to put six dimes on the Minicomputer and another to write the number below the Minicomputer.



T (pointing to "0.60"): Is there another way we could write this number?

If no one suggests writing "0.6", remove the pennies' board and repeat the question. If necessary, tell the students that "0.6" is another way to write this number. Write this equality on the board.

$$0.60 = 0.6$$

Of course, someone could suggest writing ".6" or ".60"; that is, writing nothing to the left of the decimal point. However, you should not write the number this way on the board.

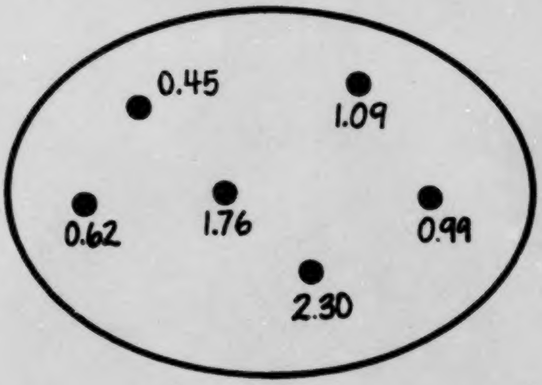
Exercise 3

Tell the following or a similar story to your class. Choose one of your students to star in this story.

N 27

T: Tony decides to spend some of his savings to buy some new ornaments for his family's Christmas tree. Tony finds six ornaments he would like to buy.

Ask the student who is the star of this story to describe six ornaments. After each ornament is described, assign it one of the prices in the string picture below, and record the corresponding number inside a string on the board. Continue this activity until six ornaments have been chosen and you have this string picture on the board.



T: Which ornament is the most expensive?

S: The ornament that costs $2.30.

T: Which ornament is the least expensive?

S: The ornament that costs $0.45.

T: Do you think the bill for all six ornaments will be more than $5? Why?

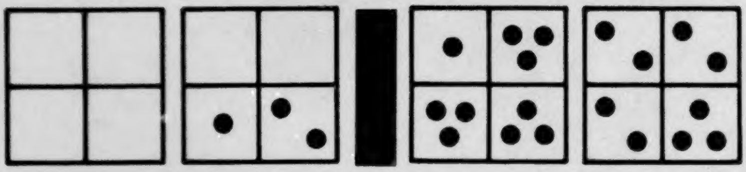
A student might indicate that the three (or four) most expensive ornaments cost more than $5.

T: Do you think the bill will be more than $10? Why?

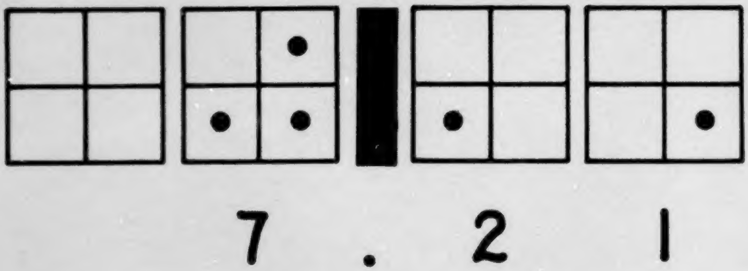
A student might indicate that three of the ornaments cost less than $1 each and the other three ornaments cost about $5, so altogether the bill should be less than $10. Conclude that the price of all six ornaments is between $5 and $10. Invite students to guess what the bill will be and record guesses between $5 and $10 on the board.

T: Let's use the Minicomputer to determine exactly how much Tony will have to pay.

Invite students to put the prices of the ornaments on the Minicomputer. You may find it convenient to move all the checkers to the corners of squares after a number is put on the Minicomputer to avoid confusion when the next number is put on. After all the numbers have been put on the Minicomputer, you will have this configuration (assuming standard configurations were used for all the numbers).



Invite students to make plays. Say the corresponding number stories if your students do not. Continue until the standard configuration for 7.21 is obtained. Ask a student to write the number below the Minicomputer.



T: How much will Tony have to pay for the ornaments?

N 27

S: $7.21.

T: Could we have solved this problem another way?

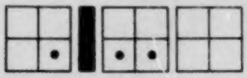
Very likely someone will suggest using the addition algorithm. Write the corresponding addition problem on the board and emphasize that all the decimal points must be lined up so you will be adding the pennies together, the dimes together, and the dollars together. Invite a student to solve the problem.

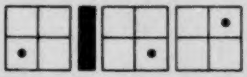
$$\begin{array}{r}
{}^{3}\;\,{}^{3}\;\;\; \\
0.62 \\
0.45 \\
1.76 \\
1.09 \\
0.99 \\
+\ 2.30 \\
\hline
7.21
\end{array}$$

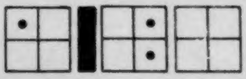
Ask the class to determine which guess was the closest to $7.21. Worksheets N27*, **, ***, and **** are available for individual work.

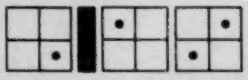
Name ____________________ **N 27 ***

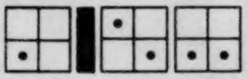
What number is on the Minicomputer?

 = 1.3

 = 2.14

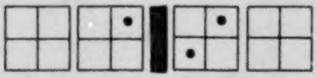
 = 8.5

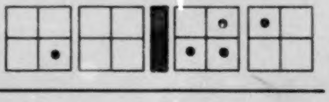
 = 1.86

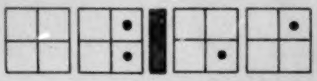
 = 2.93

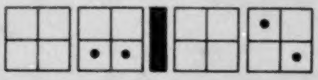
Name ____________________ **N 27 ****

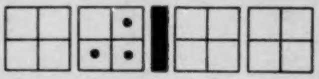
Put these numbers on the Minicomputer.

4.60 = 

10.78 = 

5.14 = 

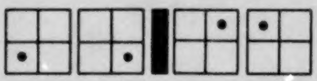
3.09 = 

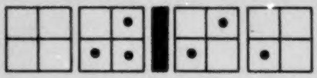
7.0 = 

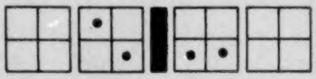
Standard configurations are given here; however, many solutions are possible.

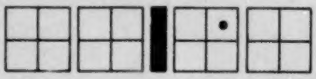
Name ____________________ **N 27 *****

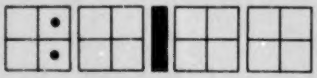
Put these numbers on the Minicomputer.

21.48 = 

7.62 = 

9.3 = 

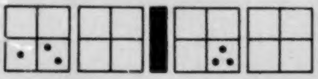
0.4 = 

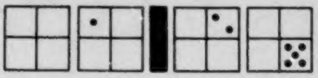
50 = 

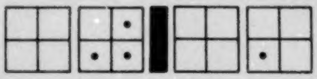
Standard configurations are given here; however, many solutions are possible.

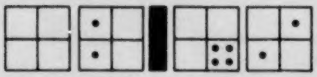
Name ____________________ **N 27 ******

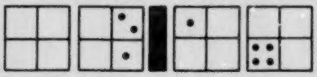
What number is on the Minicomputer?

 = 40.3

 = 8.85

 = 7.02

 = 10.46

 = 9.88

N28 A 10 x and + 1 Arrow Road

CAPSULE LESSON SUMMARY

Use ⑩-checkers to make 10 x calculations on the Minicomputer. Build a road from 0 to 52 using 10 x and +1 arrows. Discuss where to use a 10 x arrow if a shortest road is desired. Ask how to extend the road to 523.

MATERIALS

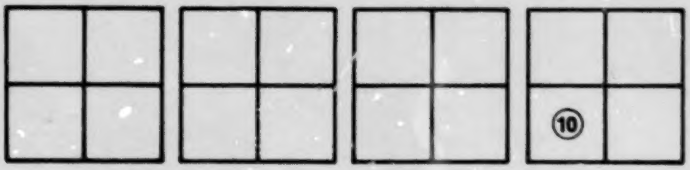
Teacher: Minicomputer set; ⑩-checkers; colored chalk
Student: Paper; pencil; colored pencils

DESCRIPTION OF LESSON

Exercise 1: 10 x on the Minicomputer

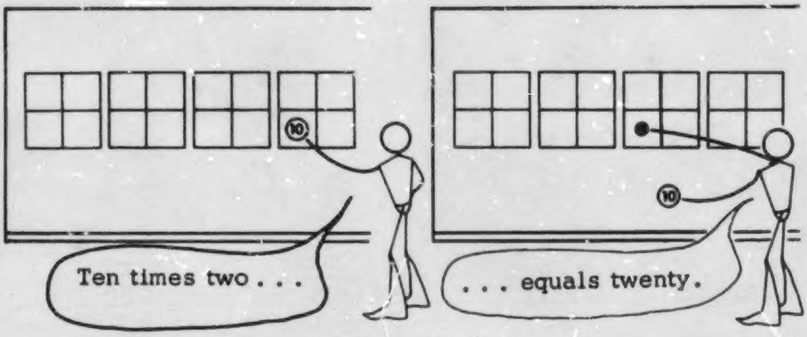
T: 10 x 3 = . . . ? (30)
10 x 5 = . . . ? (50)
10 x 9 = . . . ? (90)
10 x 0 = . . . ? (0)
10 x 10 = . . . ? (100)
10 x 2 = . . . ? (20)

Put a ⑩-checker on the 2-square of the Minicomputer.



T: Ten checkers on this red square (point to the 2-square) is the same as one checker on this red square (point to the 20-square).

Make a "10 × 2 = 20" play.



Write "10 × 2 = 20" on the board and remove the checker from the Minicomputer.

T: What number is 10 × 20?

S: 200.

Write this number story on the board.

$$10 \times 2 = 20$$
$$10 \times 20 = 200$$

Put a ⑩-checker on the 20-square of the Minicomputer and make a "10 × 20 = 200" play. Remove the checker from the Minicomputer.

T: What number is 10 × 200?

S: 2,000.

N 28

Write this number story on the board.

10 × 2 = 20
10 × 20 = 200
10 × 200 = 2,000

Put a ⑩-checker on the 200-square of the Minicomputer and make a "10 × 200 = 2,000" play.

T: Do you see any patterns?

Encourage many different responses. If any of your students suggest that when you multiply by 10 you simply add a "0" to the right of the numeral, reinforce this idea but do not suggest it yourself.

NOTE: Adding a "0" to the right of the numeral works only when an integer is multiplied by 10. For example, 10 × 7.9 is not 7.90.

Remove the checker from the Minicomputer and continue this sequence of calculations with "10 × 2,000".

10 × 2 = 20
10 × 20 = 200
10 × 200 = 2,000
10 × 2,000 =

T: What number is 10 × 2,000?

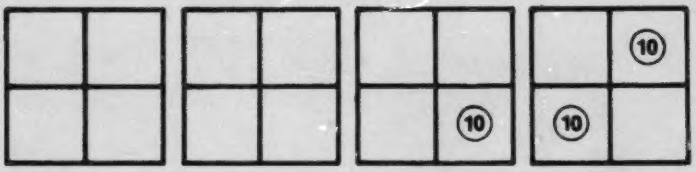
S: 20,000.

If none of your students answers correctly, ask if someone can write the answer on the board. Then let another student read the number aloud.

$$\begin{aligned} 10 \times 2 &= 20 \\ 10 \times 20 &= 200 \\ 10 \times 200 &= 2{,}000 \\ 10 \times 2{,}000 &= 20{,}000 \end{aligned}$$

Continue this sequence with "10 x 20,000 = 200,000".

Erase the board and then put this configuration on the Minicomputer.



T (pointing to the tens' board): What number is 10 x 10?

S: 100.

Write "10 x 10 = 100" on the board.

T (pointing to the ones' board): What number is 10 x 6?

S: 60.

Write "10 x 6 = 60" and "10 x 16" on the board.

$$\begin{aligned} 10 \times 10 &= 100 \\ 10 \times 6 &= 60 \\ 10 \times 16 & \end{aligned}$$

N 28

T: The number on the Minicomputer is 10 × 16. What number is this?

S: 160.

T: How do you know?

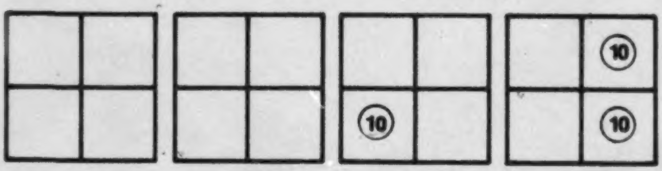
Encourage the students to express themselves. Some might have calculated "100 + 60". Commend any student who thought of writing a "0" to the right of "16".

Complete the number story.

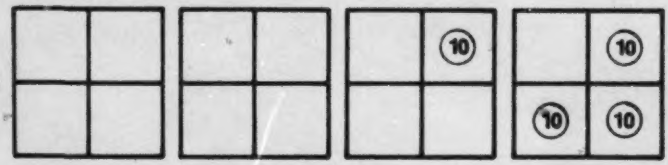
$$10 \times 10 = 100$$
$$10 \times 6 = 60$$
$$10 \times 16 = 160$$

Repeat this activity with the following calculations.

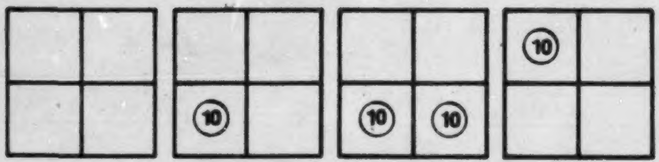
10 × 25 =



10 × 47 =



10 × 238 =

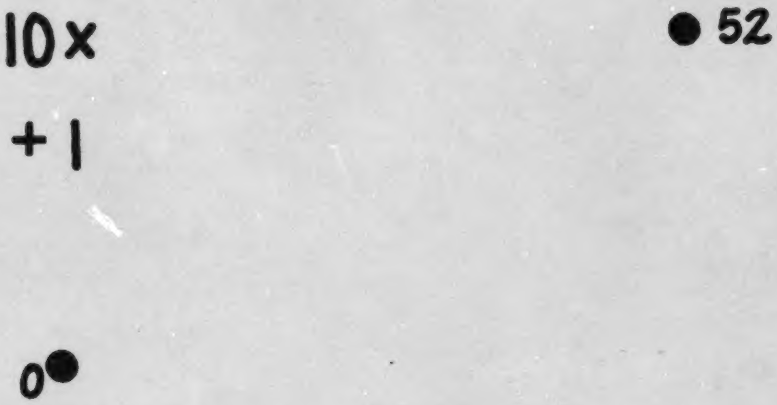


If the students have difficulty with any of these calculations, ask them to make the appropriate plays to determine which number is on the Minicomputer.

Erase the board and take away the Minicomputer.

Exercise 2: A 10 x and +1 Arrow Road

Write this information on the board.



T: Do you think we can build a road from 0 to 52 with only 10 x and +1 arrows?

S: Sure, we could use all +1 arrows.

T: How many +1 arrows would we need to go from 0 to 52?

S: Fifty-two.

T: That would be a very long road. Let's try to find a shorter one.

N 28

Begin a road from 0 to 52 with the class. Follow the suggestions of your students; a sample dialogue is given here.

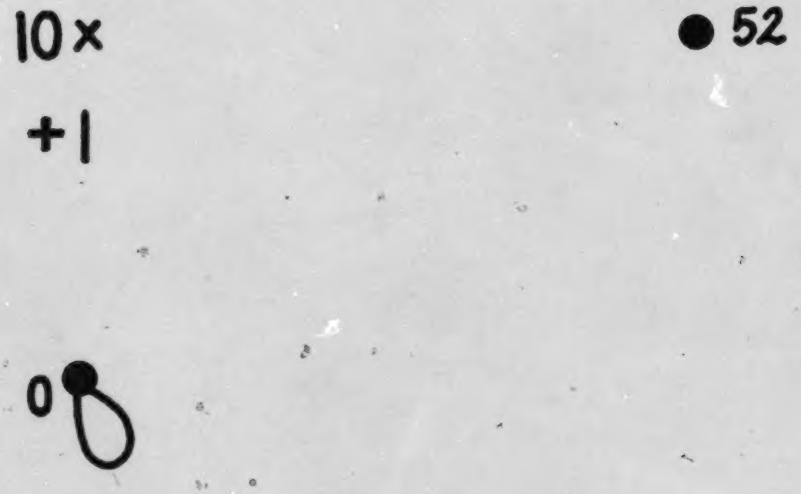
T: We start our road at 0. Do you want the first arrow to be a 10 × arrow or a +1 arrow?

S: A 10 × arrow.

T: What number is 10 × 0?

S: 0.

Draw a red loop at 0.



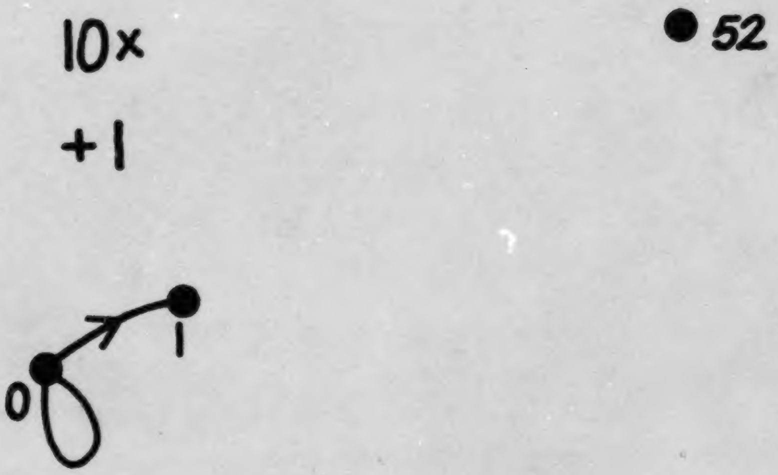
S: Draw a blue arrow starting at 0.

T: What are the blue arrows for?

S: +1.

T: What number is 0 + 1?

S: 1.



S: Now draw a red arrow.

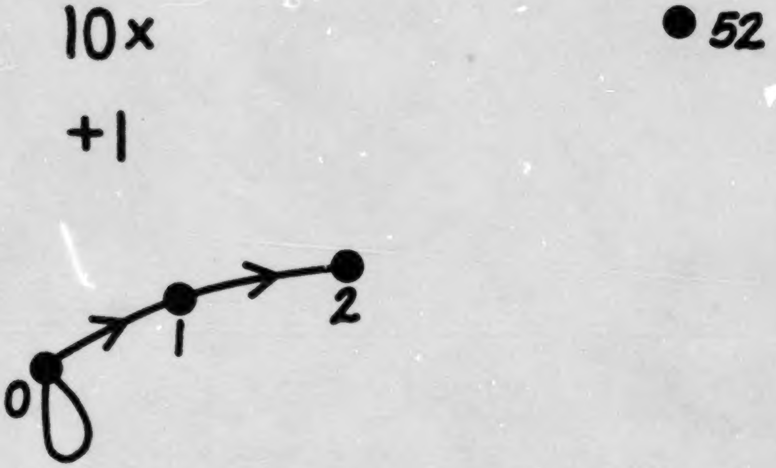
T: Would that help us to build a road to 52?

S: 10 × 1 = 10, so we would be getting closer to 52.

S: After we get to 10 we won't be able to use any more red arrows, because 10 × 10 = 100 and 100 is more than 52.

S: We should draw another blue arrow.

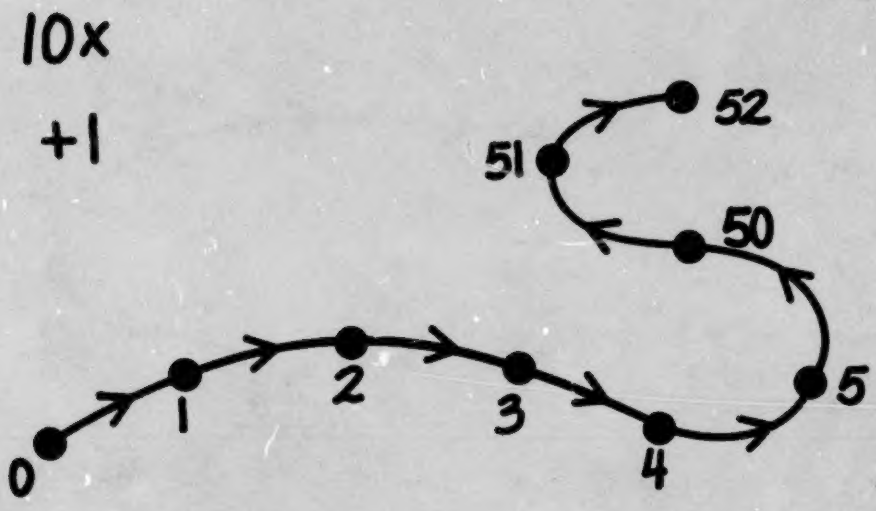
N 28



If your students want to draw a 10x arrow before they reach 5, ask them to look ahead and try to see what would happen if they drew a 10x arrow at that time. When your class decides that a shortest road has blue arrows until it reaches 5, ask them to build a road themselves on their papers.

Allow the students to work independently to build a road from 0 to 52 (permitting if they wish roads which are longer than necessary). A student might use only +1 arrows, although the road would be quite long. Ask students who finish quickly to extend their roads to 523 and then, if there is sufficient time, to 5,231.

After a few minutes, ask a student with a short road from 0 to 52 to copy it on the board.



Discuss this road collectively.

T: How many red arrows are in this road? (One)

How many blue arrows . . . ? (Seven)

How many arrows are there altogether? (Eight)

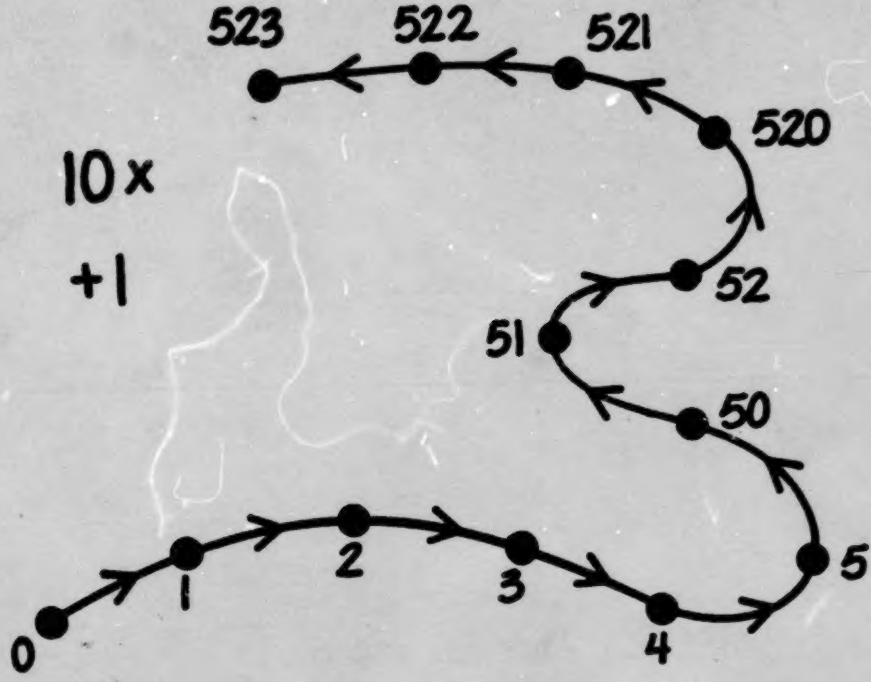
Did any one build a road from 0 to 52 with less than eight arrows?

The shortest possible 10× and +1 arrow road from 0 to 52 has eight arrows.

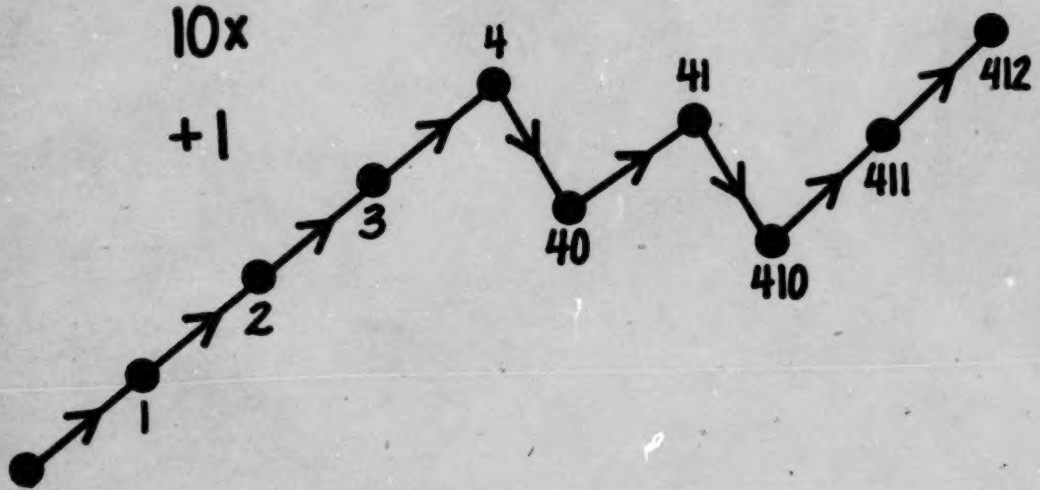
Draw a dot for 523 on the board near the arrow picture.

T: We could extend this road until it meets 523 using +1 arrows, but is there a shorter road?

Several students will have already solved this problem. Encourage them to explain their solutions to the class. Invite a student to extend the arrow road to 523 on the board.


523
522
521
520
10 x
+1
52
51
50
0
1
2
3
4
5

If there is time remaining, ask the students to build a 10 x and +1 arrow road from 0 to 412. A shortest such road has nine arrows, but do not insist that all students find shortest roads.


10x
+1
0
1
2
3
4
40
41
410
411
412

N29 Multiplication and Addition Problems #2

CAPSULE LESSON SUMMARY

Represent 39 on the Minicomputer as "$40 + \hat{1}$". Then double 39 by doubling the checkers in this configuration. Conclude that $2 \times 39 = 80 - 2 = 78$. Use the same type of strategy to do other multiplication calculations. Given the answer to an addition calculation and one of the addends, find the other addend.

MATERIALS

Teacher: Minicomputer set
Student: Paper; pencil

DESCRIPTION OF LESSON

Exercise 1: Multiplication Problems

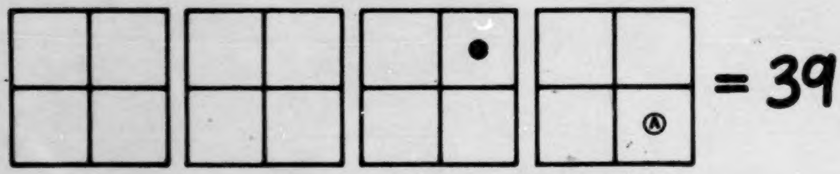
Ask each student to be prepared with a pencil and a piece of paper for doing some calculations.

T: What number is 2 × 40? (80)
$2 \times 40 = 80$, so what number is 2×39? (78)
How do you know?

Ask several students to explain how they calculated "2 × 39". Commend any student who subtracted "80 - 2". Write this number story on the board.

$$39 = 40 - 1$$

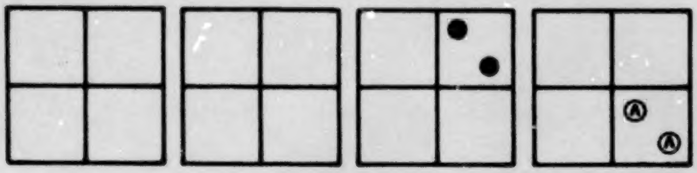
T: $39 = 40 - 1$. Who can put 39 on the Minicomputer using exactly two checkers?



N 29

T: This number is 39. We want to calculate "2 × 39".

Ask a student to put the same configuration for 39 on the Minicomputer again.



= 2 × 39

T: What number is on the tens' board?

S: 80.

T: What number is on the ones' board?

S: $\hat{2}$.

T: So what number is on the Minicomputer?

S: 78.

T: How do you know?

S: $80 + \hat{2} = 78$.

S: $80 - 2 = 78$.

Record the subtraction problem and the answer.

$$2 \times 39 = 80 - 2 = 78$$

312

Remove the checkers from the Minicomputer and erase the board.

T: What number is 2 × 50? (100)
2 × 50 = 100; so what number is 2 × 49? (98)
How do you know?

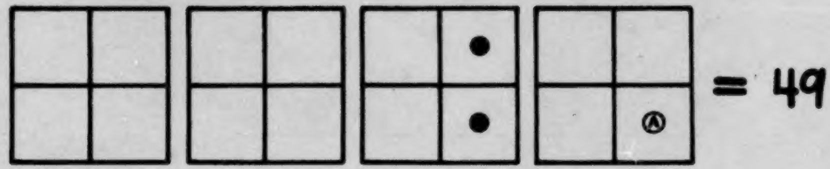
Encourage students to explain which calculations they did; when appropriate, record the calculations on the board. Commend any student who suggests calculating "100 - 2".

Write this number story on the board.

49 = 50 - 1

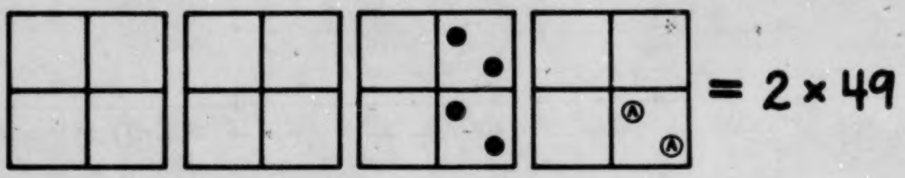
T: 49 = 50 - 1.

Ask a student to put 50 + $\hat{1}$ on the Minicomputer.



T: This number is 49. We want to calculate "2 × 49".

Ask a student to put the same configuration for 49 on the Minicomputer again.



T: What number is on the tens' board?

S: 100.

T: What number is on the ones' board?

S: $\hat{2}$.

T: What number is on the Minicomputer?

S: 98.

T: How do you know?

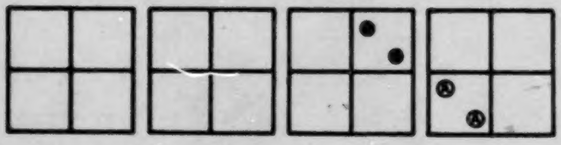
S: 100 + $\hat{2}$ = 98.

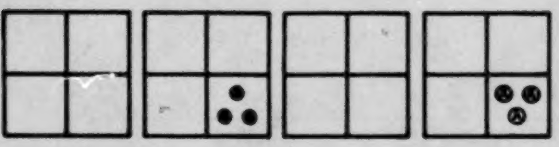
S: 100 - 2 = 98.

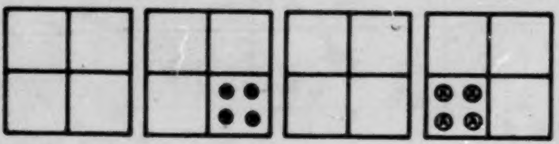
Record the subtraction problem and the answer.

$$2 \times 49 = 100 - 2 = 98$$

Remove the checkers and erase the board. Repeat this activity with the following multiplication problems.

2 × 38 =  = 80 − 4 = 76

3 × 99 =  = 300 − 3 = 297

4 × 98 =  = 400 − 8 = 392

COMMENTS: Calculating "3 × 99" and "4 × 98" provide an excellent opportunity to review the addition algorithm.

$$\begin{array}{r} 98 \\ 98 \\ 98 \\ +98 \\ \hline \end{array}$$

Encourage the use of multiplication facts when you review the addition algorithm. For example, if you were solving the addition problem shown above, you would encourage the students to think "4 × 8 = 32" instead of "8 + 8 = 16, 16 + 8 = 24, and 24 + 8 = 32".

Erase the board and take away the Minicomputer.

Exercise 2: Addition Problems

Remind the students of the addition puzzles they did in an earlier lesson. Write this problem on the board.

$$\begin{array}{r} 403 \\ + \\ \hline 698 \end{array}$$

T: 403 plus a mystery number is 698. Find the missing number.

Encourage the students to work individually. After a short period of time, ask a student with the correct answer to complete the addition problem at the board.

$$\begin{array}{r} 403 \\ +295 \\ \hline 698 \end{array}$$

N 29

Check the addition collectively to show that 403 + 295 = 698.

Continue this activity with a similar problem.

$$\begin{array}{r} 467 \\ \underline{+} \\ 892 \end{array}$$

Some of your students might think that the missing number is 435. In this case, write "435" in the space and calculate "467 + 435" to show that 435 is not correct.

Ask a student with the correct answer to complete the addition problem at the board showing that 425 is correct. Emphasize that 7 + 5 = 12, so there is 1 more ten to add in the tens' column.

$$\begin{array}{r} 467 \\ \underline{+\,425} \\ 892 \end{array}$$

Continue this activity with another addition problem.

$$\begin{array}{r} 2{,}348 \\ \underline{+\phantom{0{,}000}} \\ 5{,}891 \end{array}$$

Copy the following problems on the board and ask the students to find the missing numbers. Answers are in the boxes.

$$\begin{array}{r} 36 \\ \underline{+\boxed{36}} \\ 72 \end{array} \qquad \begin{array}{r} 238 \\ \underline{+\boxed{551}} \\ 789 \end{array} \qquad \begin{array}{r} 425 \\ \underline{+\boxed{227}} \\ 652 \end{array} \qquad \begin{array}{r} 530 \\ \underline{+\boxed{59}} \\ 589 \end{array}$$

If some students finish quickly, you may wish to challenge them with some more difficult problems.

$$\begin{array}{r} 250 \\ + \boxed{54} \\ \hline 304 \end{array} \qquad \begin{array}{r} 592 \\ + \boxed{356} \\ \hline 948 \end{array} \qquad \begin{array}{r} 3{,}457 \\ + \boxed{1{,}627} \\ \hline 5{,}084 \end{array} \qquad \begin{array}{r} 1{,}576 \\ + \boxed{725} \\ \hline 2{,}301 \end{array}$$

N30 Subtraction and Composition #3

CAPSULE LESSON SUMMARY

Decide that the composite of three -10 and a -9 arrow is a -39 arrow. Do subtraction calculations which involve the function -39. Use arrow pictures to describe various methods of doing these calculations and other subtraction calculations such as "74 - 28" and "111 - 43".

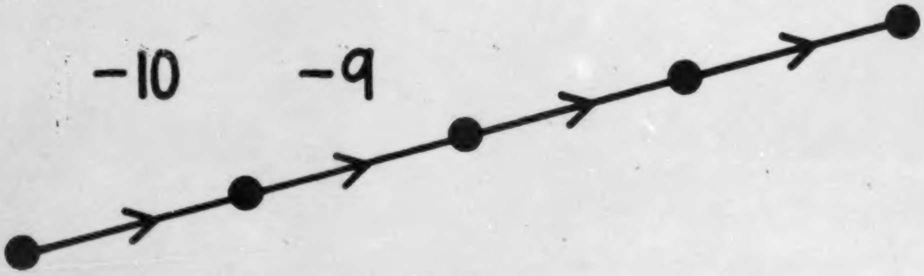
MATERIALS

Teacher: Colored chalk
Student: Paper; pencil; colored pencils

DESCRIPTION OF LESSON

Begin this lesson with five minutes of mental arithmetic activities involving the functions -10 and -20.

Draw this arrow picture on the board.



T: Where is the smallest number in this arrow picture?

The students should point to the dot on the far right.

T: Where is the largest number in this arrow picture?

The students should point to the dot on the far left.

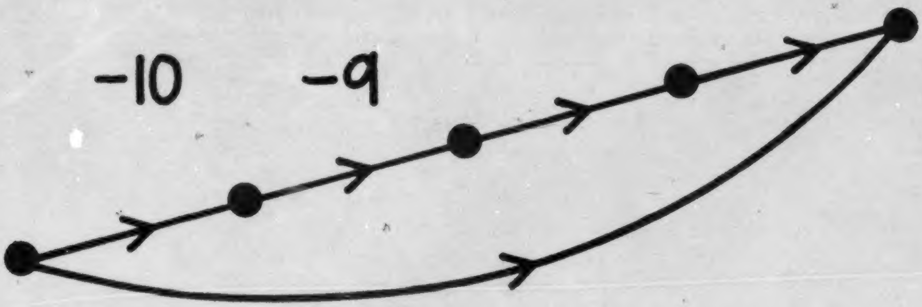
T: How do you know where the smallest and largest numbers are in this arrow picture?

The students should explain that when you subtract 10 or 9 from a number you get a smaller number.

T (pointing to the dot on the far right): How much smaller is this number than the starting number?

S: 39.

Draw a green arrow from the starting dot to the ending dot.



T: What could this green arrow be for?

S: -39.

T: How do you know?

S: Subtracting 10 three times is the same as subtracting 30, and then subtracting 9 more is the same as subtracting 39.

Write "-39" in green near the arrow picture.

N 30

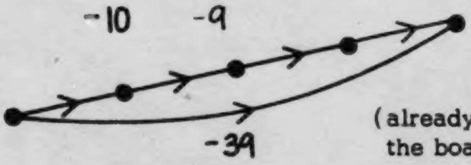
Ask students to do calculations which involve the function - 39 and to explain their methods. Use the following problems or choose similar ones.

90 - 39
93 - 39
100 - 39
40 - 39
42 - 39
0 - 39
81 - 39
67 - 39
158 - 39

When methods are suggested, draw the corresponding arrow pictures on the board. Do not label the dots. Each arrow picture should describe an approach to subtracting 39 which can be used to do any of the preceding calculations.

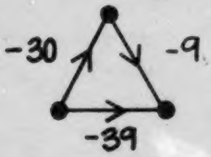
The following arrow pictures describe methods which students often give for subtracting 39.

Subtract 10 three times and then subtract 9.

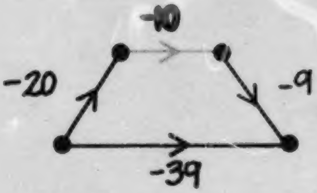


(already on the board)

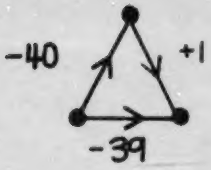
Subtract 30 and then subtract 9.



Subtract 20, then subtract 10, and then subtract 9.



Subtract 40 and then add 1.



Mention the method of subtracting 40 and then adding 1 if none of your students suggests it.

Erase the board and write this subtraction problem on the board.

$$74 - 28$$

Instruct the students to solve this problem individually and to draw arrow pictures to explain their solutions. Ask students with distinct solutions to draw them on the board. Briefly discuss the various arrow pictures and conclude that 74 - 28 = 46.

Erase the board and repeat this activity asking your students to calculate "111 - 43". If some students finish quickly, ask them to calculate "242 - 157" and to show their methods in arrow pictures.

N 31

N31 Road Map of Virginia

CAPSULE LESSON SUMMARY

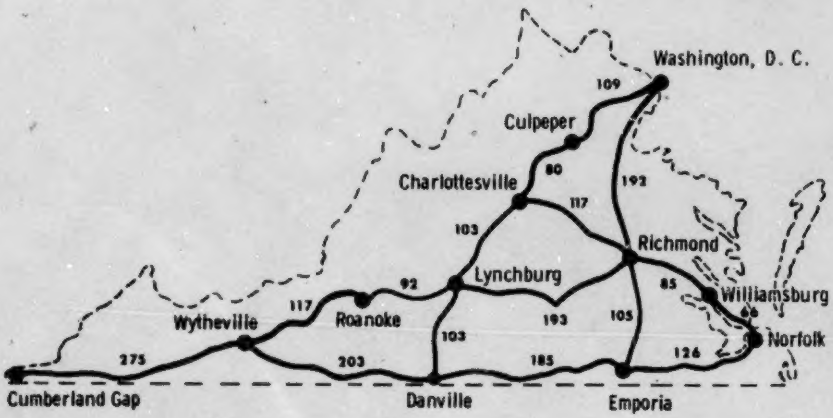
Find shortest routes between cities in Virginia. Decide which of two cities is closer to a third. Plan a trip which starts at a particular city and which is approximately 500 km long.

MATERIALS

Teacher: World of Numbers Poster #2 for UPG-III; tape
Student: Worksheet N31; paper; pencil

DESCRIPTION OF LESSON

Distribute copies of Worksheet N31 and display World of Numbers Poster #2.


Washington, D. C.
109
Culpeper
80
192
Charlottesville
117
103
Richmond
Lynchburg
92
85
Williamsburg
66
117
Wytheville
Roanoke
193
105
103
Norfolk
275
203
185
126
Cumberland Gap
Danville
Emporia

NOTE: If you and your students are in Virginia, adjust the following dialogue so that it is appropriate.

T: This is a map of one of the states of the United States of America. What is the name of this state? Do you recognize any of these cities?

You may need to tell your class that this state is Virginia. Encourage the students to share what they know about Virginia and the cities indicated on this map. Tell the class that the numbers show how far it is in kilometers from one city to another.

T: How far is it from Danville to Lynchburg (trace the appropriate road) according to this map? (103 kilometers)

. . . from Culpeper to Charlottesville? (80 kilometers)

. . . from Wytheville to Cumberland Gap? (275 kilometers)

Problem 1: Charlottesville to Williamsburg

Abbreviate the problem by writing this expression on the board.

Charlottesville to Williamsburg?

T: What is the shortest route from Charlottesville to Williamsburg?

S: Go from Charlottesville to Richmond and then on to Williamsburg.

Ask a student to trace this route on the poster; then record it on the board.

Charlottesville to Williamsburg?
Charlottesville to Richmond to Williamsburg is ___ km.

T: How long is this route from Charlottesville to Williamsburg? Write your answer on a piece of paper.

Look at many of the students' papers and help students who are having difficulty. Ask a student to answer aloud and complete the sentence on the board.

Charlottesville to Williamsburg?
Charlottesville to Richmond to Williamsburg is 202 km.

T: How did you calculate the distance from Charlottesville to Williamsburg?

S: I calculated "117 + 85".

Write this addition problem on the board.

$$\begin{array}{r} 117 \\ +\ 85 \\ \hline \end{array}$$

Ask for a volunteer to solve this problem at the board and to explain each step aloud.

T: Is this the shortest route from Charlottesville to Williamsburg? How do you know?

Let students comment on why they believe this to be the shortest route, but do not expect well-formed answers involving calculations.

Erase the board before going on to Problem 2.

Problem 2: Williamsburg to Emporia

Abbreviate the problem on the board.

Williamsburg to Emporia?

T: What is the shortest route from Williamsburg to Emporia?

Some students might think that the route through Richmond is the shortest, while other students might claim that the route through Norfolk is shorter. Ask students to trace these routes on the poster, and record them on the board. Encourage the students to calculate the lengths of both routes so tnat their lengths can be compared.

Williamsburg to Emporia?
Williamsburg to Richmond to Emporia is ____ km.
Williamsburg to Norfolk to Emporia is ____ km.

Allow a few minutes for individual work; then invite two students to solve the appropriate problems at the board.

$$\begin{array}{r} \overset{1}{8}5 \\ +105 \\ \hline 190 \end{array} \qquad \begin{array}{r} \overset{1}{6}6 \\ +126 \\ \hline 192 \end{array}$$

Complete the sentences on the board and conclude that the shortest distance from Williamsburg to Emporia is 190 km.

Williamsburg to Emporia?
Williamsburg to Richmond to Emporia is 190 km.
Williamsburg to Norfolk to Emporia is 192 km.

T: How much longer is the route from Williamsburg to Emporia through Norfolk then the route through Richmond?

S: 2 km.

Erase the board before going on to Problem 3.

Problem 3: Wytheville to Washington, D.C. vs Wytheville to Norfolk

T: Which city is closer to Wytheville — Washington, D.C. or Norfolk?

On the board abbreviate your question as follows.

Wytheville to Washington, D.C.?

Wytheville to Norfolk?

The students very likely will not be sure which city is closer to Wytheville, so ask them to calculate both distances. Allow a few minutes for independent work; then ask the students to describe the shortest routes from Wytheville to Washington, D.C. and from Wytheville to Norfolk. Ask for the length of each of these routes. Record the information on the board.

Wytheville to Washington, D.C.?
Wytheville to Roanoke to Lynchburg to Charlottesville
to Culpeper to Washington, D.C. is 501 km.

Wytheville to Norfolk?
Wytheville to Danville to Emporia to Norfolk is 514 km.

T: Which distance is shorter?

S: 501 km is less than 514 km.

$$501 < 514$$

T: Is Washington, D.C. or Norfolk closer to Wytheville?

S: Washington, D.C.

T: How much farther is it from Wytheville to Norfolk than it is from Wytheville to Washington, D.C.?

S: 13 kilometers.

Erase the board before going on to Problem 4.

Problem 4: A trip starting at Lynchburg

T: I want everyone to start at Lynchburg and to plan a trip which is approximately 500 km long. Try to make the length of your trip as close to 500 km as possible. You may end your trip in any of these cities.

After allowing several minutes for independent work, record one of the students' trips on the board. Try to choose a trip which is at least 550 km long for the collective discussion. An example is given here.

T: Elizabeth has planned a trip from Lynchburg to Richmond to Emporia to Danville and back to Lynchburg.

Lynchburg to Richmond to Emporia to Danville to Lynchburg is ___ km.

T: Did anyone else plan the same trip? How long is this trip?

S: 586 km.

Lynchburg to Richmond to Emporia to Danville to Lynchburg is 586 km.

T: Is this trip more or less than 500 km long?

S: More.

T: How much more than 500 km?

S: 86 km.

T: Is there another trip we could take starting at Lynchburg which is closer to 500 km long?

Perhaps a student will suggest this trip.

Lynchburg to Roanoke to Wytheville to Cumberland Gap is 484 km.

T: Is this trip more or less than 500 km long?

S: Less.

T: How much less . . . ?

S: 16 km.

If necessary, suggest this addition problem as a way to calculate how much less 484 is than 500.

$$\begin{array}{r} 484 \\ +\underline{} \\ 500 \end{array}$$

Conclude that 484 km is closer to 500 km than 586 km is to 500 km.

T: Is there another trip we could take starting at Lynchburg which is still closer to 500 km long?

Continue this activity until at least one trip with a length between 480 km and 520 km is found. There are several such trips. Compare all the trips you have recorded on the board and decide which of them has a length closest to 500 km.

N32 Subtraction Roads #1

CAPSULE LESSON SUMMARY

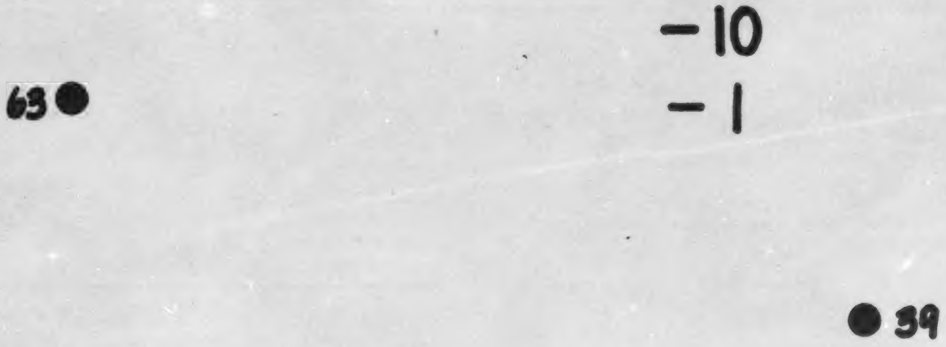
Build roads from one number to another using only subtraction arrows. For each road find the composite of the arrows in the road and ask for the corresponding subtraction number story.

MATERIALS

Teacher: Colored chalk
Student: Colored pencils; Worksheets N32*, **, ***, and ****

DESCRIPTION OF LESSON

Write this information on the board.

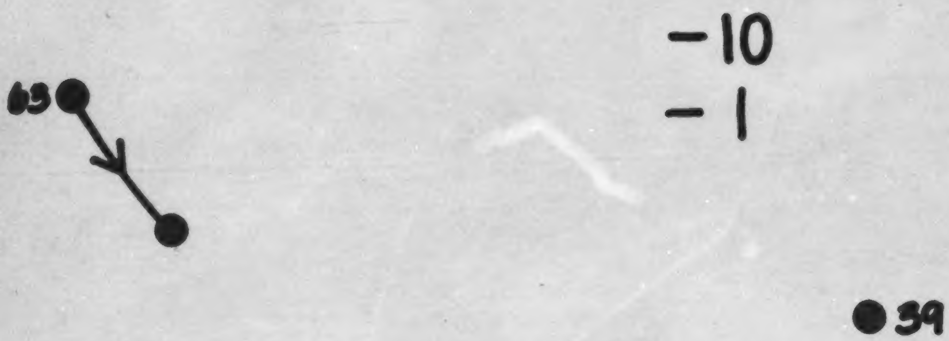


T: Could we build a road from 63 to 39 with just -10 and -1 arrows?

S: Yes, we could use all -1 arrows.

Build a road from 63 to 39 with the class. Follow the suggestions of your students; a sample dialogue is given here.

S: Let's start with a -10 arrow.



T: What number is 63 - 10?

S: 53.

Ask students to explain how they calculated "63 - 10".

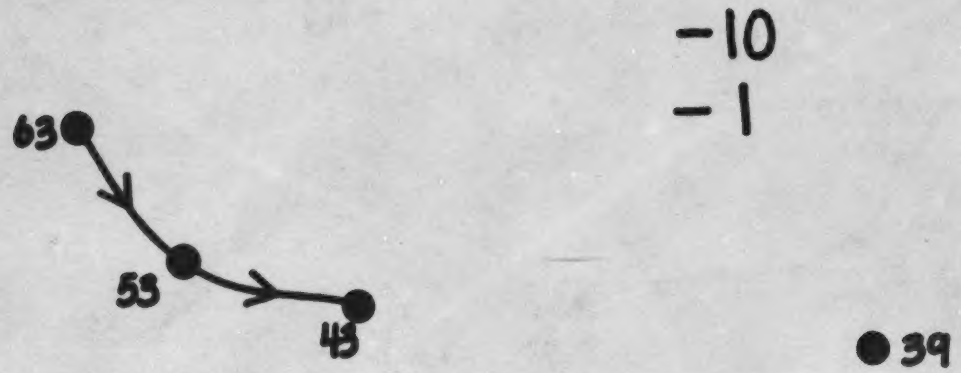
T (pointing to "63"): 63 has 6 tens and 3 ones. If we subtract 1 ten, there will be 5 tens and 3 ones left, or 53.

Label the ending dot of the red arrow "53".

T: Would it help to draw another -10 arrow? (Yes)
What number is 53 - 10?

S: 43.

T: 53 has 5 tens. If we subtract 1 ten, there will be 4 tens and 3 ones left, or 43.



T: Would it help to draw another -10 arrow?

S: No.

T: Why not?

S: 43 - 10 = 33. We can't go from 33 to 39 because we are using only subtraction arrows.

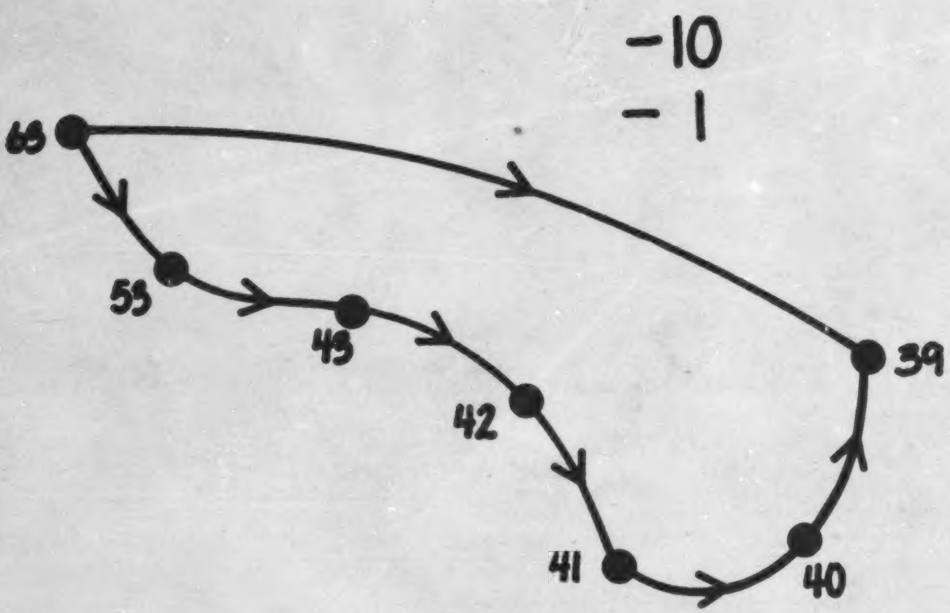
S: We need to draw some -1 arrows.

T: How many -1 arrows should we draw?

S: Four.

Ask a student to complete the arrow road. Draw a green arrow from 63 to 39. (See the illustration on the next page.)

N 32



T: What could this green arrow be for?

S: - 24.

Write "- 24" in green near the arrow picture.

T: What subtraction story could we write about this green arrow?

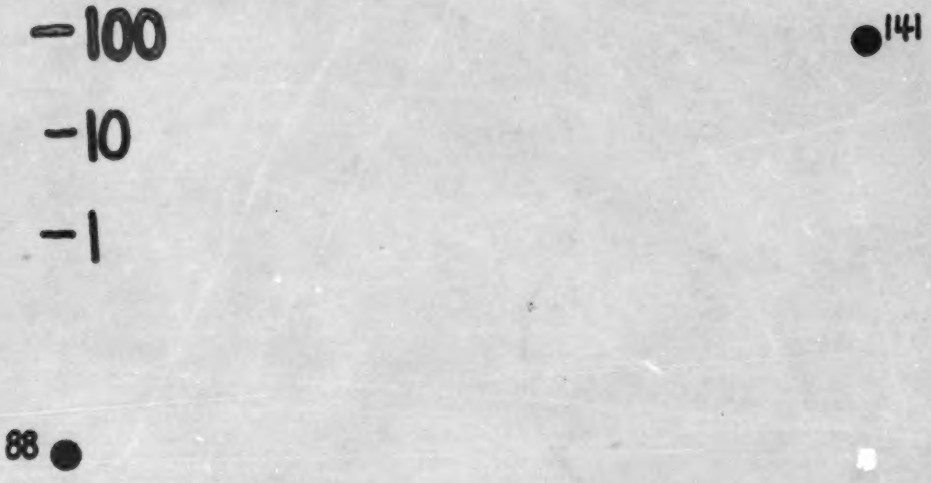
S: "63 - 24 = 39".

If necessary, provide this number story yourself. Ask for volunteers to record it on the board in both horizontal and vertical formats.

$$63 - 24 = 39$$

$$\begin{array}{r} 63 \\ -\,24 \\ \hline 39 \end{array}$$

Erase the board and then draw this arrow picture.



T: How should we start if we want to build a yellow-red-blue arrow road between 88 and 141?

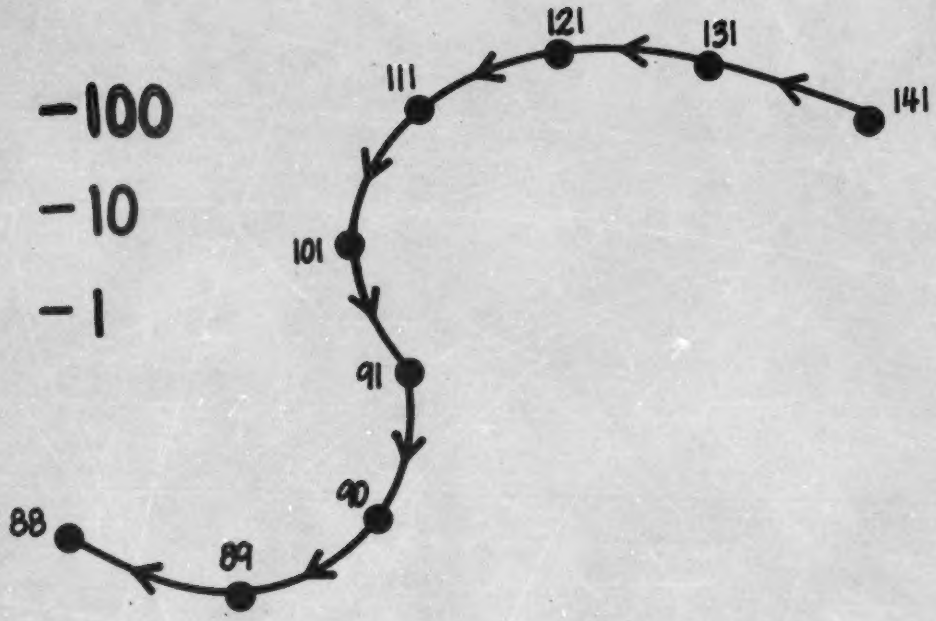
Encourage the students to express themselves. Direct the discussion so that the class concludes that the arrow road should begin at 141 because it is impossible to go from 88 to a larger number using only subtraction arrows. Observe that the -100 arrow is useless because 141 - 100 = 41 and we cannot go from 41 to 88 using subtraction arrows.

T: How many -10 arrows could there be in our arrow road from 141 to 88?

It is possible to build a road from 141 to 88 with as many as five -10 arrows.

Distribute copies of Worksheet N32* and ask the students to build a road from 141 to 88 individually. After a few minutes ask a student with a shortest road to draw it on the board. One of the many possible shortest roads is shown here. (See the illustration on the next page.)

N 32



Draw a green arrow from 141 to 88 on the board.

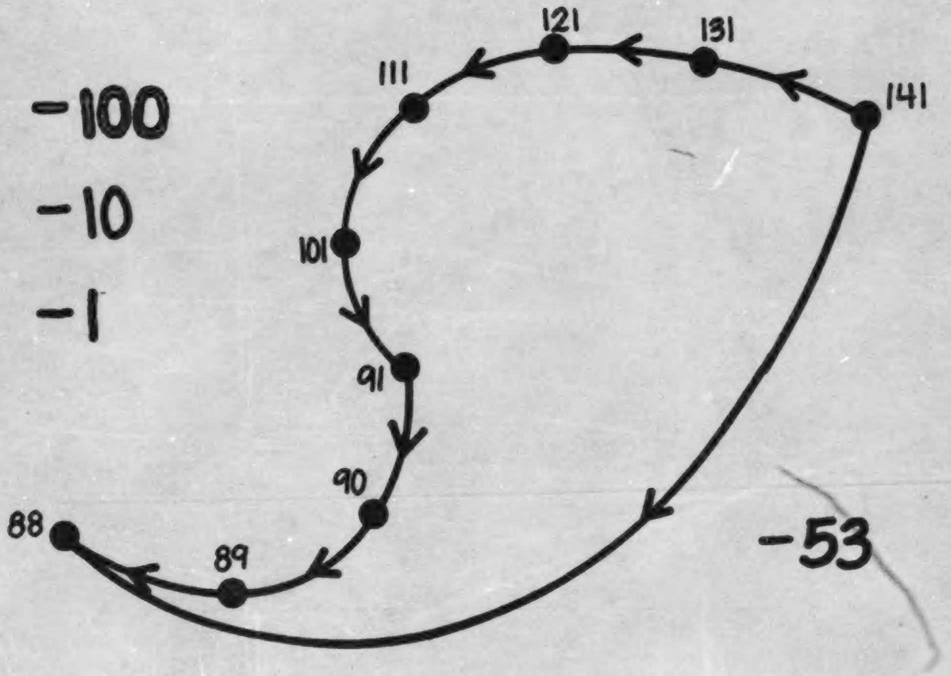
T: What could this green arrow be for?

S: - 53.

T: How do you know?

S: There are five - 10 arrows and three - 1 arrows. 5 × 10 = 50 and 50 + 3 = 53.

Write "- 53" in green near the arrow picture on the board. Ask the students to copy the green arrow and key on their worksheets. (See the illustration on the next page.)


-100
-10
-1
121
131
111
141
101
91
90
88
89
-53

T: What subtraction story could we write about this green arrow?

S: "141 - 53 = 88".

Ask for volunteers to record this number story on the board in both horizontal and vertical formats.

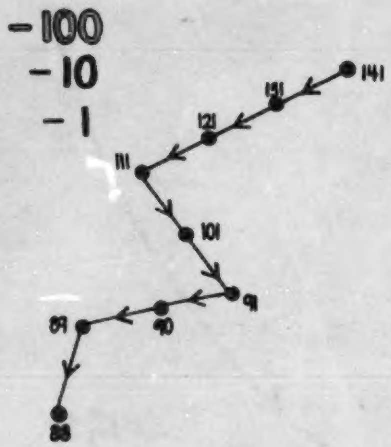
$141 - 53 = 88$

$$\begin{array}{r}141\\-\ 53\\\hline 88\end{array}$$

Worksheets N32**, ***, and **** are available for individual work.

Name ____________________ N 32 *

Build a yellow-red-blue road from 141 to 88.

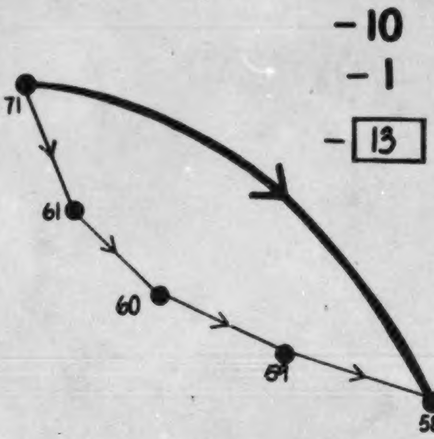


-100
-10
-1
141
131
121
111
101
91
90
89
88

Many solutions are possible, but a shortest road has eight arrows.

Name ____________________ N 32 **

Build a red-blue road from 71 to 58.
Fill in the box for the grey arrow.

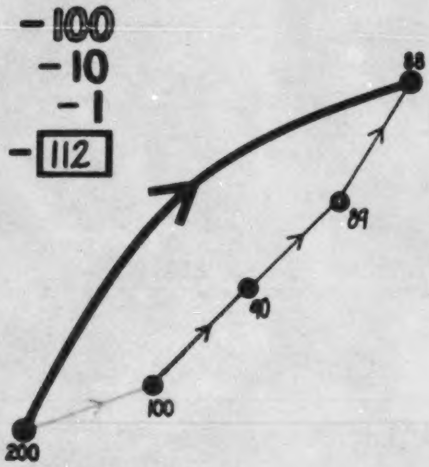


-10
-1
- 13
71
61
60
59
58

Other roads are possible, but a shortest road has four arrows.

Name ____________________ N 32 ***

Build a yellow-red-blue road from 200 to 88.
Fill in the box for the grey arrow.

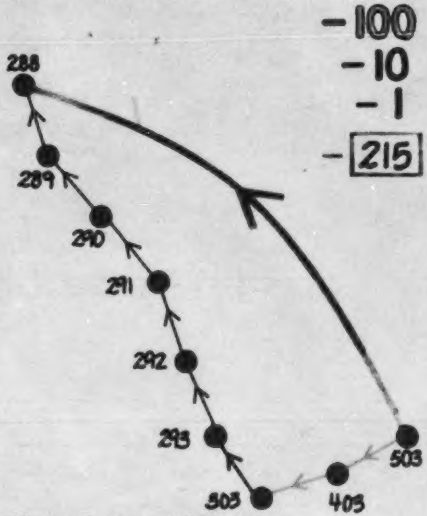


-100
-10
-1
- 112
88
89
90
100
200

Other roads are possible, but a shortest road has four arrows.

Name ____________________ N 32 ****

Build a yellow-red-blue road from 503 to 288.
Fill in the box for the grey arrow.



-100
-10
-1
- 215
288
289
290
291
292
293
503
403
303

Other roads are possible, but a shortest road has eight arrows.

N33 Subtraction Problems

CAPSULE LESSON SUMMARY

Discuss methods of subtracting 8, of subtracting 6, and of subtracting 16; for example,

20 - 8 = (20 - 10) + 2 = 12

30 - 6 = (30 - 10) + 4 = 24

60 - 16 = (60 - 10) - 6 = 44

Estimate where 336 - 167 is on the number line. Draw a -100 arrow starting at 336 to help arrive possibly at a better estimate. Use the Minicomputer to do the calculation and then locate the answer on the number line. Continue with similar subtraction problems.

MATERIALS

Teacher:	Minicomputer set
Student:	Paper; pencil; Minicomputer set

DESCRIPTION OF LESSON

Exercise 1

T: What number is 20 - 10? (10)

20 - 10 = 10; so what number is 20 - 8? (12)

How did you calculate the answer?

Let students explain their methods. Encourage the idea that an easy way to subtract 8 is to subtract 10 and then add 2.

Continue this activity with the following calculations.

30 - 8 (22)
40 - 8 (32)
50 - 8 (42)
60 - 8 (52)
80 - 8 (72)

100 - 8 (92)
140 - 8 (132)
170 - 8 (162)
200 - 8 (192)
1,000 - 8 (992)

N 33

T: What number is 20 - 10? (10)
20 - 10 = 10; so what number is 20 - 6? (14)
What number is 30 - 6? (24)
How did you calculate the answer?

Let students explain their methods. Encourage the idea that an easy way to subtract 6 is to subtract 10 and then add 4.

Continue this activity with the following calculations.

40 - 6 (34)
50 - 6 (44)
70 - 6 (64)
100 - 6 (94)

T: What number is 60 - 10? (50)
60 - 10 = 50; so what number is 60 - 16? (44)
How do you know?

Let several students explain their methods.

S: 60 - 10 = 50; 50 - 6 = 44.

S: 60 - 6 = 54; 54 - 10 = 44.

S: 60 - 20 = 40 and 40 + 4 = 44.

Continue this activity with the following calculations.

30 - 16 (14)
70 - 16 (54)
20 - 16 (4)
90 - 16 (74)

Record the last calculation on the board.

$$\begin{array}{r} 90 \\ -16 \\ \hline 74 \end{array}$$

T: We just determined that 90 - 16 = 74. What number is 100 - 16?

S: 84.

Write this calculation on the board.

$$\begin{array}{r} 100 \\ -\ 10 \\ \hline \end{array}$$

T: What number is 100 - 10?

S: 90.

$$\begin{array}{r} 100 \\ -\ 10 \\ \hline 90 \\ -\ \ \ 6 \\ \hline \end{array}$$

T: What number is 90 - 6?

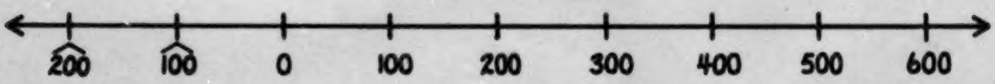
S: 84.

T: 100 - 16 = 84.

$$\begin{array}{r} 100 \\ -\ 10 \\ \hline 90 \\ -\ \ \ 6 \\ \hline 84 \end{array} \qquad\qquad \begin{array}{r} 100 \\ -16 \\ \hline 84 \end{array}$$

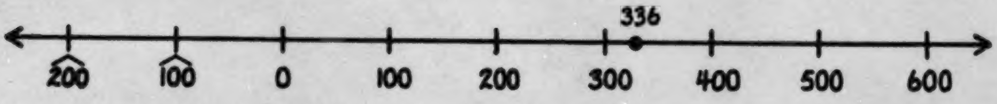
N 33

Erase the board and then draw this number line.


$\hat{200}$
$\hat{100}$
0
100
200
300
400
500
600

T: Approximately where would 336 be on this number line?

Invite students to point to where they think 336 is on the number line; then locate and draw a dot for 336 yourself.


336
$\hat{200}$
$\hat{100}$
0
100
200
300
400
500
600

Write this subtraction problem on the board.

$$\begin{array}{r} 336 \\ -167 \\ \hline \end{array}$$

T: Before we calculate "336 - 167", let's estimate the answer.

Invite several students to point to where they think 336 - 167 is on the number line. Students very likely will indicate points between 100 and 300.

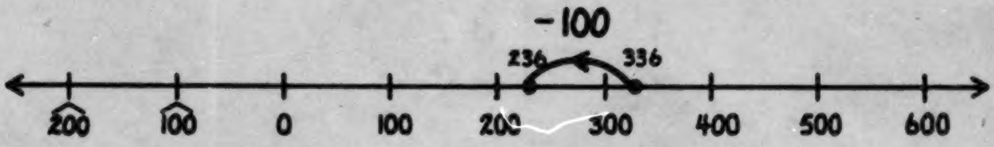
T: I would like to get a better estimate of 336 - 167. What number is 336 - 100?

S: 236.

T: 336 has 3 hundreds. If we subtract 1 hundred, there will be 2 hundreds left; 336 - 100 = 236. Approximately where is 236 on this number line?

Invite a student to point to where (approximately) 236 is on the number line; then locate and draw a dot for 236 yourself.

Draw a -100 arrow from 336 to 236.



T: 336 - 100 = 236. Is 336 - 167 more or less than 200?

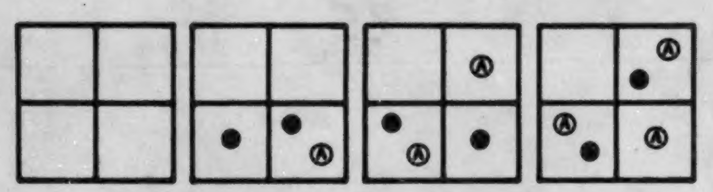
S: It is less than 200 because we still need to subtract 67 from 236.

T: We know that 336 - 167 is less than 200. Try to calculate "336 - 167" on your own and then we will calculate the answer together on the Minicomputer.

Allow a few minutes for independent work. Look at many of the students' answers.

T: How could we calculate "336 - 167" on the Minicomputer?

S: Put 336 on the Minicomputer with regular checkers and 167 with negative checkers. Then make plays.



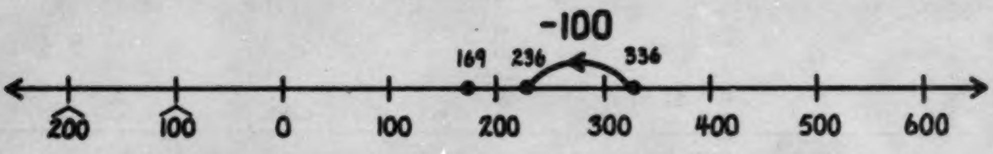
N 33

Invite students to make plays until the standard configuration for 169 is obtained. Conclude that 336 - 167 = 169 and complete the subtraction problem on the board.

$$\begin{array}{r} 336 \\ -167 \\ \hline 169 \end{array}$$

T: Approximately where is 169 on this number line?

Invite a student to point to where (approximately) 169 is on the number line; then locate and draw a dot for 169 yourself.



Commend any students who correctly solved this subtraction problem on their own and ask them to share their methods with the class. Record the calculations on the board.

Repeat this activity with another subtraction problem.

$$\begin{array}{r} 409 \\ -253 \\ \hline \end{array}$$

Exercise 2

Write the following problems on the board. Ask the students to solve as many of them as they can and to use whichever methods they prefer. Distribute individual Minicomputer sets to students who want to use them.

*

No backward plays needed on the Minicomputer.

$$\begin{array}{r}253\\-\ 42\\\hline\end{array}\qquad\begin{array}{r}367\\-106\\\hline\end{array}\qquad\begin{array}{r}639\\-438\\\hline\end{array}$$

**

One or two backward plays needed.

$$\begin{array}{r}345\\-281\\\hline\end{array}\qquad\begin{array}{r}575\\-268\\\hline\end{array}\qquad\begin{array}{r}201\\-120\\\hline\end{array}$$

***

More than two backward plays needed.

$$\begin{array}{r}248\\-196\\\hline\end{array}\qquad\begin{array}{r}315\\-287\\\hline\end{array}\qquad\begin{array}{r}934\\-265\\\hline\end{array}$$

N 34

N34 Multiplication and Composition

CAPSULE LESSON SUMMARY

Draw a picture with 2 x 7 dots. Then double the number of dots and ask which calculations are suggested by the picture. Decide that a 2 x arrow followed by a 2 x arrow is the same as a 4 x arrow. Put 17 on the Minicomputer, double 17, and then double the result. Notice that the number now on the Minicomputer is 4 x 17. Show this in an arrow picture.

MATERIALS

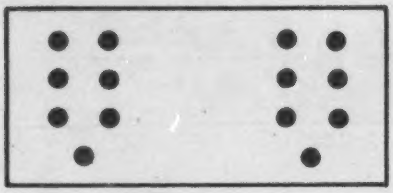
Teacher: Colored chalk; Minicomputer set

Student: Paper; colored pencils

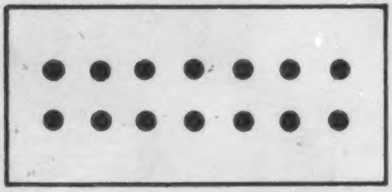
DESCRIPTION OF LESSON

T: On your paper draw 2 x 7 dots. For the moment, I am not interested in calculating "2 x 7". I only want you to draw a dot picture with exactly 2 x 7 dots.

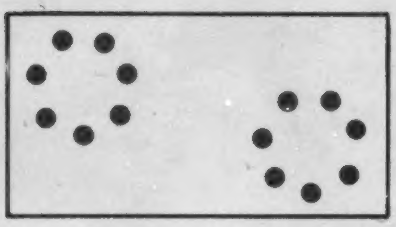
Look at the students' pictures and select several of them in which it is easy to see that there are 2 x 7 dots. When most of the students have completed their pictures, show those which you have selected to the class and ask for volunteers to describe each of them. Sample pictures and corresponding student descriptions are given here.



S: There are two groups of dots. Each group has seven dots.

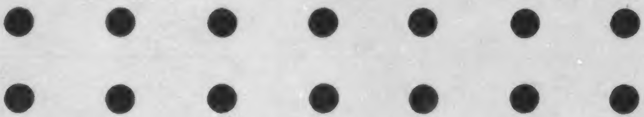


S: There are two rows of dots with seven dots in each row.



S: There are two circles and each circle has seven dots.

Draw this picture on the board.



T: What calculations does this dot picture suggest?

S: "2 x 7 = 14".

S: "7 x 2 = 14".

S: "7 + 7 = 14".

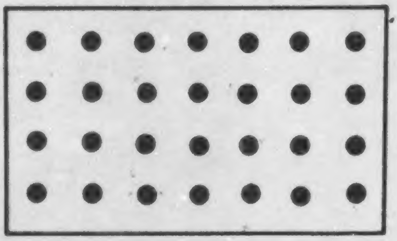
S: "2 + 2 + 2 + 2 + 2 + 2 + 2 = 14".

T: Copy this picture on the other side of your paper.

Allow the students some time to complete this task.

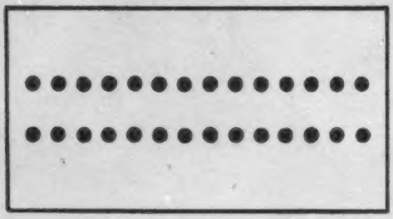
T: Now continue drawing dots until your picture has twice as many dots as the picture on the board.

When most of the students have completed their pictures, discuss a few of them with the class as you did earlier.



S: There are four rows with seven dots each.

N 34



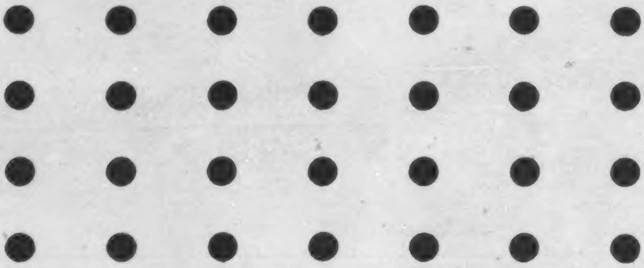
S: There are two rows with 14 dots each.

T: I am going to continue drawing dots on the board until there are twice as many dots as there are now. I'll draw two more rows of seven dots below the two rows already on the board. How many dots will I have in my picture? How do you know?

S: Twenty-eight dots, because there will be four rows of seven dots and 4 x 7 = 28.

S: There are fourteen dots on the board now and 2 x 14 = 28.

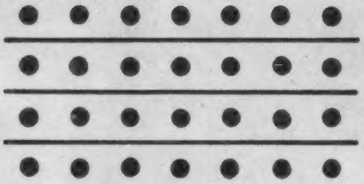
Extend the dot picture on the board.



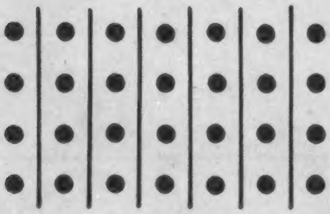
T: What calculations does this dot picture suggest?

Record the corresponding number stories on the board as calculations are suggested. Relate each number story to the dot picture or ask for volunteers to do so. Several examples are given in the next illustrations. Do not draw any lines on the board; they are included in these illustrations to show how the collection of dots could be partitioned.

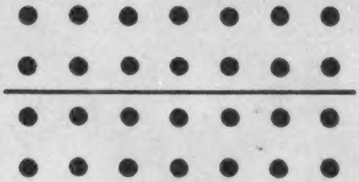
S: "4 x 7 = 28".



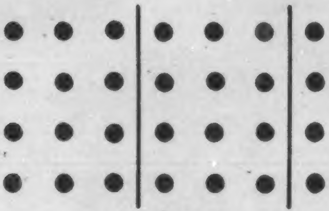
S: "7 x 4 = 28".



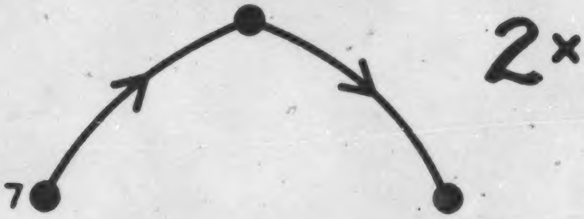
S: "2 x 14 = 28".



S: "12 + 12 + 4 = 28".

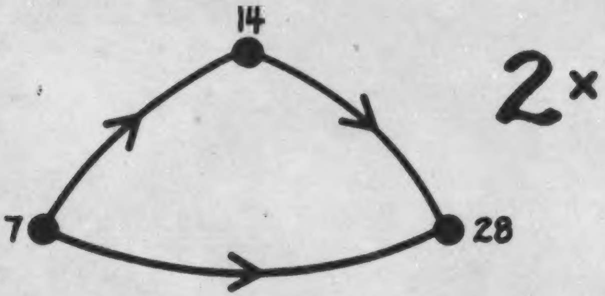


Erase the dot picture and then draw this arrow picture on the board.



N 34

Ask for volunteers to label the dots. Draw a red arrow from 7 to 28.



T: What could this red arrow be for?

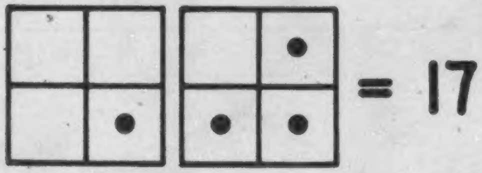
S: 4 x .

T: That's right. 4 x 7 = 28.

Write "4 x" in red near the arrow picture. Trace the arrows using motions described in Lesson L2 Composition #1 as you say:

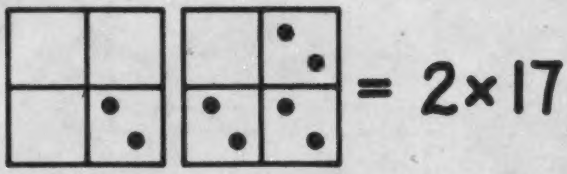
T: 2 x followed by 2 x is the same as 4 x .

Erase the board the display two Minicomputer boards. Ask a student to put 17 on the Minicomputer.

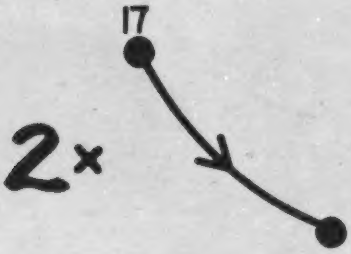


T: How could we add more checkers so that 2 x 17 will be on the Minicomputer?

S: Put another checker with each of the checkers already on the Minicomputer.



Draw this arrow picture on the board near the Minicomputer.



T: First we put 17 on the Minicomputer, and then we doubled it. Now 2 x 17 is on the Minicomputer. For the moment, I don't want to calculate "2 x 17". What could we do to put 2 x this new number on the Minicomputer?

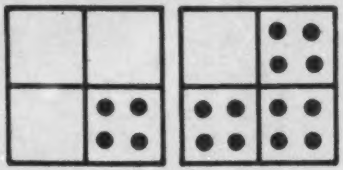
Let the students discuss this problem and make suggestions.

S: We need to put another pair of checkers with each pair of checkers on the Minicomputer.

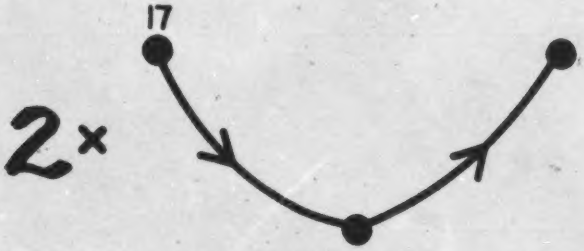
S: We should put 2 x 17 on the Minicomputer again.

N 34

Ask a student to double the number on the Minicomputer by putting this 2 x 17 configuration on again.



Draw another blue arrow.

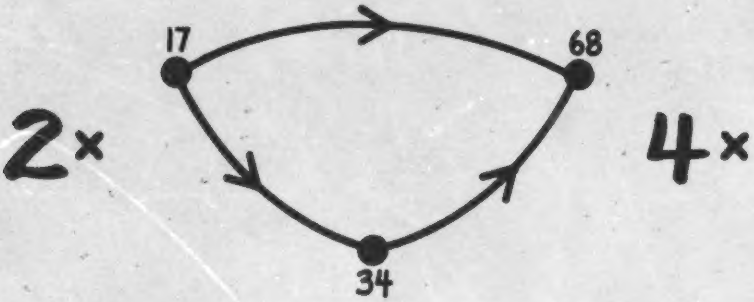


T: First we doubled 17 and then we doubled it again. How many times is 17 on the Minicomputer now?

S: Four.

T: The number on the Minicomputer is 4 x 17.

Ask for a volunteer to draw a red 4 × arrow in the picture and label it "4 ×". Call on students to label the dots.



Trace the appropriate arrows as you say:

T: 2 x 17 = 34 and 2 x 34 = 68, so 4 x 17 = 68.

Erase the labels for the dots and take away the Minicomputer. Use the arrow picture in a brief 4 x mental arithmetic activity.

T: What number is 4 x 11? How do you know?

S: 44, I added "11 + 11 + 11 + 11".

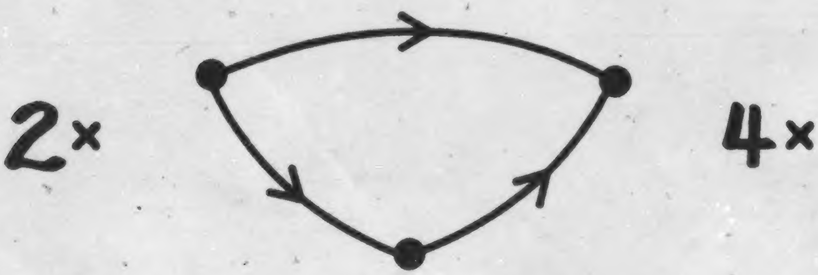
T: You are thinking of this calculation.

Write this addition problem on the board.

$$\begin{array}{r} 11\\ 11\\ 11\\ +\,11\\ \hline 44 \end{array}$$

S: 2 x 11 = 22 and 2 x 22 = 44.

T: You are thinking of this arrow picture (point to the picture on the board).



N 34

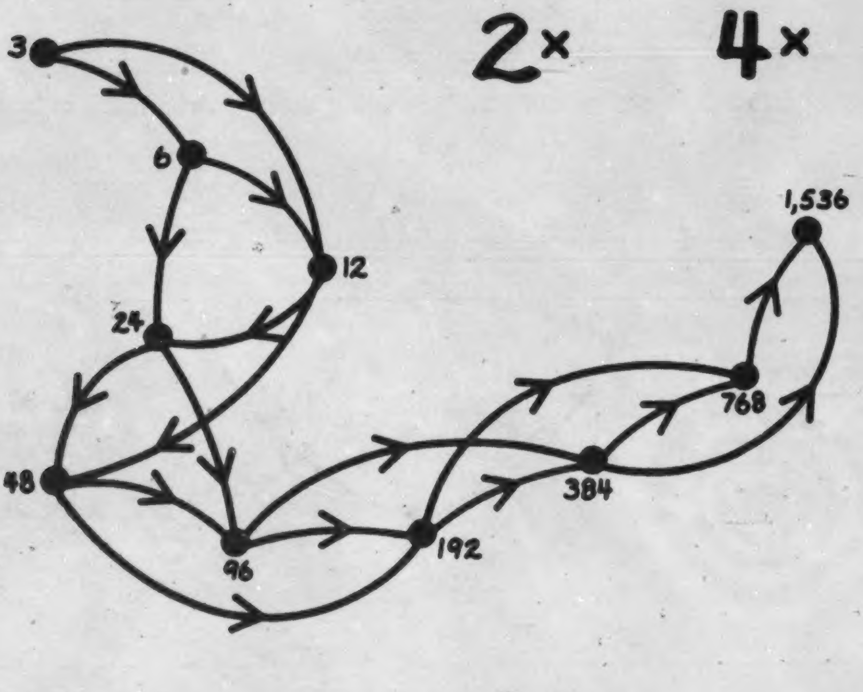
Demonstrate this calculation with the arrow picture. Point to the starting dot and say "11"; then trace the appropriate arrows as you say:

T: 2 x 11 = 22 and 2 x 22 = 44, so 4 x 11 = 44.

Erase everything on the board except the arrow picture and repeat this activity with these calculations.

4 × 25 (100)
4 × 6 (24)
4 × 27 (108)
4 × 55 (220)

Ask the students to start with any whole number and build a 2 x arrow road. After the students have several arrows in their 2 x arrow roads, ask them where they can draw 4 x arrows between numbers already in their picture. One example of a student's work is shown here.



N35 Minicomputer Tug of War #2

CAPSULE LESSON SUMMARY

Given configurations consisting of one or more ⑩-checkers, replace each ⑩-checker with a regular checker on the square of the same color on the next board to the left. Complete the corresponding number stories. Play Minicomputer Tug of War.

MATERIALS

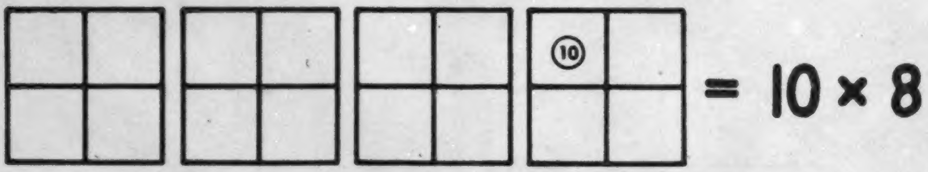
Teacher: Minicomputer set; ⑩-checkers; colored chalk
Student: None

DESCRIPTION OF LESSON

Exercise 1: 10 x on the Minicomputer

Display four Minicomputer boards.

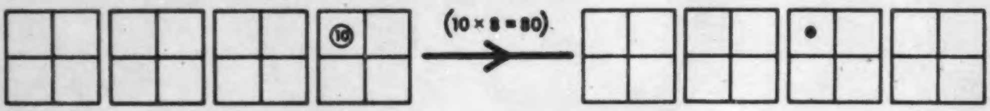
T: Who can put 10 x 8 on this Minicomputer using a ⑩-checker?



T (pointing to the ⑩-checker): We can use a ⑩-checker to show that there are ten checkers on the same square. Ten checkers on the 8-square is 10 x 8. Which play can we make with this ⑩-checker?

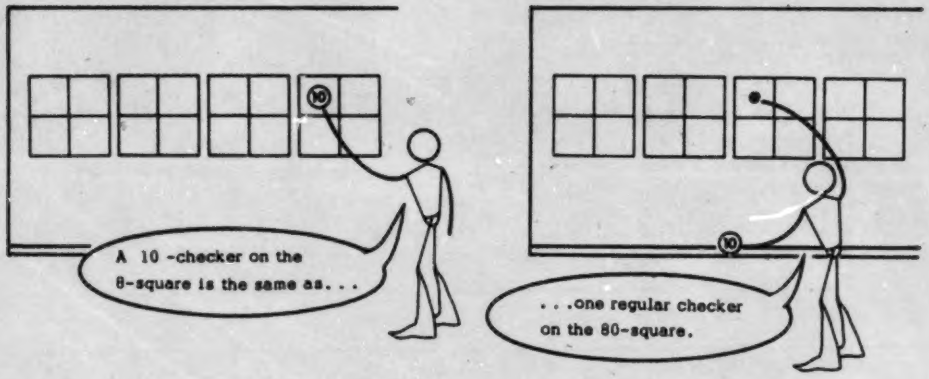
S: "10 x 8 = 80".

Invite a student to make the play.



N 35

Repeat the play yourself.

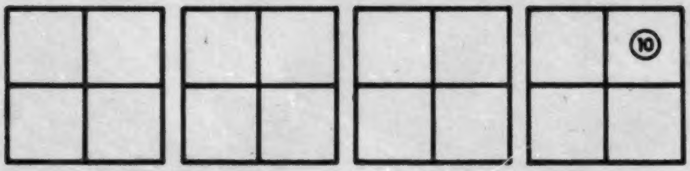


10
A 10 -checker on the 8-square is the same as. . .
10
. . .one regular checker on the 80-square.

Record "10 x 8 = 80" on the board.

$$10 \times 8 = 80$$

Put a ⑩-checker on the 4-square.



T: What calculation does this ⑩-checker suggest?

S: "10 x 4".

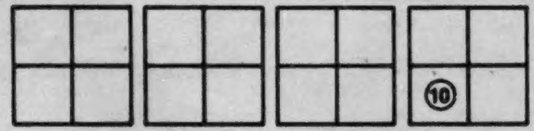
T: The number on the Minicomputer is 10 x 4. Who can make a play with this ⑩-checker?

The volunteer should remove the ⑩-checker from the 4-square and put a regular checker on the 40-square. As the play is made, say the "10 x 4 = 40" number story yourself if the student does not.

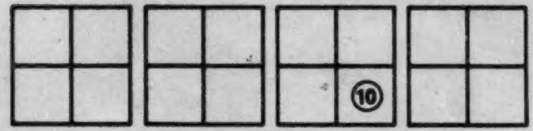
Record "10 x 4 = 40" on the board below "10 x 8 = 80".

$$10 \times 8 = 80$$
$$10 \times 4 = 40$$

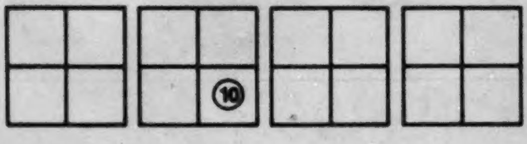
Repeat this activity with these configurations.



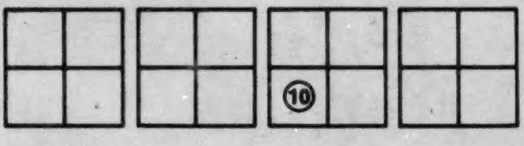
= $10 \times 2 = 20$



= $10 \times 10 = 100$

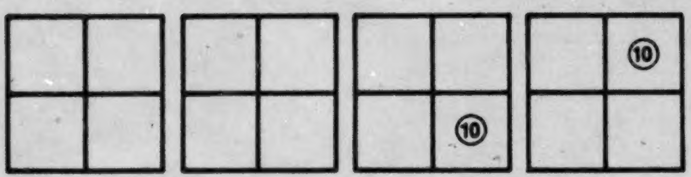


= $10 \times 100 = 1{,}000$



= $10 \times 20 = 200$

Put this configuration on the Minicomputer.



T: What calculation do these ⑩-checkers suggest?

S: "10 x 14".

T: What number is 10 x 14?

S: 140.

Write "10 x 14 = 140" on the board below "10 x 20 = 200". Ask the students to explain how they know that 10 x 14 = 140 and encourage a variety of responses. A sample dialogue is given here.

S: 10 x 10 = 100 and 10 x 4 = 40 and 100 + 40 = 140, so 10 x 14 = 140.

Write these calculations on the board.

$$10 \times 10 = 100$$
$$10 \times 4 = 40$$
$$10 \times 14 = 140$$

S: I made the plays in my head.

Make the "10 x 10 = 100" and "10 x 4 = 40" plays on the Minicomputer to aid the student's explanation.

S: I knew it would be 140 because I added a "0" to the end of "14". When we multiply by 10, the answer always ends in 0.

Point to the column of number stories on the board.

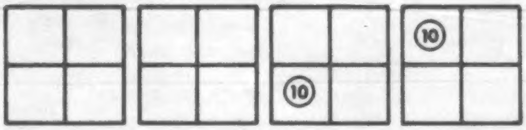
$$10 \times 8 = 80$$
$$10 \times 4 = 40$$
$$10 \times 2 = 20$$
$$10 \times 10 = 100$$
$$10 \times 100 = 1{,}000$$
$$10 \times 20 = 200$$
$$10 \times 14 = 140$$

T: Is it true that each time we multiplied by 10 the answer ended in 0?

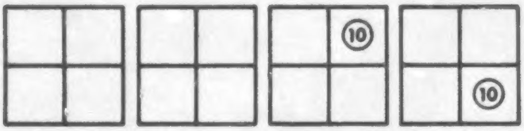
S: Yes.

NOTE: This observation is true because all the examples are integers; however, it cannot be applied to non-integer decimals, for example: $10 \times 0.8 = 8$ and $10 \times 0.08 = 0.8$.

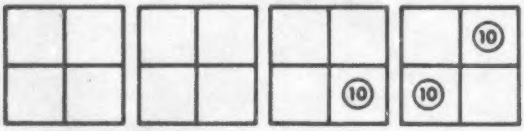
Continue this activity with these configurations.



$= 10 \times 28 = 280$



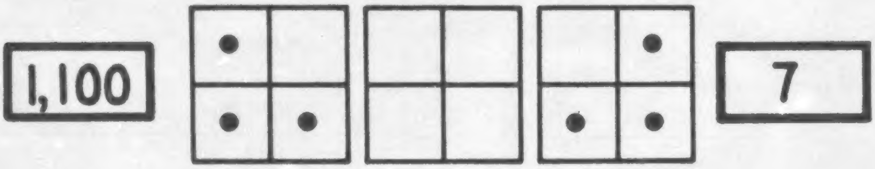
$= 10 \times 41 = 410$



$= 10 \times 16 = 160$

Exercise 2: Minicomputer Tug of War

Play several games of Minicomputer Tug of War as explained in Lesson W4 FESTIVAL OF PROBLEMS #1 (Lesson 1). You may wish to use a variation of the game with this starting configuration.



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N36 Subtraction Roads #2

CAPSULE LESSON SUMMARY

Begin with mental arithmetic activities involving the functions - 10, - 20, and - 100. Discuss what kinds of arrows could be used to build a road between 232 and 89. Then build a road using - 100, - 10, and - 1 arrows. Find the composite of the arrows in the road and ask for the corresponding subtraction number story.

MATERIALS

Teacher: Colored chalk

Student: Worksheets N36*, **, and ***; colored pencils

DESCRIPTION OF LESSON

Exercise 1: Mental Arithmetic

Use the following activity involving the functions - 10, - 20, and - 100 or a similar activity which is appropriate for the abilities of your students. Try to involve all the students by intermixing calculations of various difficulties.

T: I'll say a number and then you tell me which number is 10 less than my number. My first number is 16.

S: 6 is 10 less than 16.

T: 16 - 10 = 6. My next number is 26.

S: 16; 26 - 10 = 16.

Continue this activity with the following or similar numbers.

36	(26)	102	(92)
56	(46)	120	(110)
106	(96)	123	(113)
10	(0)	200	(190)
50	(40)	1,000	(990)
100	(90)	1,005	(995)

T: I'll say a number and then you tell me which number is 20 less than my number.

20	(0)	100	(80)
24	(4)	110	(90)
34	(14)	120	(100)
50	(30)	136	(116)
49	(29)	171	(151)
67	(47)	1,000	(980)
80	(60)	1,001	(981)

T: I'll say a number and then you tell me which number is 100 less than my number.

136	(36)	1,000	(900)
179	(79)	1,053	(953)
182	(82)	824	(724)
340	(240)	901	(801)
206	(106)	718	(618)
500	(400)	1,400	(1,300)

If these activities have been easy for many of your students, continue with the following, more difficult activity. Otherwise, go on to Exercise 2.

T: What number is 110 less than 200? How do you know?

S: 90. 200 - 100 = 100 and 100 - 10 = 90, so 200 - 110 = 90.

S: I subtracted 10 first and then subtracted 100. 200 - 10 = 190 and 190 - 100 = 90.

Continue this activity with the following or similar numbers.

210	(100)	800	(690)
150	(40)	611	(501)
153	(43)	254	(144)
390	(280)	479	(369)
398	(288)	1,000	(890)

Exercise 2

Draw two dots on the board. Label one of them "232" and the other "89".

T: Today we are going to build a road between 232 and 89. Would you prefer to use addition, subtraction, or multiplication arrows?

Encourage the students to discuss this problem. A possible dialogue is given here.

S: We could start at 89 and use addition arrows.

T: Could we go from 89 to 232 using only addition arrows?

S: Yes, if we choose the right kinds of addition arrows.

S: We could start at 232 and use subtraction arrows.

T: Could we go from 232 to 89 using only subtraction arrows?

S: Yes, if we choose the right kinds of subtraction arrows.

T: Could we build a road between 232 and 89 using only "2 x" arrows?

S: No, because 2 x 89 = 178 and 2 x 178 is more than 232.

T: If we could use a 2 x arrow, what other kind of arrow would be helpful?

S: We could start at 89 with a 2 x arrow and then draw lots of + 1 arrows until we reach 232.

There are other correct answers. Perhaps a clever student will suggest using + 2 arrows or + 6 arrows or even a + 54 arrow.

Write "- 100" in yellow near the picture on the board.

T: Could we build a road between 232 and 89 using only - 100 arrows?

S: If we start at 232 and draw one - 100 arrow, we would get to 132; then we would need other kinds of subtraction arrows to get to 89 because 132 - 100 = 32.

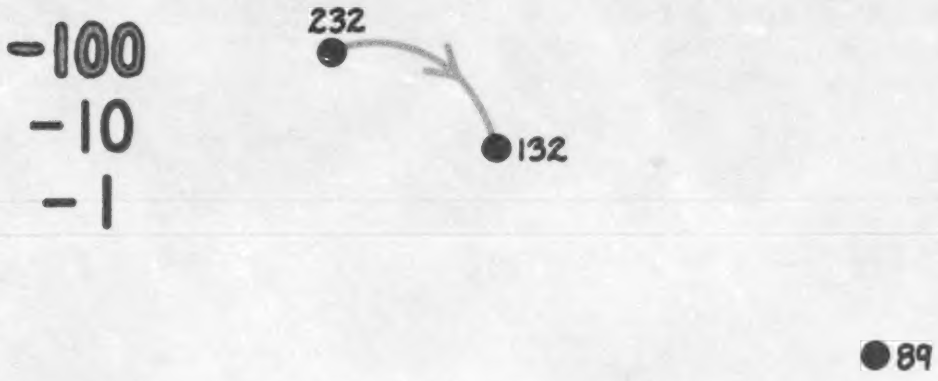
S: If we start at 232 and draw two - 100 arrows, we would get to 32; then we would need some addition arrows to get to 89.

Write "- 10" in red and "- 1" in blue near the arrow picture.

T: Let's build a road between 232 and 89 using only - 100, - 10, and - 1 arrows. Where should we start?

S: We must start at 232 because we are using only subtraction arrows.

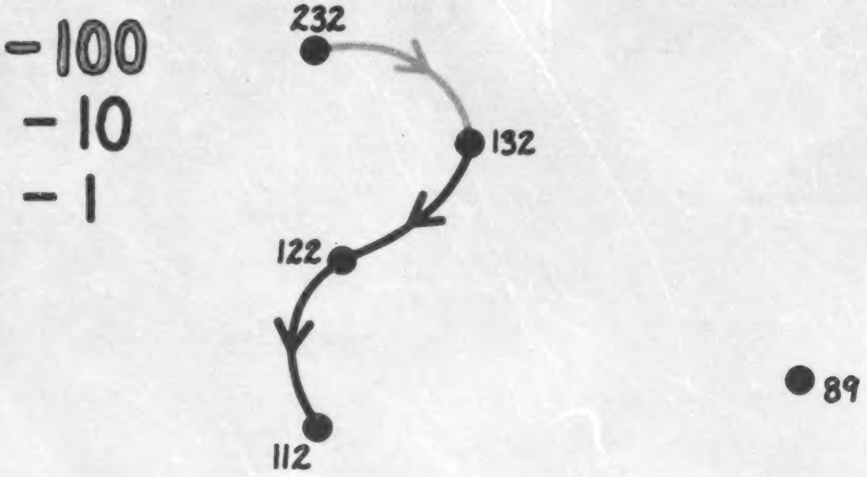
Invite a student to draw the first arrow. The student could use any of three types of arrows successfully. A possible sequence of arrows and corresponding dialogue are given here.



T: We can't draw another - 100 arrow, because it would take us below 89. We can use only - 10 and - 1 arrows now.

N 36

Continue letting students add arrows to the road.

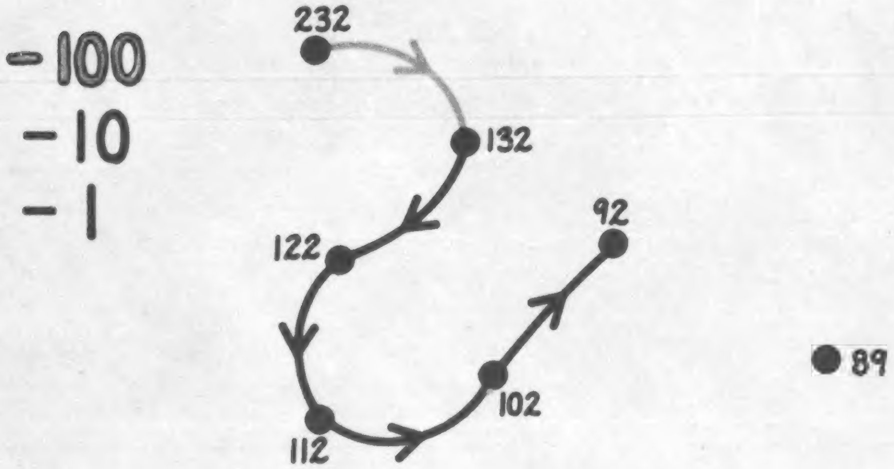

-100
-10
-1
232
132
122
112
89

T. How many more - 10 arrows will we be able to use in our road?

Allow several students to answer.

T: Let's continue building our road and see how many more - 10 arrows we can draw.

Invite students to continue drawing - 10 arrows.


-100
-10
-1
232
132
92
122
89
102
112

T: Can we use any more - 10 arrows in our road?

S: No, we're too close to 89.

T: How many - 10 arrows did we use altogether?

S: Four.

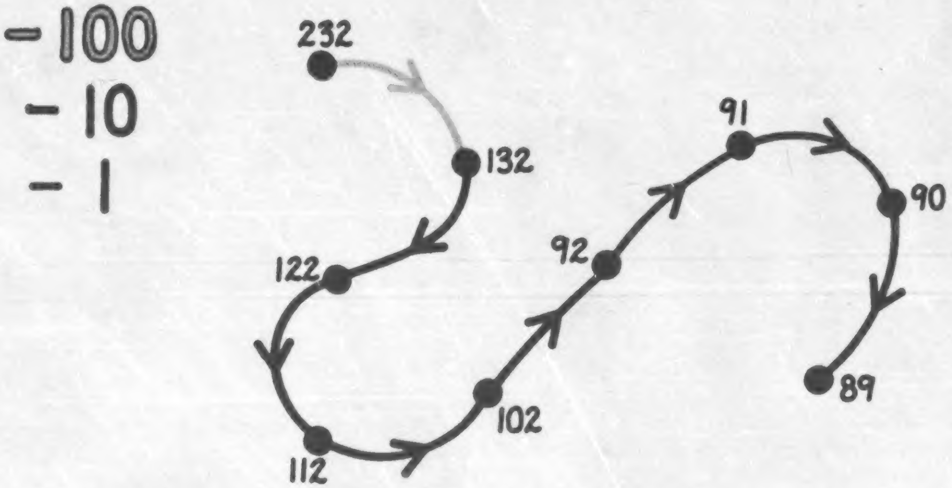
T: What kind of arrows should we use now?

S: - 1 arrows.

T: How many - 1 arrows do we need to complete our road?

S: Three.

Let a student complete the arrow road.


- 100
- 10
- 1
232
132
122
112
102
92
91
90
89

Draw a green arrow from 232 to 89.

N 36

T: I drew a green arrow from 232 to 89. What could this green arrow be for?

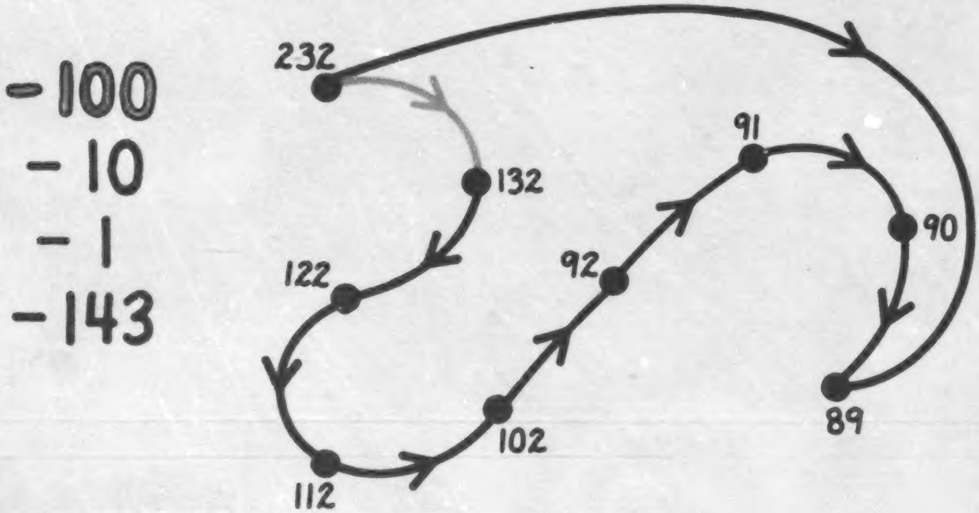
S: - 143.

T: How do you know?

S (pointing to each arrow in turn): Take away 100, 110, 120, 130, 140, 141, 142, 143.

S: I know it could be for - 143 because there is one yellow arrow, four red arrows, and three blue arrows.

Write "- 143" in green near the arrow picture.



Write the subtraction problem on the board.

$$\begin{array}{r} 232 \\ -143 \\ \hline \end{array}$$

T: What number is 232 - 143? How do you know?

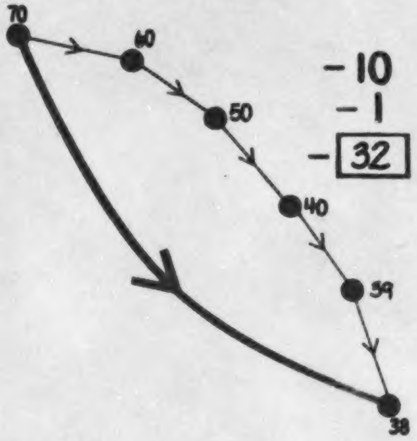
S: It's 89. I just looked at the arrow picture. The green arrow is for - 143, so 232 - 143 = 89.

Worksheets N36*, **, and *** are available for individual work.

Name

N 36 *

Build a red-blue road from 70 to 38.
Fill in the box for the grey arrow.

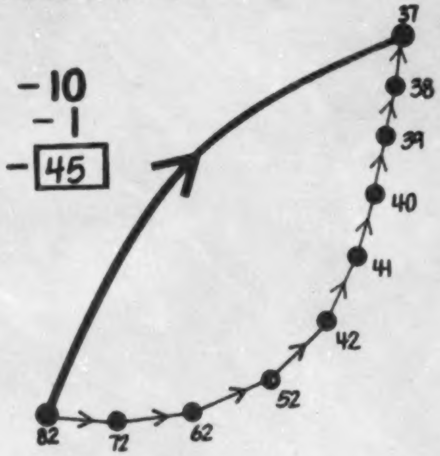

70
60
50
40
39
38
-10
-1
-32

Other roads are possible, but a shortest road has five arrows.

Name

N 36 **

Build a red-blue road from 82 to 37.
Fill in the box for the grey arrow.

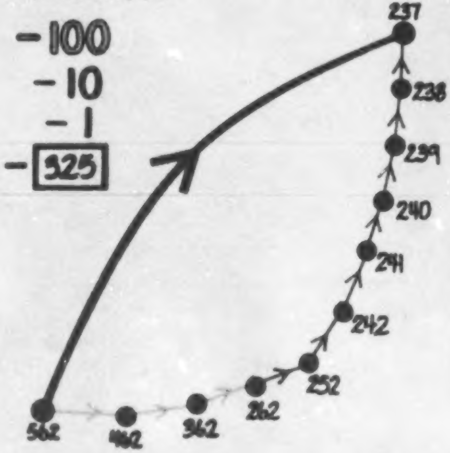

37
38
39
40
41
42
52
62
72
82
-10
-1
-45

Other roads are possible, but a shortest road has nine arrows.

Name

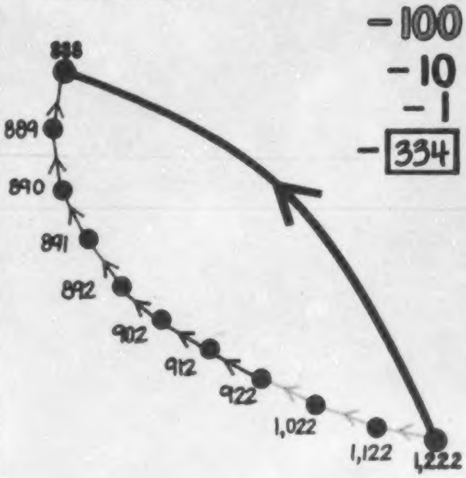
N 36 ***

Build a yellow-red-blue road from 562 to 237.
Fill in the box for the grey arrow.


237
238
239
240
241
242
252
262
362
462
562
-100
-10
-1
-325

Other roads are possible, but a shortest road has ten arrows.

Build a yellow-red-blue road from 1,222 to 888.
Fill in the box for the grey arrow.

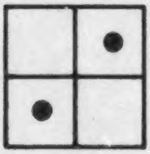

888
889
890
891
892
902
912
922
1,022
1,122
1,222
-100
-10
-1
-334

Other roads are possible, but a shortest road has ten arrows.

APPENDIX A: STANDARD CONFIGURATION AND STANDARD MINICOMPUTER MOVES

A number can be represented by putting checkers[*] on the squares of the Minicomputer. Each square has a numerical (positional) value, so that the number represented by a checker depends on which square the checker is placed. If several checkers are put on the Minicomputer, the number shown is the sum of the numbers represented by the checkers. A number can be put on the Minicomputer in a variety of ways, but the representation which uses at most one checker on each square and uses checkers to represent a digit 9 or less on each board is usually the easiest to read. In this case, we say that the number is in standard configuration. Standard configurations for the numbers 1-9 soon become as familiar to the students as the usual numerals so that they no longer need to do any mental calculation for such configurations.

For example,



. . . is seen as a name for 6 without doing the calculation "4 + 2 = 6".

Doing calculations on the Minicomputer involves putting one or more checkers on the various squares and making "plays" to get a specific configuration. In this context, a play is an exchange of checkers in a configuration which does not change the number represented. Some plays are natural to the design of the Minicomputer; we call these standard or usual plays (these are described in detail in this section). There are advantages to establishing such a set of plays:

---

[*] In this appendix we refer only to whole numbers and, therefore, use only regular (positive) checkers.

- They become routine, and a large group of students with varying abilities can follow a calculation without needing to verify every play which is made.
- They lead to standard configurations of numbers.

These are the standard Minicomputer moves.

1 + 1 = 2
10 + 10 = 20
100 + 100 = 200
1,000 + 1,000 = 2,000

two checkers on the white square
of a board
=
one checker on the red square
of the same board

2 = 1 + 1
20 = 10 + 10
200 = 100 + 100
2,000 = 1,000 + 1,000

one checker on the red square
of a board
=
two checkers on the white square
of the same board

2 + 2 = 4
20 + 20 = 40
200 + 200 = 400
2,000 + 2,000 = 4,000

two checkers on the red square
of a board
=
one checker on the purple square
of the same board

4 = 2 + 2
40 = 20 + 20
400 = 200 + 200
4,000 = 2,000 + 2,000

one checker on the purple square
of a board
=
two checkers on the red square
of the same board

4 + 4 = 8

40 + 40 = 80

400 + 400 = 800

4,000 + 4,000 = 8,000

two checkers on the purple square
of a board
=
one checker on the brown square
of the same board

8 = 4 + 4

80 = 40 + 40

800 = 400 + 400

8,000 = 4,000 + 4,000

one checker on the brown square
of a board
=
two checkers on the purple square
of the same board

8 + 2 = 10

80 + 20 = 100

800 + 200 = 1,000

8,000 + 2,000 = 10,000

one checker on each of the brown
and red squares of a board
=
one checker on the white square
of the next board to the left

10 = 8 + 2

100 = 80 + 20

1,000 = 800 + 200

10,000 = 8,000 + 2,000

one checker on the white sqaure
of a board
=
one checker on each of the brown
and red squares of the next board
to the right[*]

---

[*] This provides the link that motivates the introduction of the decimal numbers. In particular, we have that one checker on the white square of the ones' board equals one checker on each of the brown and red squares of a special board (the tenths' board - or as it is often referred to in the lessons, the dimes' board) located to the right of the ones' board and separated from it by a special barrier.

After many experiences with the Minicomputer, some students might suggest "non-standard" plays, e.g. "8 + 4 = 10 + 2". When students suggest such plays, ask them to convince the class that the plays are indeed correct. There will be a natural acceptance or rejection of non-standard plays by the students, sometimes depending on the situation.

Some "non-standard" plays will be introduced for specific purposes; for example, ten checkers on a square can be exchanged for one checker on the same color square of the next board to the left. Such plays serve to reinforce the support that the Minicomputer gives to decimal place value or to provide insight into a certain kind of problem. They are inviting in particular situations, but do not play the same "routine" role that the usual plays do.

APPENDIX B: MENTAL ARITHMETIC

Mental arithmetic activities are short, fast-moving question and answer sessions where students are asked to calculate mentally, preferably without the aid of paper and pencil. Often a mental arithmetic session consists of several sequences in which the answers to one or more questions lead to the answer of another, more difficult question; for example,

2 × 6 = ? (12)
2 × 10 = ? (20)
2 × 16 = ? (32)

There are specific lessons which call for mental arithmetic; however, we suggest you try to include such activities for five to ten minutes at least twice a week. Use mental arithmetic as a warm-up activity to a lesson, as a transition between two exercises in a lesson, as a conclusion to a lesson, or as a quick change of pace at any time during the day.

Why is Mental Arithmetic Important?

1) It develops mental computational skills.
2) It provides an opportunity to involve and help students at all ability levels.
3) It creates an awareness of patterns and mathematical relationships.
4) It helps students to be able eventually to recall arithmetic facts easily.
5) It provides an opportunity to keep alive and to reinforce concepts previously introduced.
6) It helps the students to see that there is often more than one way to solve a problem.

Hints for Mental Arithmetic Activities

1) Vary the level of difficulty of questions throughout a session.
2) Involve many students.
3) Follow a pattern for awhile and then start a new pattern.
4) Keep a brisk pace, although you should allow students time for thought.
5) Give extra attention to a student who shows signs of improvement.
6) Illustrate, whenever possible, the value of estimation in making calculations.
7) Occasionally ask a student to explain an answer.
8) Occasionally use the chalkboard to emphasize a pattern.
9) Occasionally ask students to whisper an answer to you so that many students will have an opportunity to answer.

SUGGESTED MENTAL ARITHMETIC ACTIVITIES

This section contains specific examples that illustrate patterns in arithmetic. You should elaborate and expand upon ideas depending upon the abilities and interests of your students. As you work with your class on the regularly scheduled mathematics lessons, you might make a note of computational skills you would like to improve; then plan a short mental arithmetic activity which develops those skills. Do not expect everyone of your students to be good at all of these suggested activities, even at the end of the year.

Counting Activities

1) Count by twos, threes, fours, fives, or tens collectively; everyone counting together.

2) Count by twos, threes, fours, fives, or tens as a group activity with each student saying one number. Use a natural seating order in your class (such as going up and down rows) and ask each student in turn to say the number which comes next. If you are counting by threes, the first student will say "3"; the second will say "6"; and so on. Repeat the counting but start with a different student.

3) Count by twos, threes, fours, fives, or tens, starting at 1 or any number other than 0.

4) Count backward by twos, fives, or tens. Do this immediately following a similar forward count.

5) Play "BUZZ". The students count by ones starting at 0 but skip every fourth number by saying "buzz". This should be done rapidly. This can be done collectively or as a group activity with each student in turn saying the number that comes next or "buzz".

More challenging variations of this activity involve:

- skipping every sixth number rather than every fourth number;
- starting the counting at 2 and skipping every fourth number;
- counting by twos or fives and skipping every fourth number.

Addition and Subtraction

The first fourteen examples are lists of calculations to be done mentally. Each list suggests a pattern in arithmetic which students should begin to use in doing such calculations.

1) 6 + 4
6 + 5
6 + 6
6 + 7

8 + 2
8 + 3
8 + 4
8 + 5

30 + 20
30 + 21
30 + 22
30 + 23

7 + 4
7 + 14
7 + 24
7 + 34

9 + 5
9 + 15
9 + 25
9 + 35

21 + 35
21 + 45
21 + 55
21 + 65

702 + 100
702 + 200
702 + 300

536 + 100
536 + 200
536 + 300

600 + 213
600 + 313
600 + 413

2) 10 - 4
10 - 5
10 - 6
10 - 7

40 - 1
40 - 2
40 - 3
40 - 4

100 - 10
100 - 11
100 - 12
100 - 13

44 - 10
44 - 20
44 - 30
44 - 40

52 - 3
52 - 13
52 - 23
52 - 33

186 - 10
186 - 20
186 - 30
186 - 40

307 - 100
307 - 200
307 - 300

520 - 105
520 - 205
520 - 305

1,000 - 100
1,000 - 200
1,000 - 300

3) 2 + 5
20 + 50
200 + 500
2,000 + 5,000
2,000,000 + 5,000,000

8 + 8
80 + 80
800 + 800
8,000 + 8,000
8,000,000 + 8,000,000

23 - 10
230 - 100
2,300 - 1,000

51 - 10
510 - 100
5,100 - 1,000

98 - 20
980 - 200
9,800 - 2,000

4) 4 + 10
14 + 10
24 + 10
44 + 10

7 + 10
17 + 10
117 + 10
1,017 + 10

6 + 10
18 + 10
33 + 10
71 + 10

5 + 20
15 + 20
25 + 20
35 + 20

3 + 20
13 + 20
113 + 20
1,013 + 20

19 + 20
100 + 20
600 + 20
619 + 20

16 - 10
26 - 10
36 - 10
46 - 10

100 - 10
200 - 10
600 - 10
1,000 - 10

40 - 20
50 - 20
60 - 20
90 - 20

5) 10 + 6
20 + 6
30 + 6
40 + 6

13 + 14
23 + 14
33 + 14
43 + 14

19 + 3
29 + 3
39 + 3
49 + 3

7 - 5
17 - 5
27 - 5
37 - 5

10 - 3
20 - 3
30 - 3
40 - 3

100 - 8
200 - 8
300 - 8
400 - 8

6) 5 + 3
8 - 3
7 + 6
13 - 6

10 + 20
30 - 20
90 + 40
130 - 40

106 + 105
211 - 105
150 + 150
300 - 150

7) 16 + 5
5 + 16

54 + 12
12 + 54

3 + 2 + 5
3 + 5 + 2
5 + 3 + 2

8) 6 + 10
6 + 9

20 - 10
20 - 9

8 + 10
8 + 9

43 - 10
43 - 9

27 + 100
27 + 99

126 - 100
126 - 99

9) 50 + 40
8 + 2
58 + 42

50 - 20
8 - 6
58 - 26

30 + 20
5 + 7
35 + 27

90 - 30
7 - 2
97 - 32

40 + 60
9 + 4
49 + 64

60 - 10
9 - 4
69 - 14

10) 5 + 2
5 + 1
5 + 0
$5 + \hat{1}$
$5 + \hat{2}$

16 + 2
16 + 1
16 + 0
$16 + \hat{1}$
$16 + \hat{2}$

64 + 2
64 + 1
64 + 0
$64 + \hat{1}$
$64 + \hat{2}$

11) 12 - 5
$12 + \hat{5}$

21 - 4
$21 + \hat{4}$

145 - 30
$145 + \widehat{30}$

12) $2 + \hat{3}$
$20 + \widehat{30}$
$200 + \widehat{300}$

$\hat{5} + \hat{1}$
$\widehat{50} + \widehat{10}$
$\widehat{500} + \widehat{100}$

$8 + \hat{5}$
$80 + \widehat{50}$
$800 + \widehat{500}$

13) $10 - 6$ $\quad$ $11 - 4$ $\quad$ $27 - 13$

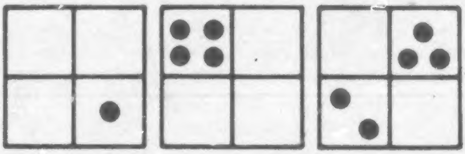
$6 - 10$ $\quad$ $4 - 11$ $\quad$ $13 - 27$

14) $7 + \hat{9}$ $\quad$ $3 + \hat{8}$ $\quad$ $12 + \widehat{15}$

$\hat{7} + 9$ $\quad$ $\hat{3} + 8$ $\quad$ $\widehat{12} + 15$

15) Choose some numbers and ask the students to add 1 to each number. For example, if you say "108", a student responds "109". You can vary this activity by asking the students to add 2, 3, 5, or 10 to each of your numbers or to subtract 1, 2, 3, or 10 from each of your numbers.

16) Conduct an activity similar to an exercise entitled "Transforming a Number" in Lesson N1. Put six to ten checkers on the ones' and the tens' boards. Move one checker from some square (it makes no difference which square you choose) to another square. Ask the students how much larger or smaller the new number is. For example, suppose this configuration is on the Minicomputer.



Move a checker from the 4-square to the 10-square. When the students tell you that the new number is larger, ask them how much larger this number is than the previous number. (6 larger)

17) Ask the students to give various names for a particular number. Encourage them to suggest names involving subtraction and also names involving addition of three numbers. For example, names for 20 include: $16 + 4$; $30 - 10$; $4 + 4 + 12$; $21 + \hat{1}$; 4×5; $\frac{1}{2} \times 40$.

Multiplication

The first fifteen examples are lists of calculations to be done mentally. Each list suggests a pattern in arithmetic which students should begin to use in doing such calculations.

1)
3 + 3
2 × 3
5 + 5
2 × 5

7 + 7
2 × 7
24 + 24
2 × 24

100 + 100
2 × 100
200 + 200
2 × 200

4 + 4 + 4
3 × 4
5 + 5 + 5
3 × 5

8 + 8 + 8
3 × 8
12 + 12 + 12
3 × 12

2 + 2 + 2 + 2
4 × 2
11 + 11 + 11 + 11
4 × 11

2)
1 × 5
2 × 5
3 × 5
4 × 5

1 × 20
2 × 20
3 × 20
4 × 20

5 × 3
6 × 3
7 × 3
8 × 3

3)
2 × 3
2 × 30
2 × 300
2 × 3,000
2 × 3,000,000

2 × 7
2 × 70
2 × 700
2 × 7,000
2 × 7,000,000

3 × 6
3 × 60
3 × 600
3 × 6,000
3 × 6,000,000

4)
2 × 4
2 × 44
2 × 444
2 × 4,444

3 × 3
3 × 33
3 × 333
3 × 3,333

2 × 5
2 × 55
2 × 555
2 × 5,555

5) 2 x 3
3 x 2
2 x 30
30 x 2

8 x 5
5 x 8
8 x 50
50 x 8

2 x 16
16 x 2
3 x 12
12 x 3

6) 2 x 10
2 x 9
2 x 20
2 x 19

2 x 50
2 x 49
2 x 100
2 x 99

3 x 10
3 x 9
3 x 20
3 x 19

7) 2 x 10
2 x 3
2 x 13

2 x 40
2 x 2
2 x 42

2 x 20
2 x 6
2 x 26

2 x 60
2 x 4
2 x 64

2 x 100
2 x 7
2 x 107

2 x 200
2 x 30
2 x 1
2 x 231

8) 10 x 5
10 x 50
10 x 500

10 x 8
10 x 80
10 x 800

10 x 12
10 x 120
10 x 1,200

20 x 3
20 x 30
20 x 300

20 x 5
20 x 50
20 x 500

20 x 8
20 x 80
20 x 800

9) $6 + 6$

2×6

$\frac{1}{2} \times 12$

$10 + 10$

2×10

$\frac{1}{2} \times 20$

$50 + 50$

2×50

$\frac{1}{2} \times 100$

$11 + 11 + 11$

3×11

$\frac{1}{3} \times 33$

$20 + 20 + 20$

3×20

$\frac{1}{3} \times 60$

$13 + 13 + 13$

3×13

$\frac{1}{3} \times 39$

10) $\frac{1}{2} \times 10$

$\frac{1}{2} \times 100$

$\frac{1}{2} \times 1{,}000$

$\frac{1}{2} \times 10{,}000$

$\frac{1}{2} \times 10{,}000{,}000$

$\frac{1}{2} \times 24$

$\frac{1}{2} \times 240$

$\frac{1}{2} \times 2{,}400$

$\frac{1}{2} \times 24{,}000$

$\frac{1}{2} \times 24{,}000{,}000$

$\frac{1}{3} \times 6$

$\frac{1}{3} \times 60$

$\frac{1}{3} \times 600$

$\frac{1}{3} \times 6{,}000$

$\frac{1}{3} \times 6{,}000{,}000$

11) $\frac{1}{2} \times 20$

$\frac{1}{2} \times 30$

$\frac{1}{2} \times 40$

$\frac{1}{2} \times 50$

$\frac{1}{2} \times 6$

$\frac{1}{2} \times 16$

$\frac{1}{2} \times 26$

$\frac{1}{2} \times 36$

$\frac{1}{2} \times 10$

$\frac{1}{2} \times 100$

$\frac{1}{2} \times 108$

$\frac{1}{2} \times 148$

12) 2×13

$\frac{1}{2} \times 26$

2×35

$\frac{1}{2} \times 70$

2×104

$\frac{1}{2} \times 208$

3×10

$\frac{1}{3} \times 30$

3×8

$\frac{1}{3} \times 24$

3×105

$\frac{1}{3} \times 315$

13) $\frac{1}{2} \times 40$

$\frac{1}{2} \times 8$

$\frac{1}{2} \times 48$

$\frac{1}{2} \times 100$

$\frac{1}{2} \times 12$

$\frac{1}{2} \times 112$

$\frac{1}{3} \times 90$

$\frac{1}{3} \times 6$

$\frac{1}{3} \times 96$

14) $\frac{1}{2} \times 14$

$\frac{1}{2} \times 15$

$\frac{1}{2} \times 20$

$\frac{1}{2} \times 21$

$\frac{1}{2} \times 7$

$\frac{1}{2} \times 9$

$\frac{1}{2} \times 11$

$\frac{1}{2} \times 13$

$\frac{1}{2} \times 100$

$\frac{1}{2} \times 101$

$\frac{1}{2} \times 102$

$\frac{1}{2} \times 103$

15) 2×3

2×2

2×1

2×0

$2 \times \hat{1}$

$2 \times \hat{2}$

$2 \times \hat{3}$

2×30

2×20

2×10

2×0

$2 \times \widehat{10}$

$2 \times \widehat{20}$

$2 \times \widehat{30}$

3×3

3×2

3×1

3×0

$3 \times \hat{1}$

$3 \times \hat{2}$

$3 \times \hat{3}$

16) Ask for a multiple of 3 (or 4) between two other numbers; for example, ask for a multiple of 3 between 20 and 25.

Estimation

Ask the students to estimate a calculation and to explain their answers. For example,

T: Is 218 + 157 more than 300? How do you know?

S: Yes, because 200 + 100 = 300.

T: Is it more than 500? How do you know?

S: No, because 300 + 200 = 500. 218 is less than 300 and 157 is less than 200, so 218 + 157 must be less than 500.

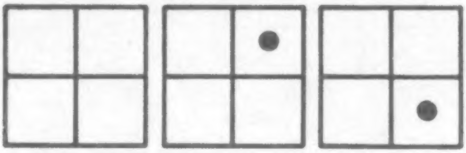
T: Is it more than 400?

S: No, because 18 + 57 is less than 100.

During the lessons, students are often asked to estimate a number displayed on the Minicomputer. See Lesson N33 for an example.

Other Mental Arithmetic Activities

1) Use the Minicomputer to give visual clues for addition, subtraction, and multiplication problems. For example, suppose this configuration is on the Minicomputer.



T: What number is on the Minicomputer? (41)

Hold a checker over the 8-square.

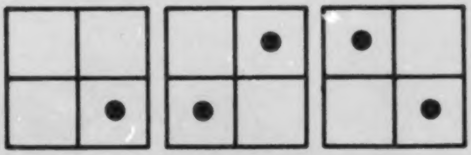
T: What number is 41 + 8? (49)

Hold a checker over the 10-square.

T: What number is 41 + 10? (51)

After a few such clues, hold a checker over the square only if the class has difficulty.

The next example involves subtraction.



T: What number is on the Minicomputer? (169)

Lift the checker temporarily from the 8-square.

T: What number is 169 - 8? (161)

Lift the checker temporarily from the 20-square.

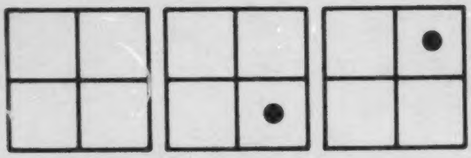
T: What number is 169 - 20? (149)

Lift the checkers temporarily from the 20-square and the 1-square.

T: What number is 169 - 21? (148)

After a few such clues, lift the checkers only if the class has difficulty.

he next example involves multiplication.



:   What number is on the Minicomputer? (14)

ift the checkers.

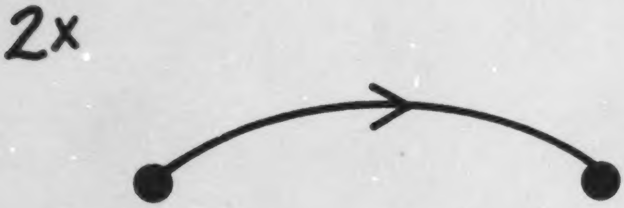
:   What number is 10 × 14?

:   140.

ove the checker on the 10-square to the 100-square; move the checker on the -square to the 40-square.

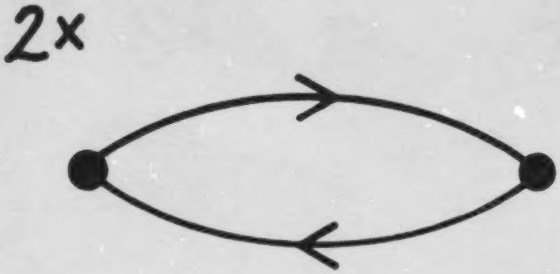
epeat this activity starting with other starting configurations.

2) Use an arrow picture to pose mental arithmetic questions. For example, draw this arrow picture on the board.



sk what number is at the right if the number at the left is, for example, 7.

Repeat this activity several times and then draw the return arrow.



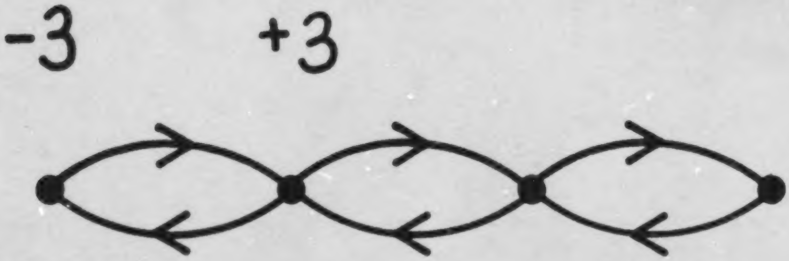
T: What could this blue arrow be for?

S: $\frac{1}{2}$ x.

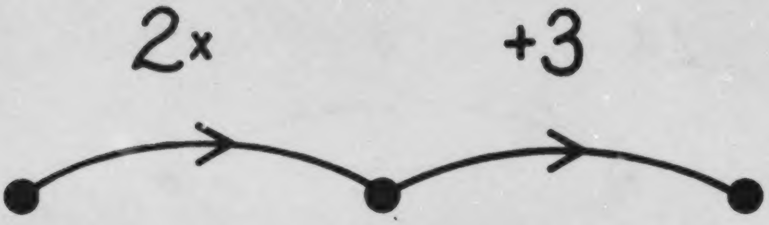
Ask the students what number is at the left if the number at the right is, for example, 10. Repeat this activity several times.

There are many variations of this activity; for example,

- use larger starting and ending numbers;
- use functions other than 2 x and $\frac{1}{2}$ x;
- use two or more arrows of the same function;



- use a composition of two functions.



3) Use the suggestion of an arrow picture to pose place value questions.

- Imagine an arrow road starting at 0 and having four + 10 arrows and seven + 1 arrows. What is the ending number?
- Imagine an arrow road starting at 0 and having two + 100 arrows, five + 10 arrows, and two + 1 arrows. What is the ending number?

4) Play "Guess My Rule". Choose a secret rule such as 3 x. A student suggests a number to you and you answer with 3 x that number. This continues until the students discover your rule; for example,

S: 5.

T: 15.

S: 1.

T: 3.

S: 100.

T: 300.

: 50.

T: 150. Does anyone know my secret rule?

S: 3 x.

T: How do you know?

S: Each time you answered a number which was 3 x the number we gave you.

Choose other secret rules such as + 3, + 5, - 2, - 4, $\frac{1}{2}$ x, 4 x, and 10 x.

Hand-Calculator Activities

Use the hand-calculator (i) to develop and reinforce counting skills; (ii) to emphasize counting patterns; and (iii) to improve estimation skills.

1) Use one or more hand-calculators in the class to support counting activities in which patterns are generated. Start with any whole number on the display of your calculator and press $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ .... Record on the board the number that appears on the display of the hand-calculator each time you press $\boxed{=}$. As soon as most of your students are able to predict the sequence quickly, you can abandon the calculator. Ask students to explain the pattern.

There are many variations of this activity, such as:

- press $\boxed{+}$ $\boxed{2}$ (or $\boxed{3}$ or $\boxed{4}$ or $\boxed{1}$ $\boxed{0}$) $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ...;
- start with a large number and press $\boxed{-}$ $\boxed{5}$ (or $\boxed{2}$ or $\boxed{1}$ $\boxed{0}$ or $\boxed{1}$ $\boxed{0}$ $\boxed{0}$) $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ....

2) Play "Let's Concentrate" using one or more hand-calculators in the class. Ask the students who have the calculators to put a number of your choice on the display and then to hide the display with one hand. Give a sequence of two operations for the students to do by pressing the appropriate keys and without looking at the display. Then ask what number is on the display and let the students check to see if they are correct. Continue with the new number and another sequence of two operations. For example,

Start with 4 on the display.

Press $\boxed{+}$ $\boxed{3}$ $\boxed{+}$ $\boxed{2}$ $\boxed{=}$. The resulting number is 9.
(Students check that 9 is on the display.)

Press $\boxed{+}$ $\boxed{1}$ $\boxed{\times}$ $\boxed{2}$ $\boxed{=}$. The resulting number is 20.
(Students check that 20 is on the display.)

Press $\boxed{-}$ $\boxed{3}$ $\boxed{+}$ $\boxed{1}$ $\boxed{=}$. The resulting number is 18.
(Students check that 18 is on the display.)

3) Play "Hand-Calculator Golf" with a small group of students, each one having a calculator. Ask the students to put a small whole number of your choice on the display of the calculators. Then let the students take turns suggesting some numbers to add or to subtract until a particular goal, such as 200, is reached.

A possible game in which 200 is the goal and four students are playing is described below.

		Display
Start with 12 on the display.		12
First player:	+ 9	21
Second player:	+ 72	93
Third player:	+ 17	110
Fourth player:	+ 7	117
First player:	+ 93	210
Second player:	- 10	200

Some variations of this game include:

- changing the starting number;
- making the goal a larger multiple of 100;
- allowing the use of $\boxed{\text{x}}$ $\boxed{\text{some number between 0 and 10}}$

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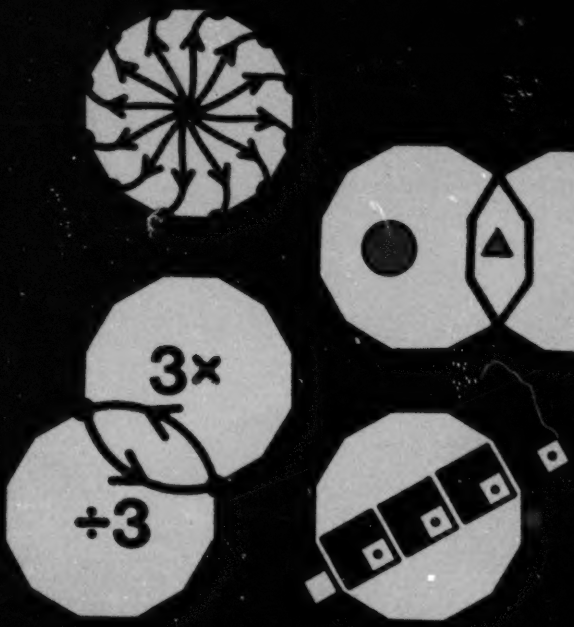
CSMP Mathematics for the Upper Primary Grades

Part III

Teacher's Guide

The Languages of Strings and Arrows
Geometry and Measurement
Workbooks



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Comprehensive School Mathematics Program
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Comprehensive School Mathematics Program

CSMP Mathematics
for the
Upper Primary Grades
Part III

The Languages of Strings and Arrows

CEMREL, Inc.

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THE LANGUAGES OF STRINGS AND ARROWS

INTRODUCTION

The language of strings is the nonverbal language of sets, that is, collections of objects put together in some way. The language of arrows is the nonverbal language of relations among objects. These languages permeate the CSMP content strands and are used, separately or together, to present an unending variety of numerical and nonnumerical situations.

Classification: The Language of Strings

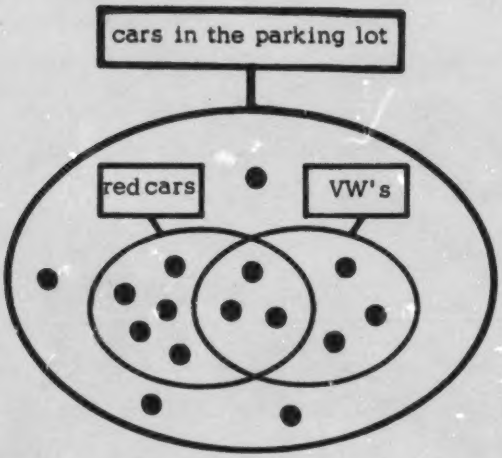
As the word implies, classifying means putting things into classes, or as the mathematician says: sets. If the mathematics of sets has any place in an elementary school mathematics curriculum, it is precisely because it can help students to understand and use the ideas of classification.

The basic idea is simple: given a set S and any object x, either x belongs to S (x is in S) or x does not belong to S (x is not in S). We represent this simple act of sorting -- "in" or "out" -- neatly by using string pictures to illustrate "in" and "out" in a dramatic way. Objects to be sorted are represented by dots, and the sets into which they are sorted are represented by drawing "strings" around dots. A dot inside (the region delineated by) a set's string is for an object in the set and a dot outside a set's string is for an object not in the set. (See, for example, Lesson L4 Multiples of 3 and 4.)

This language of strings and dots provides a precise (and nonverbal) way of recording and communicating thoughts about classification. The abilities to classify, to reason about classification, and to extract information from a classification are important skills for everyday life, for intellectual activity, in general, and for the pursuit and understanding of mathematics, in particular. Ordinarily, we are not expected to deal with such problems until our command of our natural language is fully developed and until we face classification problems in adult life. It is not that an ability to classify requires special intelligence, but that it requires a special mode of thinking. The unique quality of the language of strings is that it provides a nonverbal language that is particularly suited to the mode of thinking involved in classification. It frees young minds to think logically and creatively about situations which involve classifying and to report their thinking long before they have extensive verbal skills.

In this strand we present situations and ask carefully phrased questions to continue to advance skills in classification, always remembering that the skills grow out of such experiences. To be able to draw strings and dots is not an objective in itself; to develop the mode of thinking involved is the objective. Thus it is important for us to construct the situations carefully. The sets into which we ask students to classify objects must be determined by well-defined attributes; otherwise, the students have the added problem of deciding whether a certain object does or does not have a certain attribute. For this reason we make extensive use of A-Blocks (Attribute Blocks) and particular sets of numbers in designing classification situations. Students can immediately say whether or not a block is red, whether or not it is a square, and so on. They know whether or not a given number is less than 15, whether or not it is odd, and so on.

One reason for classifying objects is to count the objects that have a certain attribute. Suppose for example, in a parking lot there are eight red cars and six Volkswagons. If four cars in the lot are neither red nor Volkswagons, must there be eighteen cars in the lot? Suppose we count the cars in the parking lot and find there are exactly fifteen. Is this possible? A string picture immediately settles this apparent paradox.



There are three red Volkswagons.

Relations: The Language of Arrows

Relations are interesting and important to us in our everyday lives, in our careers, in school, and in scientific pursuits. We are always trying to establish, explore, and understand relations. In mathematics it is the same; to study mathematics means to study the relations among mathematical objects like numbers or geometric shapes. The tools that we use to understand the everyday world are useful to us in the world of mathematics. Conversely, the tools that we develop to help us think about mathematical things often serve us in nonmathematical situations.

A serious study of anything requires a language for representing the things under investigation. The language of arrows provides an apt language for studying and talking about relations. Arrow diagrams are a handy graphical representation of a relation, somewhat the same way that a blueprint is a handy graphical representation of a house. By means of arrow diagrams, we can represent important facts about a given relation in a simple, suggestive, pictorial way -- usually more conveniently than the same information could be presented in words. (See, for example, Lesson L11 THE LITTLE DONKEY.)

The convenience of arrow diagrams has important pedagogical consequences or introducing children to the study of relations in the early grades. A child can "read" -- and also draw -- an arrow diagram of a relation long before he or she can read or present the same information in words. The difficulty of presenting certain ideas to children lies not in their intellectual inability to grasp the ideas; rather, the limitations are often mechanical, motor problems of writing or reading about the ideas in conventional systems. Arrow diagrams have all the virtues of a good notation: they present information in a clear, natural way; they are attractive, colorful things to look at; they are easy and fun for children to draw. Another educational bonus occurs when arrow diagrams are used imaginatively. A story or problem may be captured by an arrow diagram, and at the same time, the resulting arrow diagram may suggest other, similar situations. This allows students to call upon previous experiences or to expand their creativity.

One of the purposes of this strand is to exploit the power of the language of arrows to help children think logically about relations. Again it must be remembered that our goals concern the thinking process and not the mechanism. We resist making the ability to draw prescribed arrows an objective in itself, ust as we resist viewing an arrow diagram as just another format for drill problems in arithmetic.

The general aim of The Languages of Strings and Arrows strand is to suggest situations that are inherently interesting and thought provoking, and to give children modes of thinking and appropriate languages with which they can rganize, classify, and analyze. In addition to a varied assortment of lessons concerning sets and relations, this strand includes lessons involving systematic methods for solving combinatorial (counting) problems; methods for ollecting, recording, and interpreting data in "real life" situations; and robabilistic thinking.

L1 The String Game[*] #1

CAPSULE LESSON SUMMARY

Describe each of the A-Blocks by its shape, color, and size. Introduce and play the String Game with two strings and nine possible labels. Discuss the "not" attributes. Place all the A-Blocks in a two-string picture where one string is labeled with a "not" attribute. Play the String Game allowing the possibility that the strings could have any of the sixteen labels.

MATERIALS

Teacher: The String Game with A-Blocks kit; colored chalk
Student: None

NOTE: You will need to prepare some materials in advance for this lesson. They will be used also in future lessons on the String Game.

1. Punch out a set of A-Blocks. If you have a magnetic chalkboard available, magnetize each A-Block by sticking a small piece of the magnetized material to the back. If there is no magnetic chalkboard available, you can still use a regular chalkboard. In this case, attach a loop of masking tape or some other sticky substance to the back of each A-Block so that they can be stuck to the board. Be prepared to reinforce the A-Block pieces regularly.
2. Prepare the string cards in the same manner as the A-Blocks.

DESCRIPTION OF LESSON

Exercise 1

Before the lesson begins, put all the A-Blocks in a small box (a greeting cards box is a good size).

T: In this box, I have some pieces which are different shapes, colors, and sizes. What do you think the different shapes are?

S: Squares.

---

[*] See the appendix "The String Game" for a description of the various versions of the game.

L1

S : Triangles.

S : Circles.

Students who had this program last year will be familiar with these pieces rom having played the String Game.

T : What are the different colors of the pieces?

S : Red.

S : Blue.

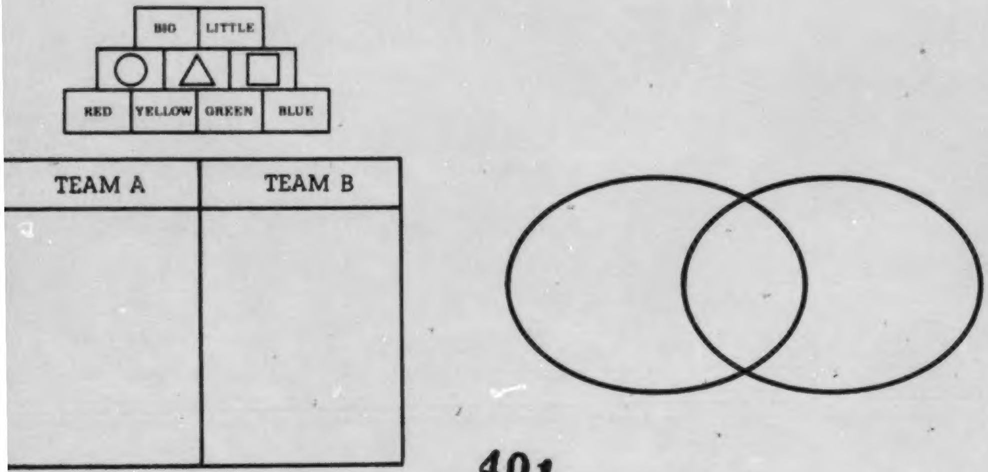
S : Green.

S : Yellow.

T : What are the two different sizes?

S : Big and little.

Tape a copy of the list of nine possible labels to the board and prepare for the String Game as in the diagram below.


BIG
LITTLE
RED
YELLOW
GREEN
BLUE
TEAM A
TEAM B

Indicate to your class which students are on Team A and which are on Team B.

T: We are going to play the String Game with shapes, but first you must help me take the game pieces out of the box. The teams take turns; someone from Team A can tell me a piece to put on Team A's side of the board, and then someone from Team B can tell me a piece to put on Team B's side of the board. Remember to describe a piece which is still in the box and to make it clear exactly which piece you want.

Alternating teams call on students to describe game pieces. Insist on complete descriptions as called for in the following discussion.

S: A big circle.

T (looking in the box): I have three big circles. Which one do you want?

S: The red one.

T: I have a big red circle (holding up this piece and placing it on the appropriate team's side of the board).

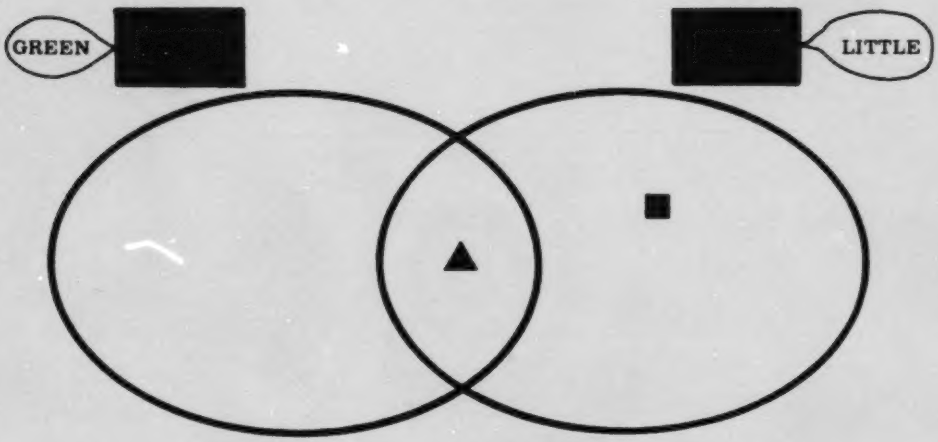
If a piece described is already on the board, point to it and ask for another. Call on weaker students early in the exercise so that there are many choices open to them. Encourage all students to be thinking about which pieces they will ask for when called upon.

Continue until all the pieces have been divided between teams. Leave them on the board for the next exercise.

L1

Exercise 2

Place the string card "GREEN" face down near the red string and the string card "LITTLE" face down near the blue string. Choose one piece from each team's choices to use as clues; for example,

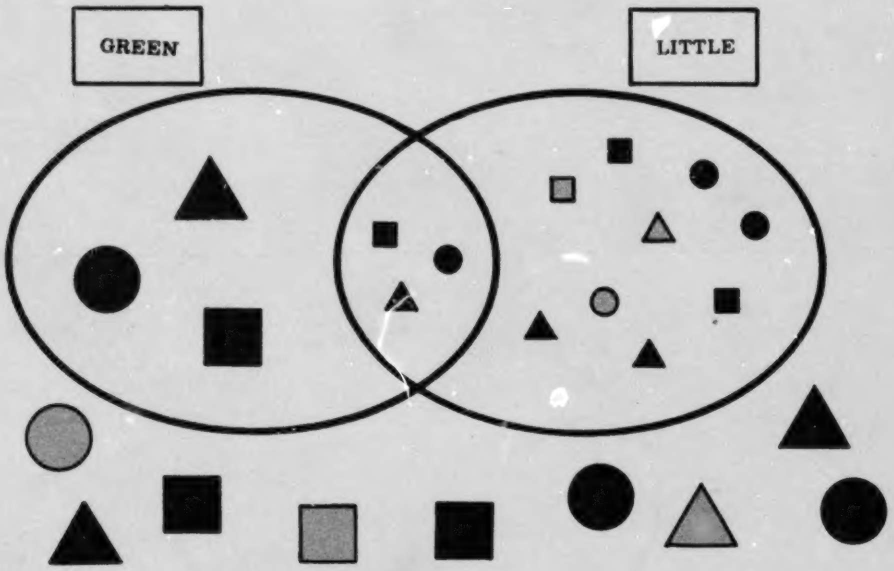


Play the String Game with your class using the same teams as in Exercise 1.

Remember that the rules of the game call for you to judge the positioning of the pieces. If a piece is correctly placed, say "yes" and immediately invite the player to place a second piece. No player should have more than two consecutive plays. If a piece is incorrectly placed, say "no" and ask the player to return the piece to his or her team's side of the board.

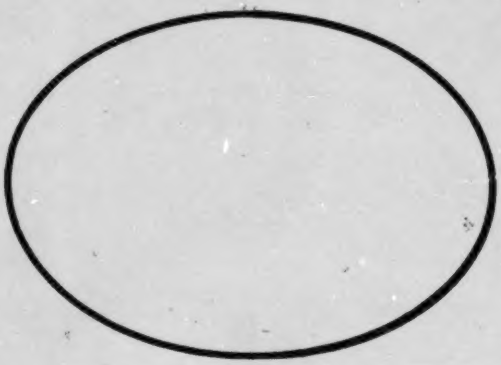
The game ends when a player correctly places the last piece from a team's choices and correctly identifies both strings. If the player gives an incorrect label for one of the strings, the next player on the opposing team has a chance to place pieces from that team's unplayed pieces or to identify the strings when all their pieces have been correctly placed.

The next picture shows correct placement for the game pieces with the strings having these particular labels.



Exercise 3

Clear the board from the last exercise. Have ready the not-cards for the discussion. Draw one string on the board.



T: I am going to show you some other possibilities for string labels.

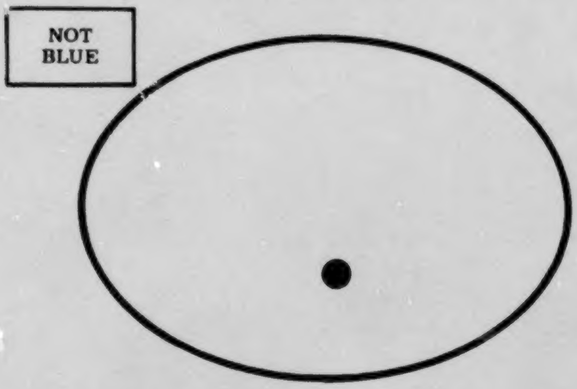
L1

Show the class the string card "NOT BLUE".

T: If the red string were for "NOT BLUE", what would be a piece which goes inside the string?

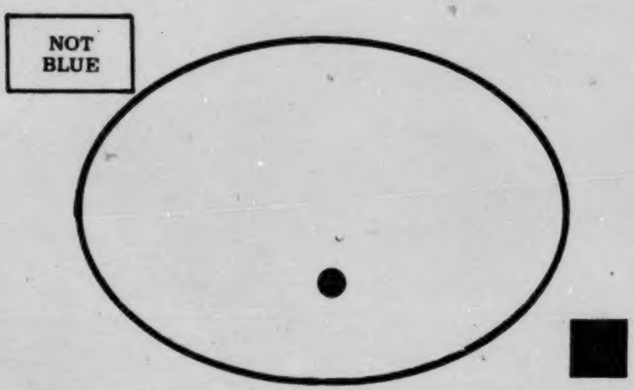
S: A small green circle.

Give the student this piece to put in the picture. If necessary, discuss a few other pieces which would be inside the string.



T: What would be a piece which goes outside the string?

S: A large blue square.



Take the A-Blocks off the board and erase the string.

T: What other new string labels do you think I have?

Show the class each not-card when it is mentioned.

S: "NOT RED".

S: "NOT GREEN".

S: "NOT YELLOW".

T: I also have a card "NOT ○". (Show the card to the class.) Who can describe a piece which is not a circle?

S: A small blue triangle.

Continue in this manner considering "NOT △" and "NOT □".

T: Do I need a card "NOT LITTLE"?

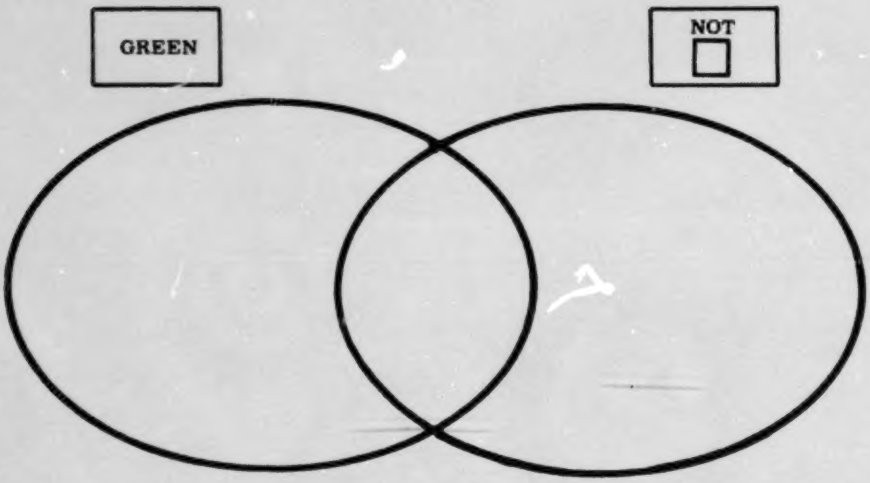
S: No, "BIG" is the same as "NOT LITTLE".

T: . . . and "NOT BIG"?

S: No, "LITTLE" is the same as "NOT BIG".

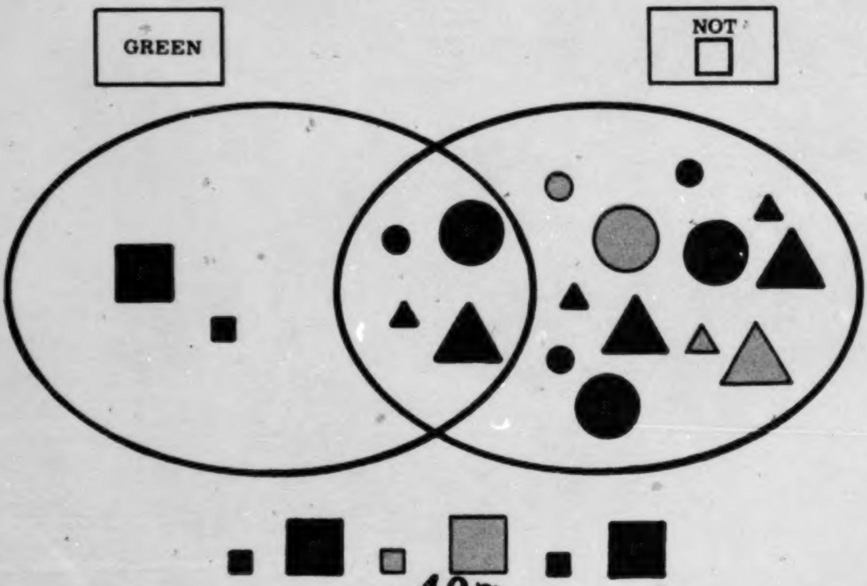
L1

Draw two overlapping strings on the board using two different colors. Next to the strings, attach these string cards face up as shown below.



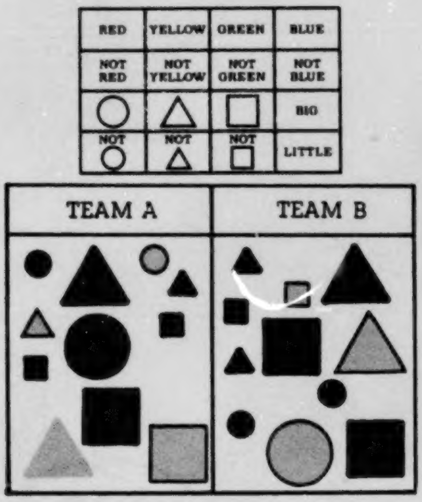
Let the students place various pieces of their choice (or yours) in the picture. After about eight to ten pieces have been put on the board, the class should be better acquainted with the new labels.

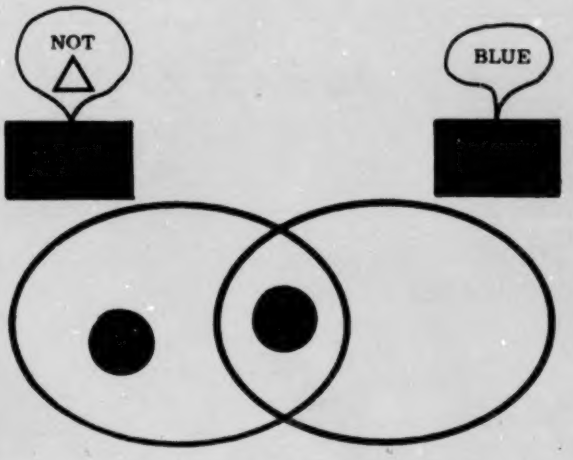
The next picture shows the correct placement of all the game pieces.



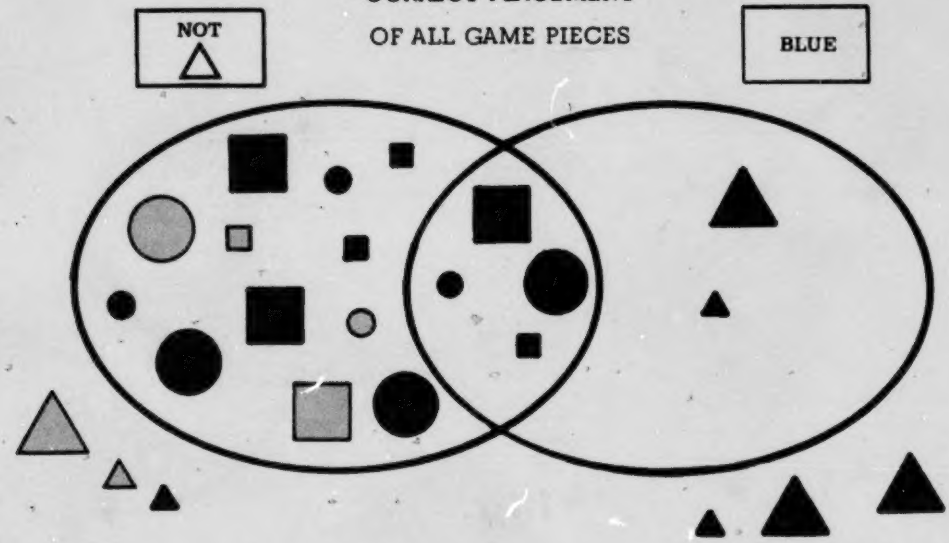
Exercise 4

Prepare your board to play a two string version of the String Game in the usual way with face down string cards. Tape the poster list of sixteen labels above the team board. Distribute game pieces for Team A and Team B on the Team board. A possible game is described in the next illustrations; the first illustration gives two starting clues and the second gives the correct placement of all the game pieces.


RED
YELLOW
GREEN
BLUE
NOT RED
NOT YELLOW
NOT GREEN
NOT BLUE
BIG
NOT
NOT
NOT
LITTLE
TEAM A
TEAM B



CORRECT PLACEMENT
OF ALL GAME PIECES



L2 Composition Games #1

CAPSULE LESSON SUMMARY

Play a composition game with colored arrows where the only rule for the game is: yellow followed by blue is red.

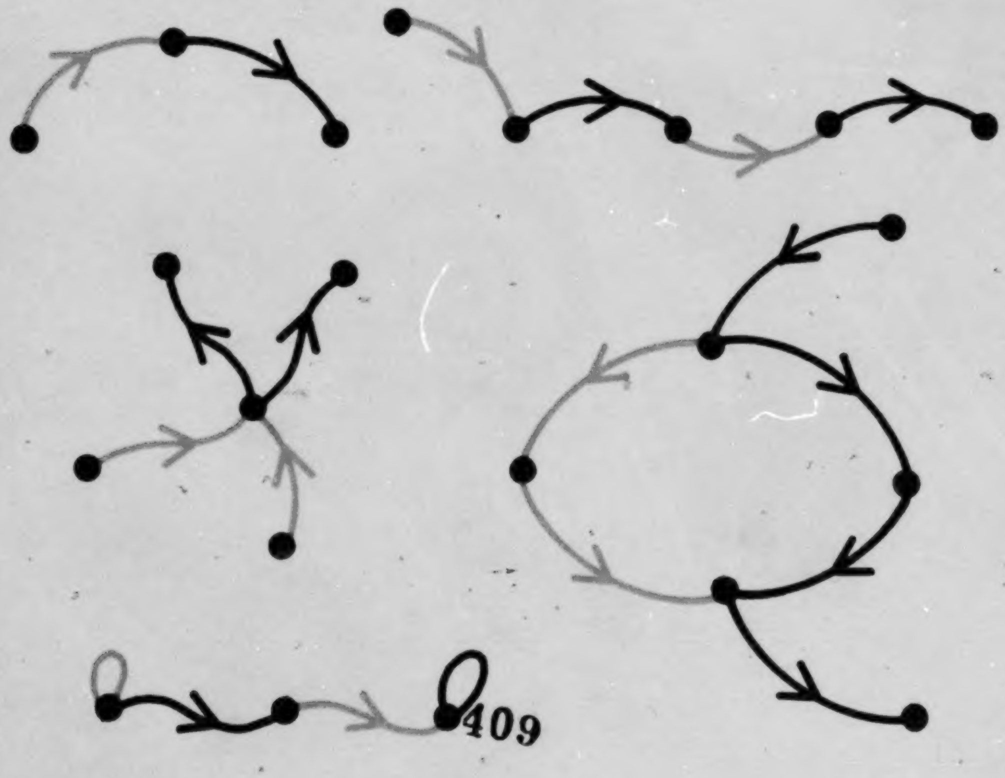
MATERIALS

Teacher: Colored chalk
Student: Worksheets L2*, **, ***, and ****; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Before the lesson begins, draw this arrow picture on the board. Spread the picture out over the entire board, and draw arrows in colors that can be easily distinguished.

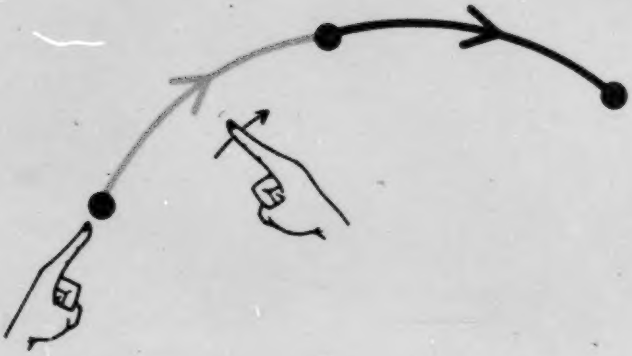


Let the students comment on the picture.

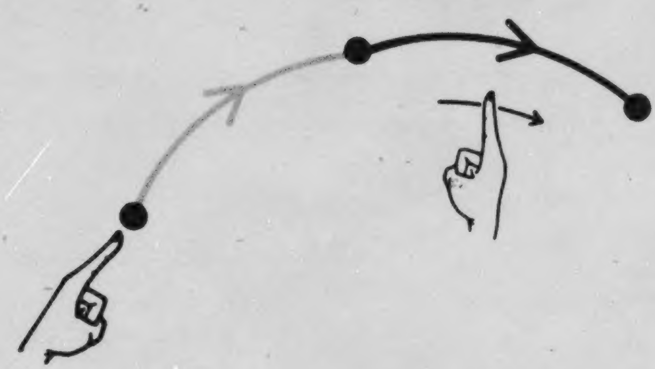
T: We are going to play a game with yellow and blue arrows. Some of you will remember this game from last year. The object of the game is to draw red arrows, but there is a rule for drawing red arrows.

Use the upper left corner of the arrow picture for your explanation. As you explain the rule stated in the left column below, make the motions described in the right column.

T:	This is the rule of the game...	Place your left forefinger on a dot at which a yellow arrow starts.
	Each time there is a yellow arrow...	With your left forefinger in place at the dot, follow the yellow arrow with your right forefinger in the direction of the arrowhead.

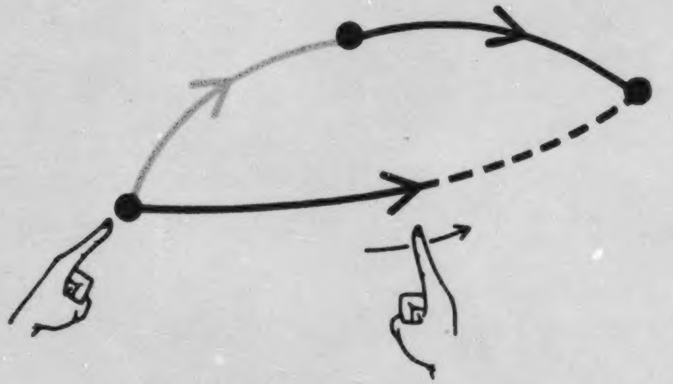


T:	...followed by a blue...	Stop the motion of your right forefinger at the middle dot, tap the dot, and then follow the blue arrow.

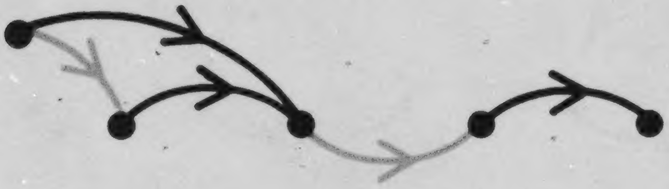


T:	...then we can draw a red arrow from the dot where the yellow arrow starts...	Tap this dot several times with your left forefinger.
	...to the dot where the blue arrow ends.	Tap this dot several times with your right forefinger.

First trace and then draw the red arrow.



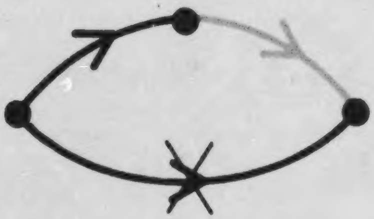
Repeat the statement of the rule using the same words and motions with the upper right portion of your picture. Trace and then draw this red arrow.



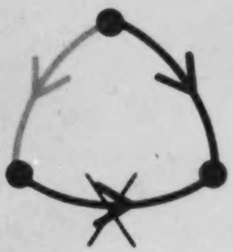
T: Where can we draw another red arrow?

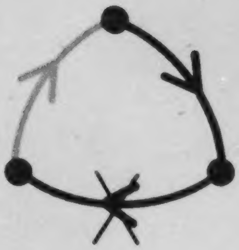
Invite students one at a time to the board to show where other red arrows can be drawn. Ask the students to first trace a yellow arrow and a blue arrow which follows it and then to trace how a red arrow can be drawn. Stop a student who starts to trace against the direction of an arrow and emphasize that the direction of an arrow must be followed. The rest of the class should help you check for mistakes. Let a student draw a red arrow if it has been correctly traced.

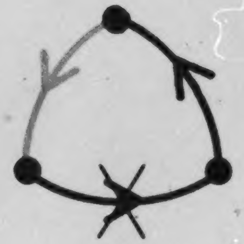
Note on common mistakes and difficulties: The most common mistake at first will occur when students confuse the rule "yellow followed by blue" with "blue followed by yellow."



Another mistake might occur when the directions of the arrows are not correctly followed.

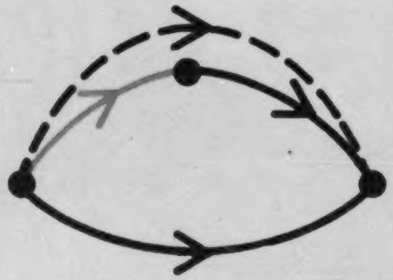






Remind students making such mistakes that they must follow the direction of the arrows. The arrows are like one-way streets and they are not allowed to go the wrong way.

If it happens that a student looks at a portion of the arrow picture and proposes to draw a red arrow as indicated by the dotted arrow, ...

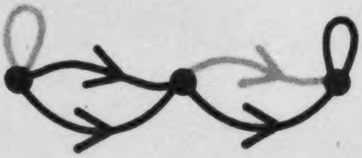


... another student will probably remark that the red arrow has already been drawn. There is nothing wrong with the dotted arrow; it is merely redundant. That portion of the picture is complete without it.

It is likely that the loops on the left side of the picture will cause difficulty. If no one suggests either of the following red arrows, suggest one of them yourself.

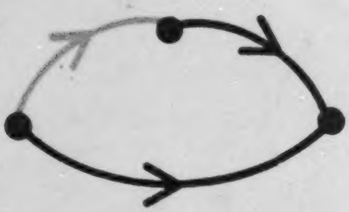
Yellow (trace around the yellow loop) followed by blue (trace the blue arrow), then red.

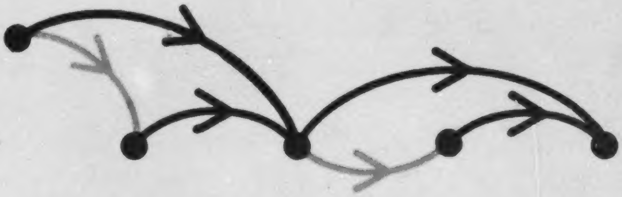
Yellow (trace the yellow arrow) followed by blue (trace around the blue loop), then red.

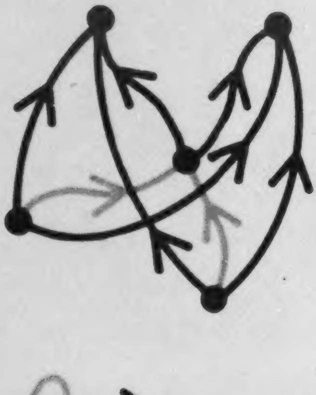


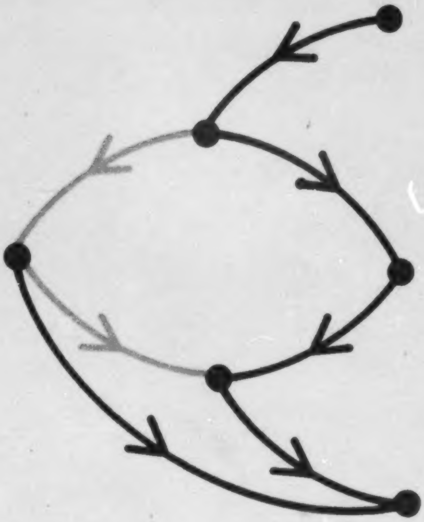
It is important to keep your left forefinger on the dot where a yellow arrow (loop) starts and then trace with your right forefinger ending at the dot where the blue arrow (loop) ends. Then a red arrow starts at your left forefinger and ends at your right forefinger.

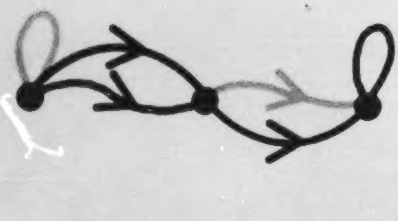
When all the red arrows have been found, your picture should look like this one.











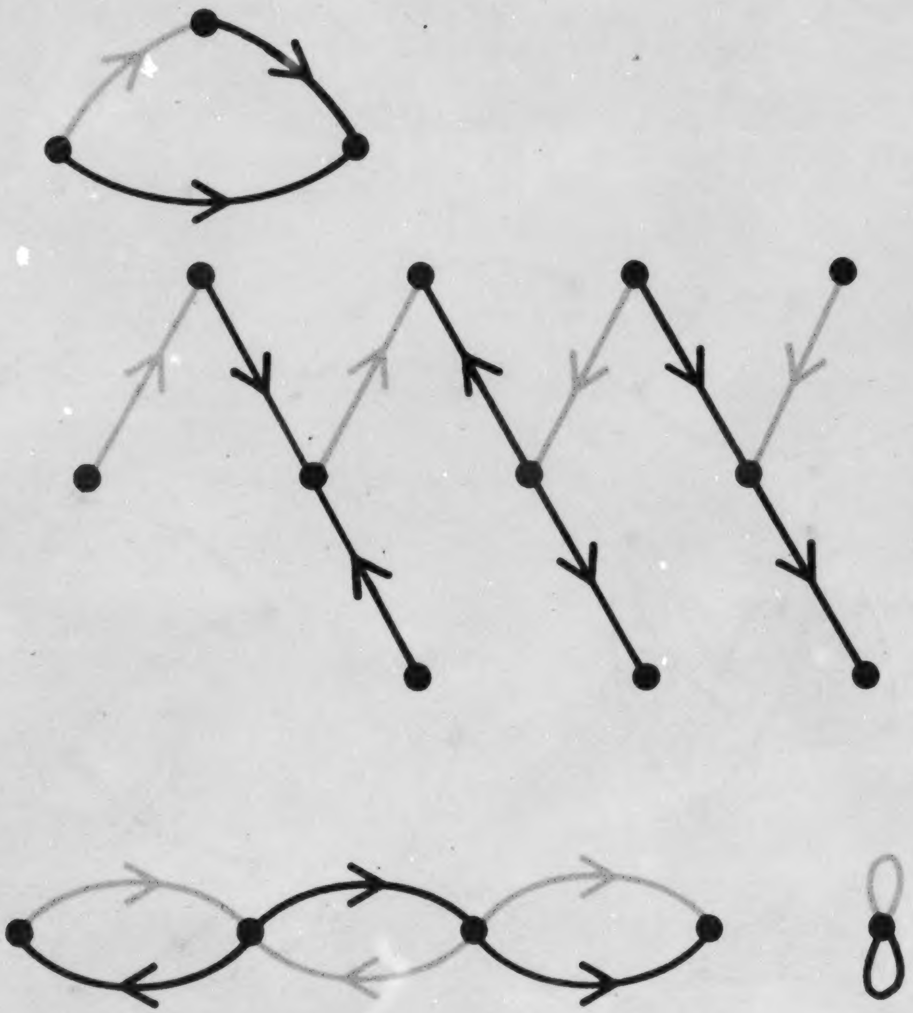
L 2

Exercise 2

Erase the board except for the upper left portion of your picture.

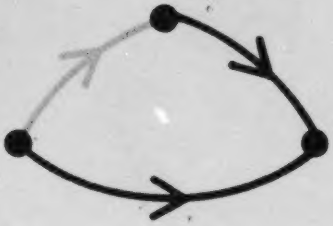
T: Now you know the rule for drawing red arrows : a yellow arrow followed by a blue arrow, then a red arrow. Watch as I draw a new picture and see how many places you can find to draw red arrows.

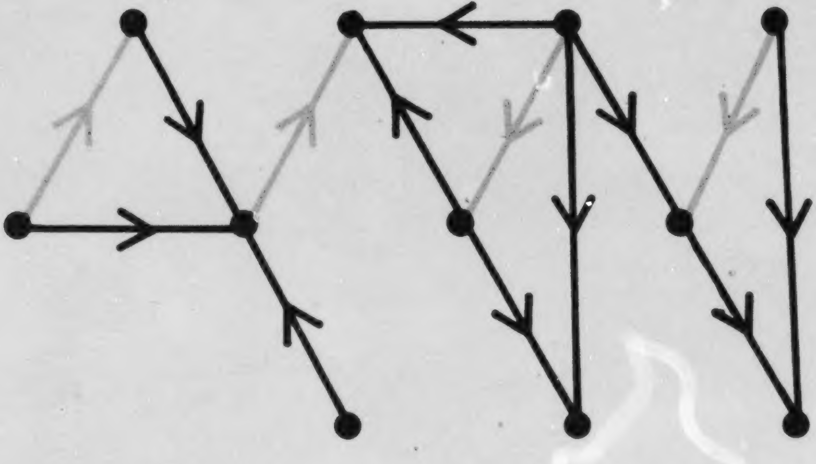
Draw this arrow picture spread out on the board.

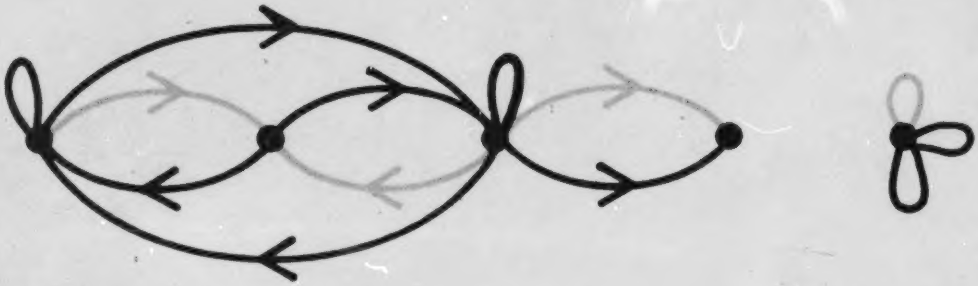


Invite students to indicate where a red arrow can be drawn by first tracing a yellow arrow and a blue arrow which follows it and then tracing the appropriate red arrow. After class agreement (including yours) on where a red arrow can be drawn, let a student draw the arrow. Try to include the entire class in the discussion and in the drawing of arrows.

When all the red arrows have been drawn, your picture should look like this one.



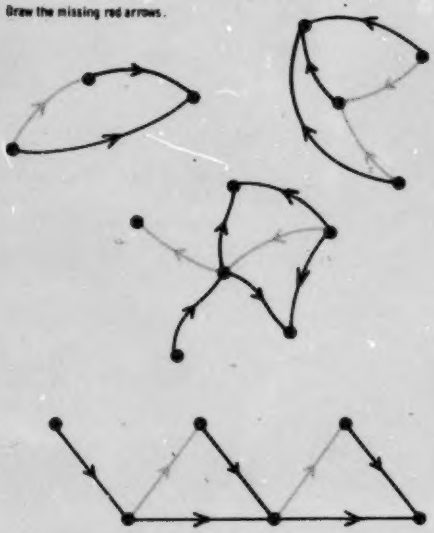




Worksheets L2*, **, ***, **** are available for individual work.

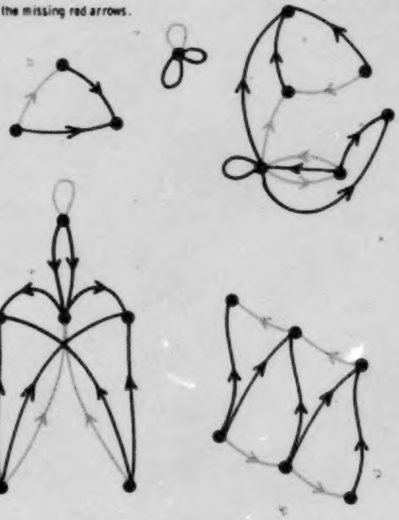
Name ____________ **L 2 ***

Draw the missing red arrows.



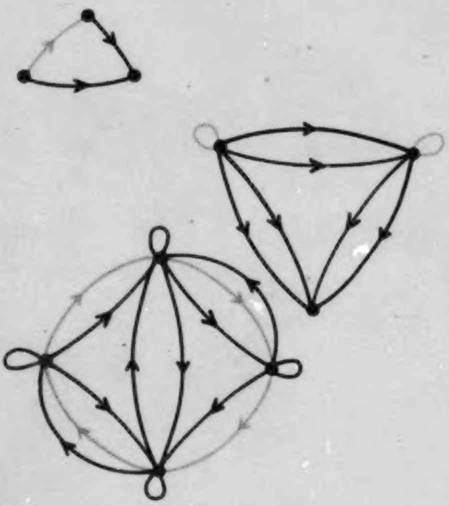
Name ____________ **L 2 ****

Draw the missing red arrows.



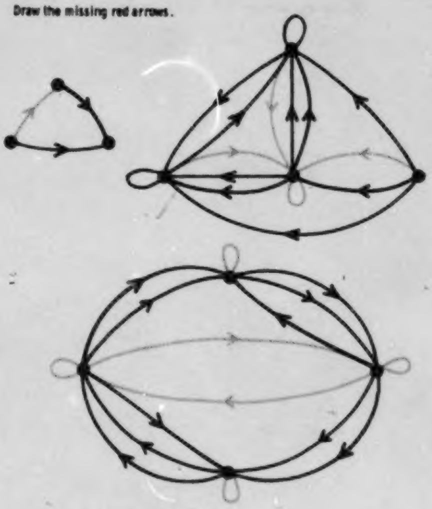
Name ____________ **L 2 *****

Draw the missing red arrows.



Name ____________ **L 2 ******

Draw the missing red arrows.



L3 Two Lists

CAPSULE LESSON SUMMARY

Determine how many people are invited to the birthday party of twins when it is known that one twin invites six friends and the other invites eight friends. Make up and solve some similar problems.

MATERIALS

Teacher: Colored chalk
Student: Paper; colored pencils

DESCRIPTION OF LESSON

Before the lesson begins, copy these lists of names using two separate slips of paper. Do not let your students see the names on these lists.

Jill's

Jason
David
Christy
Melissa
Angela
Ellen

Jack's

Sam
Jason
Alice
Russell
Christy
Mark
Laura
Ellen

T: Have you ever heard of the twins Jack and Jill?

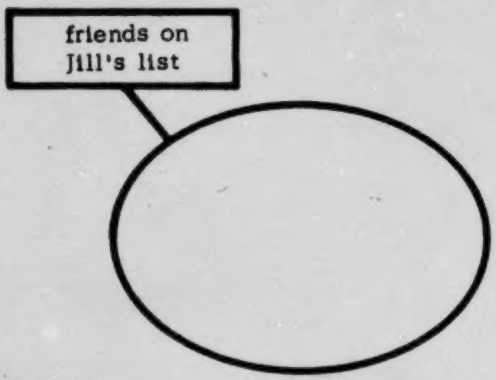
Let the students comment. (Veteran CSMP students might recall that Jack and Jill are the twins who live next to Booker's Bakery from a story in the second grade geometry lessons.)

L 3

T: Jack and Jill are going to have a birthday party. Each of them has made a list of children to invite. Jill put (look at Jill's list and count) 1, 2, 3 ..., 6 children on her list, and Jack put (look at Jack's list and count) 1, 2, 3, 4 ..., 8 children on his list. How many children will they invite altogether?

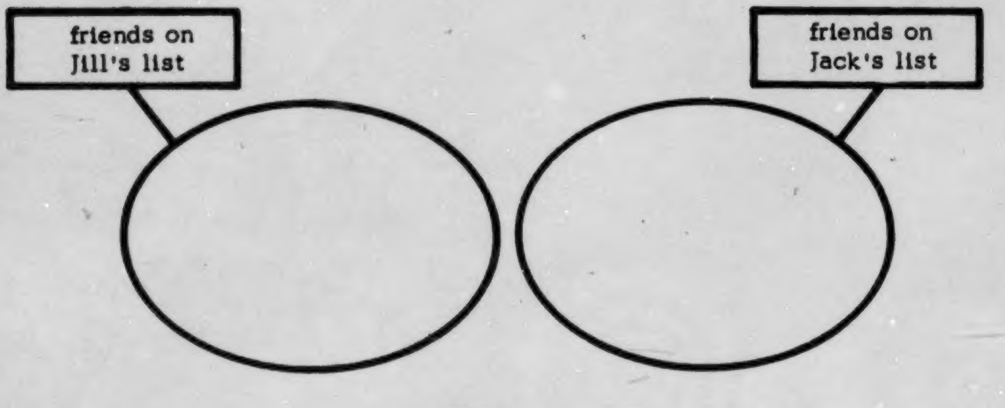
Predictions in our development classes included 13 and 14. It is very likely that your students will respond similarly by adding the number of names on each list. Perhaps some will have the insight that a name might be on both lists. On the board, record the students' predictions without comment.

T: Let's draw a picture to see if you are correct. I will draw a red string for the children on Jill's list.



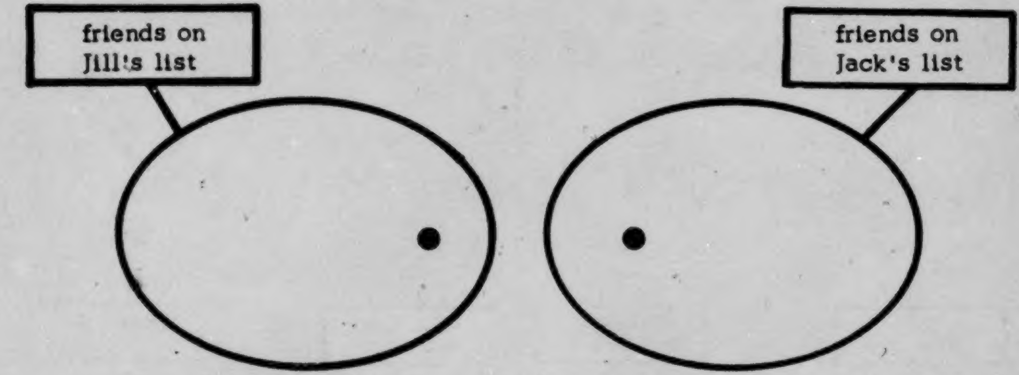
T: Who can draw a blue string for the children on Jack's list?

Let a student draw a blue string on the board. The student might draw this picture, and if so, initiate a discussion similar to that which follows.



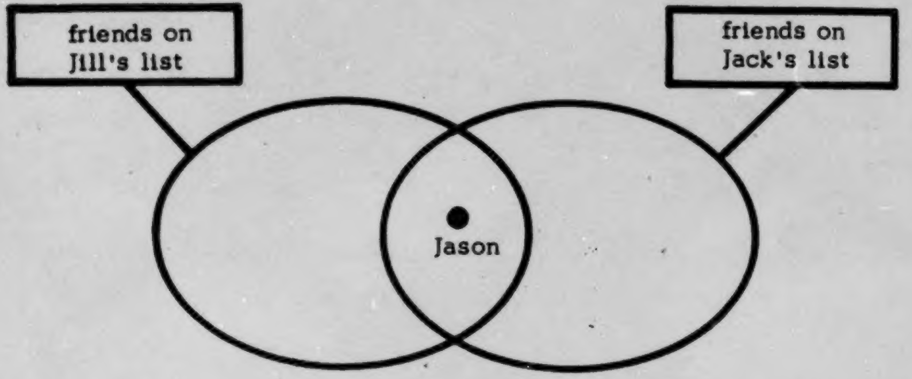
T: I have a problem. Jason's name is on both Jill's list and Jack's list. Where should we put a dot for Jason?

A student might suggest that two dots be drawn--one in each string.


friends on Jill's list
friends on Jack's list

T: But Jason is just one person. There should be only one dot for Jason. How can we solve the problem?

After perhaps a couple of other attempts, a student should suggest overlapping the strings and putting a dot in the middle. Let someone draw the strings and the dot in this manner. Label the dot "Jason".

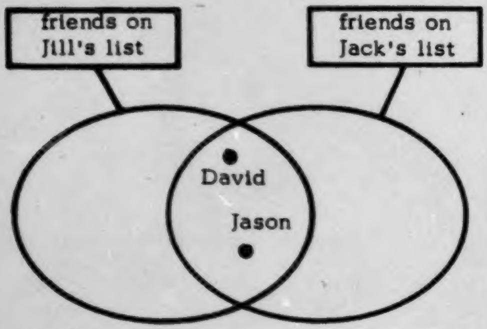

friends on Jill's list
friends on Jack's list
Jason

NOTE: If the blue string is drawn overlapping the red string the first time someone draws it, go on with the lesson without mention of the preceding problem. In this case, adjust the dialogue which follows to consider Jason first.

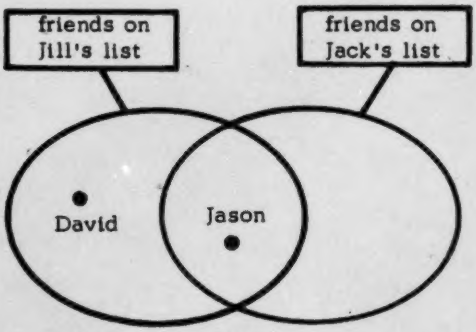
T: The first name on Jill's list is "Jason" and we have a dot for Jason in our picture. The next name is "David". Who would like to draw a dot for David?

A student could respond by guessing where to put the dot or by hesitating since at this point David could be in one of two places in the string picture.

"David" is on both
Jack's and Jill's lists.


friends on Jill's list
friends on Jack's list
David
Jason

"David" is on Jill's
list but not on Jack's.


friends on Jill's list
friends on Jack's list
David
Jason

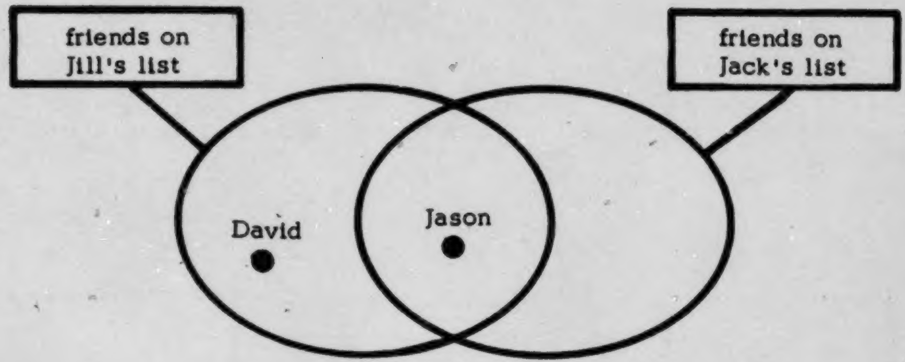
If a guess is made, ask, "How do you know where to put the dot?" Guide the discussion until it is clear that the class cannot be sure at this time where to put the dot.

T: You are right to think about the situation. Perhaps you can ask a question so you will be sure where to put the dot for David.

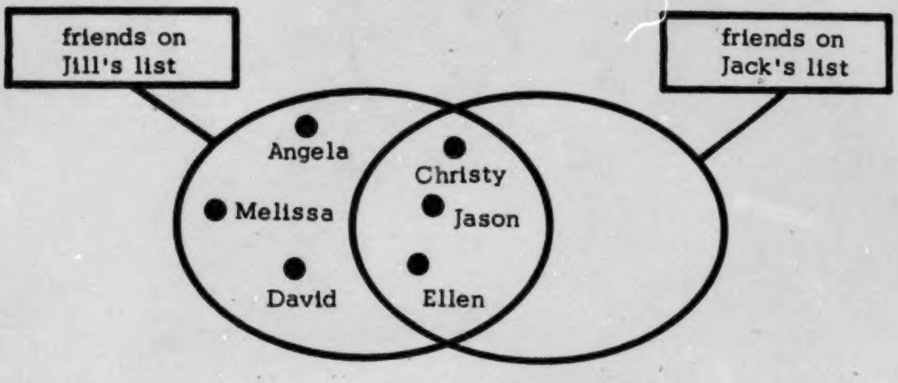
S: Is "David" on Jack's list?

T: That's a very good question. No, "David" is not on Jack's list.

Let a student draw and then label a dot for David.


friends on Jill's list
friends on Jack's list
David
Jason

Continue in this way until all of the names on Jill's list have been considered.


friends on Jill's list
friends on Jack's list
Angela
Melissa
David
Christy
Jason
Ellen

At this point you could ask the students whether they still think their predictions on the number of children who will be invited to the party are correct. If some students wish to change their predictions, ask them to explain why. Again do not comment on the correctness of their responses.

T: Now I'll look at Jack's list. How many names are on his list?

S: Eight.

T: How many children from Jack's list are already in our picture?

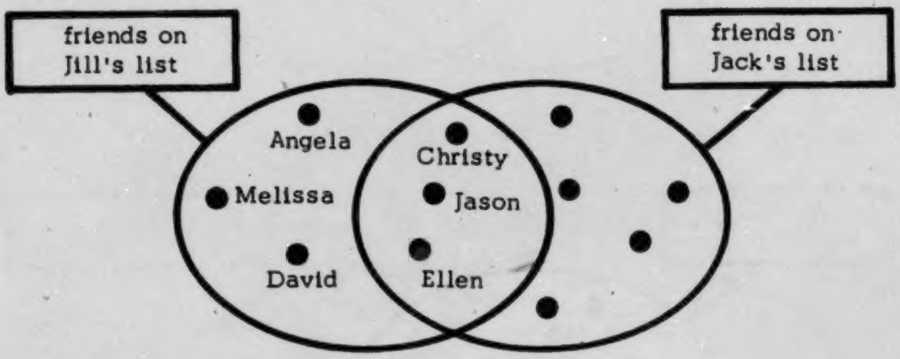
S: Three: Christy, Ellen, and Jason.

T: So how many other children did Jack put on his list?

S: Five.

T: Without knowing their names, who can put dots for these other five children in our picture?

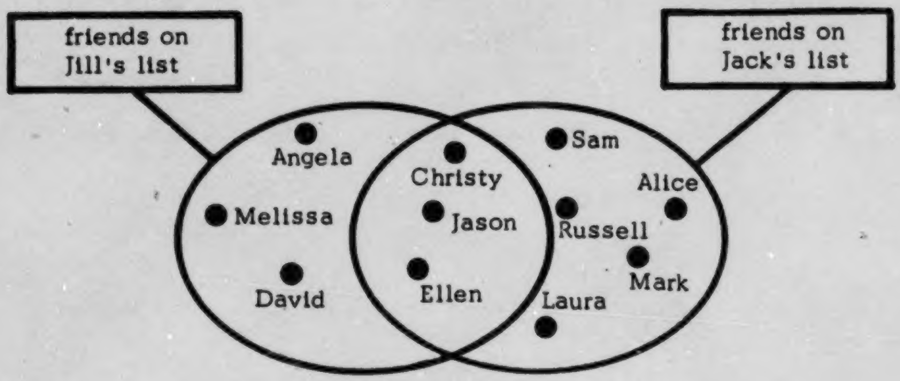
Let a student draw the dots.


friends on Jill's list
friends on Jack's list
Angela
Melissa
David
Christy
Jason
Ellen

T: How do you know they are there?

S: We found all the children whose names are on both lists when we went through Jill's list.

Read the names from Jack's list and label the dots.


friends on Jill's list
friends on Jack's list
Angela
Melissa
David
Christy
Jason
Ellen
Sam
Alice
Russell
Mark
Laura

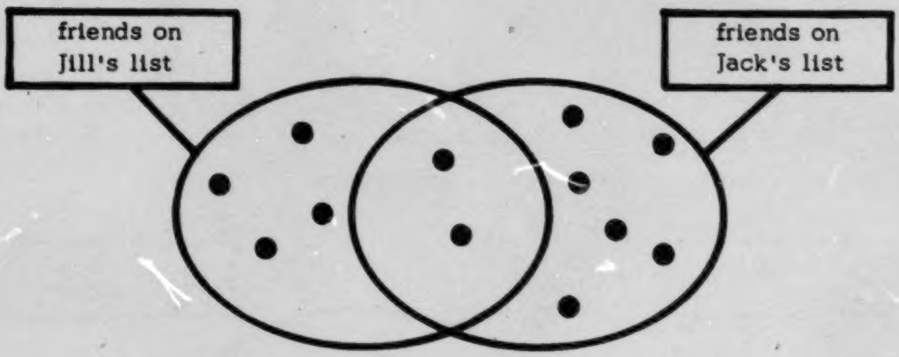
T: How many children will be invited to Jack and Jill's birthday party? Count with me as I point to each dot.

S: 1, 2, 3, ..., 11.

T: Many of you predicted fourteen children. Can anyone explain why the number of children is eleven instead of fourteen?

S: Christy, Jason, and Ellen are on both lists.

Erase the students' names from the picture; also erase one of the dots in the middle region. Put an additional dot in each string, but put neither one in the middle region.

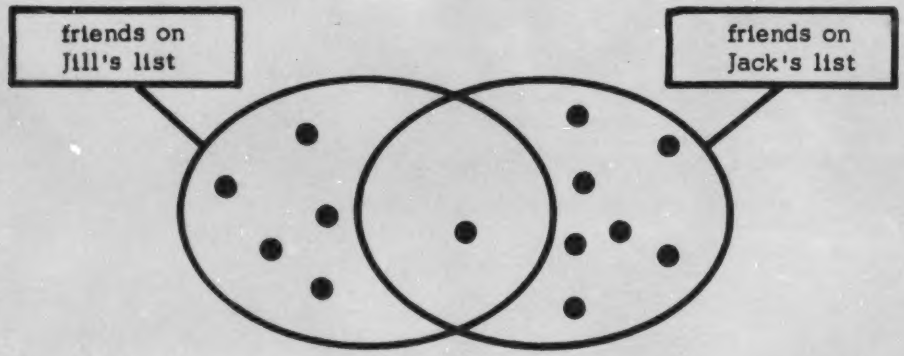


T: Suppose there had been only two names common to both lists, but there were still six names on Jill's list and eight names on Jack's list. How many children would be invited to the birthday party?

S: Twelve.

L3

Erase another dot from the middle region and replace it with an additional dot in each string (but not in the middle region).

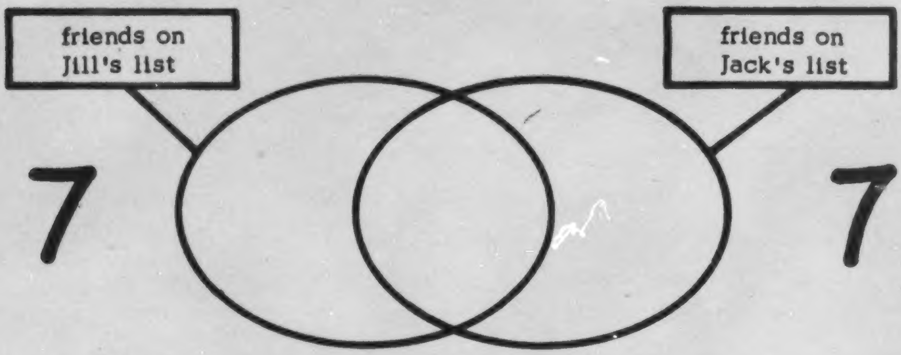

friends on Jill's list
friends on Jack's list

T: What if only one name were common to both lists ...?

S: Then thirteen children would be invited to the party.

Erase the dots in the picture and distribute unlined paper.

T: Suppose there were seven names on Jill's list and seven names on Jack's list, and altogether there were ten names. (Write "7" by each string for the students' reference.) Can you figure out where the dots would be in the strings? Draw the picture on your paper.


friends on Jill's list
friends on Jack's list
7
7

Observe the students' work. Each time you check a student's picture, ask:

T (tracing Jill's string): Are there seven dots in this string?
(tracing Jack's string): Are there seven dots in this string?
(pointing to several dots): Are there ten dots altogether?

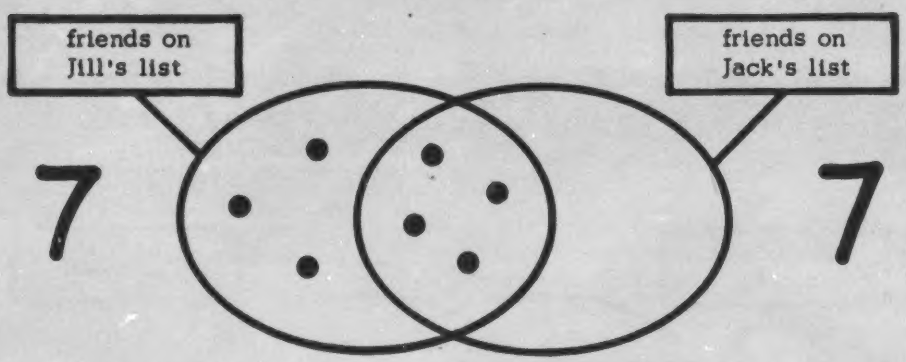
If some students finish quickly and their solutions are correct, suggest another situation. Two possibilities are given below.

a)	b)
Ten names on Jill's	Twenty-five names on Jill's
Eight names on Jack's	Fifteen names on Jack's
Thirteen names altogether	Thirty names altogether

After most of the students have found the solution to the original problem, call the class' attention to the board and go over the situation collectively.

T: Who can draw the dots on the board for Jill's string?

Let someone draw the dots in Jill's string only.



T: Are there seven dots in Jill's string? (Yes)

How many names are on Jack's list? (Seven)

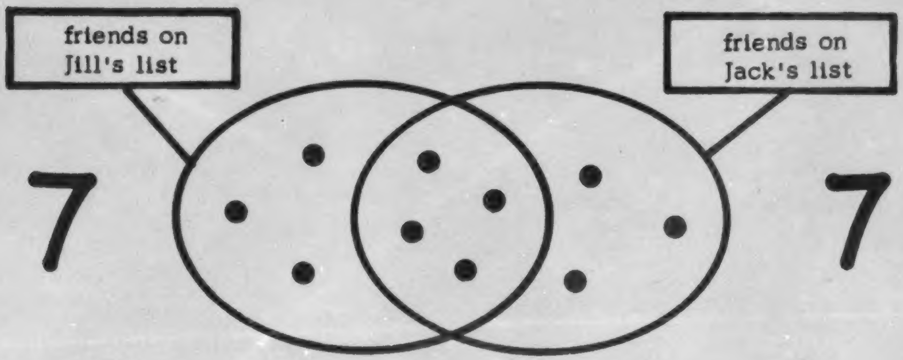
So we need seven dots in Jack's string.

How many dots are already in Jack's string? (Four)

How many more do we need? (Three)

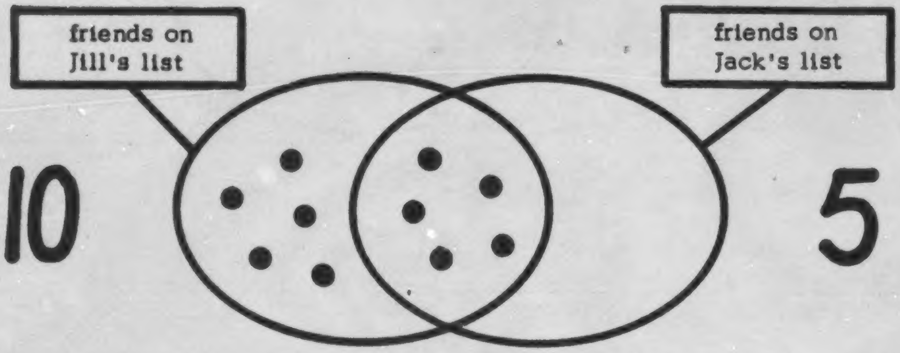
L 3

Invite a student to draw the other three dots in the picture.

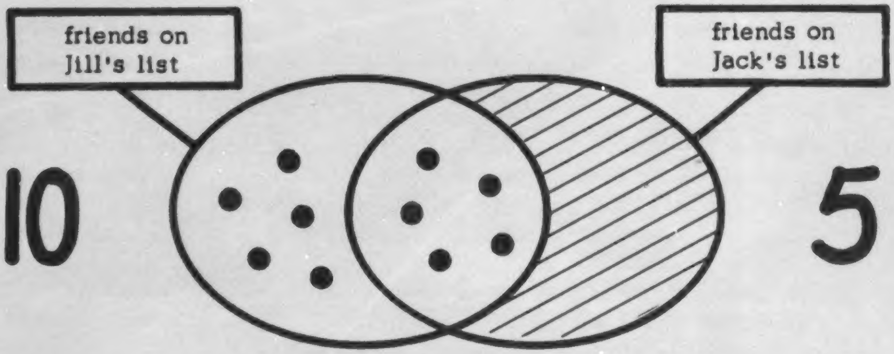

friends on Jill's list
friends on Jack's list
7
7

T: We have three (point to the region on the left) and four (point to the region in the middle) dots in Jill's string — that makes seven. We have four (point to the middle region) and three (point to the region on the right) dots in Jack's string — that makes seven.

Erase the dots in the picture. Repeat this activity using ten names on Jill's list, five on Jack's list, and ten names altogether. This is the string picture corresponding to that situation.

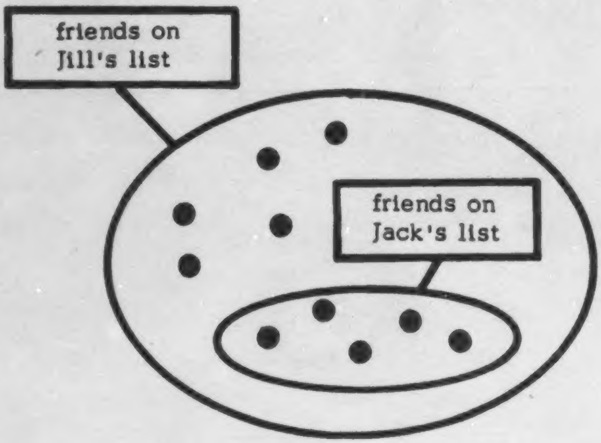

friends on Jill's list
friends on Jack's list
10
5

After the dots have been drawn in the picture, consider the empty region to the right. Students may remember that we can use hatching to indicate that there are no dots in a particular part of a picture. If no one suggests the hatching, suggest it yourself.


friends on Jill's list
friends on Jack's list
10
5

T: Do you think we could draw the strings in another way since one part of our picture is hatched?

Someone might suggest this picture.


friends on Jill's list
friends on Jack's list

L4

L4 Multiples of 3 and 4

CAPSULE LESSON SUMMARY

Use a hand-calculator to generate multiples of 3 and place the multiples of 3 in a string picture. Generate multiples of 4 on the hand-calculator and extend the string picture to include a string for multiples of 4. Discuss the various regions of the string picture. Place a selection of numbers, some given by names of the form "□ x △", in the string picture.

MATERIALS

Teacher: Hand-calculator; colored chalk
Student: Hand-calculator (if available); paper; colored pencils

DESCRIPTION OF LESSON

This lesson can be taught using as few as two hand-calculators, but use as many calculators as are available. If any of your students have their own calculators, they should be encouraged to bring them from home for this lesson.

The students with calculators can answer any of the questions relating to the calculator when the display is exposed. Anyone can answer when the display is hidden. All students should be encouraged to think just as carefully and stay just as involved as if they had calculators with hidden displays. Students with calculators can check all the answers. For different parts of the lesson, choose new students to work the calculators in order to give as many of them the experience as possible.

If you are fortunate enough to have a calculator for every student, allow a few minutes for students to experiment freely with the various keys on the calculator.

Distribute paper for students to use at their own initiative for figuring the answers to some of the questions which follow.

Exercise 1

T: Turn on the calculators. Slowly press $\boxed{+}$, $\boxed{3}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$, and continue pressing $\boxed{=}$. What numbers are appearing on the display?

S: 3, 6, 9, 12, 15, ...

T: What is the calculator doing?

S: It's counting by threes.

T: Now push the button " $\boxed{C}$ " to clear the display. What number is on the display now?

S: 0.

T: Listen carefully to my instructions. If you have a calculator, hide the display with one of your hands. Press $\boxed{+}$, $\boxed{3}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$. What number do you think is on the display?

S: 9.

T: Check the calculators.

S: 9 is right.

T: Hide the display again. Now press $\boxed{=}$ four more times. What number do you think is on the display?

S: 21.

T: Check the calculators.

S: That's right.

T: What numbers between 9 and 21 were on the display that we did not see?

S: 12.

S: 15.

S: 18.

T: Hide the display again. Press $\boxed{=}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$. What number do you think is on the display?

S: 36.

T: Check the calculators.

S: 36 is right.

T: Hide the display on the calculators. This time press $\boxed{=}$ ten times. What number do you think is on the display?

S: 66.

T: Why do you think it is 66?

S: I counted by three's.

One student in our development classes gave the following argument.

S: Four 3's is 12. Four more 3's is another 12; that makes 24. Two more 3's is 6 — so 30 more altogether.

T: Check the calculators.

S: 66.

T: How many times would we need to press $\boxed{=}$ to go from 66 to 78?

Let several students whisper their answers to you. Ask one student to respond aloud. (Four times)

T: Press $\boxed{=}$ four times and tell us the number which appears.

S: 78.

T: How do you think we could go back to 0 from 78 in such a way that all the same numbers would reappear but in reverse order?

S: Press $\boxed{-}$, $\boxed{3}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$, and so on.

T: Hide the display. Press $\boxed{-}$, $\boxed{3}$, $\boxed{=}$. What number is on the display?

S: 75.

T: Check the calculators.

S: 75 is right.

T: Keep pressing $\boxed{=}$ until 0 appears.

Now 0 is on the display. What number would appear if we press $\boxed{=}$ again?

S: $\hat{3}$.

T: ... and if we press $\boxed{=}$ again?

S: $\hat{6}$.

T: Will $\widehat{13}$ ever appear?

This question may result in disagreement.

T: What are some numbers which would appear if we kept pressing $\boxed{=}$?

S: $\hat{9}$.

S: $\widehat{12}$ and $\widehat{15}$. We missed $\widehat{13}$!

T: Turn the calculators off.

Exercise 2

T: What was the calculator doing when we continued pressing $\boxed{=}$?

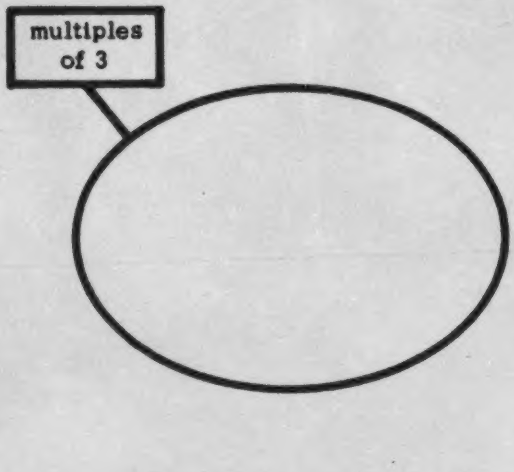
S: Counting by threes.

T: Do you know a name for the numbers that were appearing on the display?

S: Multiples of 3.

If no one suggests this name, suggest it yourself. Draw a string on the board and label it "multiples of 3".

T: This string is for the multiples of 3.



T: What are some multiples of 3?

Let about seven or eight students answer. If someone suggests a very large number, for example 2,096, say "I'm not sure. You'll have to convince me." (In this case, 2,096 is not a multiple of 3.)

T: How do you think we could get the multiples of 4 to appear on the calculator?

S: Do the same thing as we did to get multiples of 3 — only press $\boxed{4}$.

T: What keys would we press?

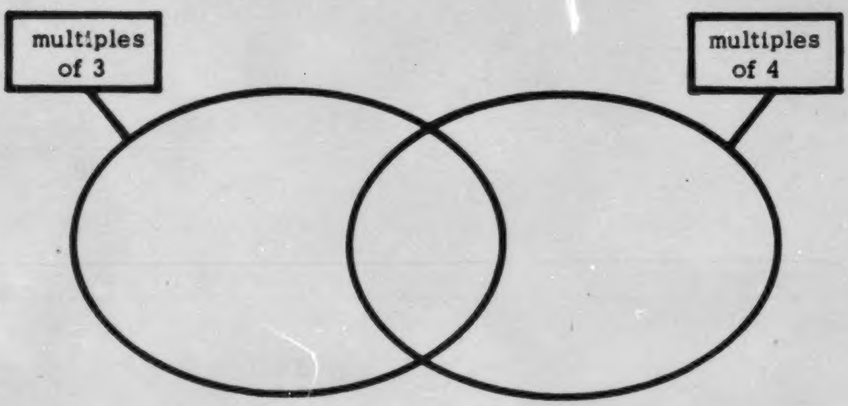
S: $\boxed{+}$, $\boxed{4}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$, and continue pressing $\boxed{=}$.

T: Let's try that. Turn the calculators on and press $\boxed{+}$, $\boxed{4}$, $\boxed{=}$, $\boxed{=}$, and so on. What numbers are appearing on the display?

S: 4, 8, 12, 16, 20, . . .

T: Who can draw a string on the board for the multiples of 4?

Let a student draw the string in a different color than the first string.



If a student draws the string so that it does not overlap the other string, ask "If we draw the strings in that way, where would 12 be in the picture?"

T: Give me some numbers which are multiples of 4.

S: 20.

T: Clear the calculators. Press $\boxed{+}$, $\boxed{4}$. How many times would we need to press $\boxed{=}$ to get 20?

S: Five times.

T: Check that on the calculator.
Give me another multiple of 4.

S: 44.

T: Does anyone know how many fours is 44?

If this question is too difficult, approach it as follows.

T: How much is 10 fours?

S: 40.

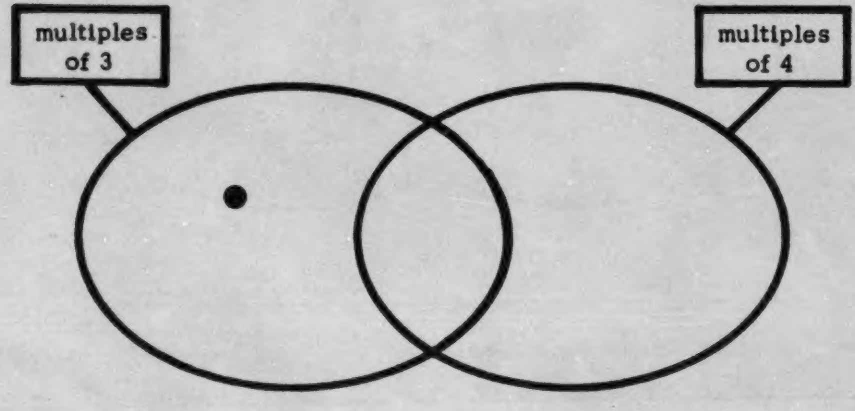
T: So how many fours is 44?

S: 11.

Continue in this way letting several students suggest numbers and explain why they are multiples of 4. At the end of the exercise, ask the students to turn off the calculators.

Exercise 3

Draw a dot in one of the regions in the string picture on the board.



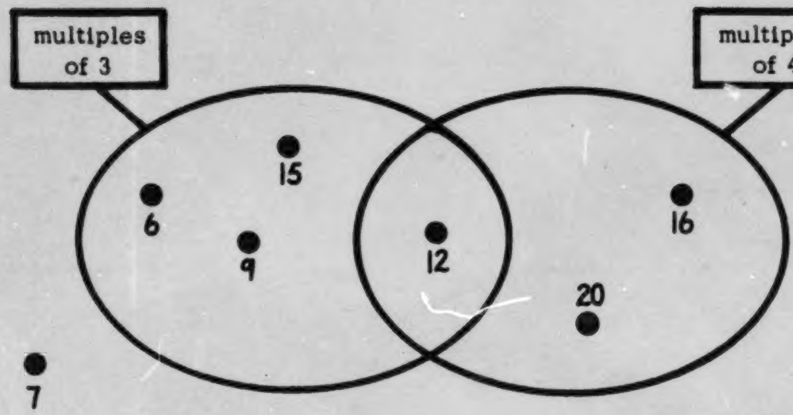
T: What number could be here (point to the dot)?

S: 9.

T: Is 9 a multiple of 3? (Yes)
Is 9 a multiple of 4? (No)

Label the dot in the picture. Draw a dot in another region. Continue in this manner until all regions have been considered. If someone gives a number which does not belong in a particular region, ask if the number is a multiple of 3 and then if it is a multiple of 4. Lead the student through these questions to discover where that number should be. Let the student draw a dot in the correct region and label it.

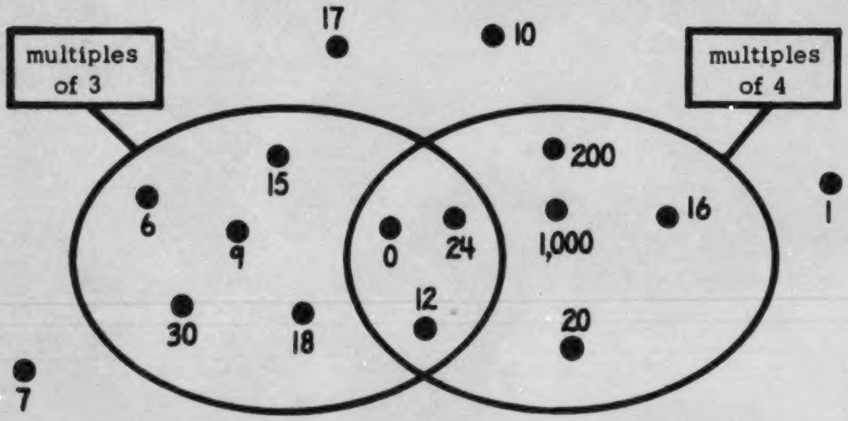
With several correct responses and several reconsidered responses, your picture might look like the following.


multiples of 3
multiples of 4
15
6
9
12
16
20
7

Now present numbers one at a time and ask students where they belong in the picture. As each is correctly located, draw a dot and label it appropriately. Use these or others depending on your students previous choices.

10	24	18
0	17	1
200	1,000	30

Check the placement of a number with the calculators when necessary. The next picture shows the correct location of the above numbers.


17
10
multiples of 3
multiples of 4
200
15
6
9
0
24
1,000
16
1
12
30
18
20
7

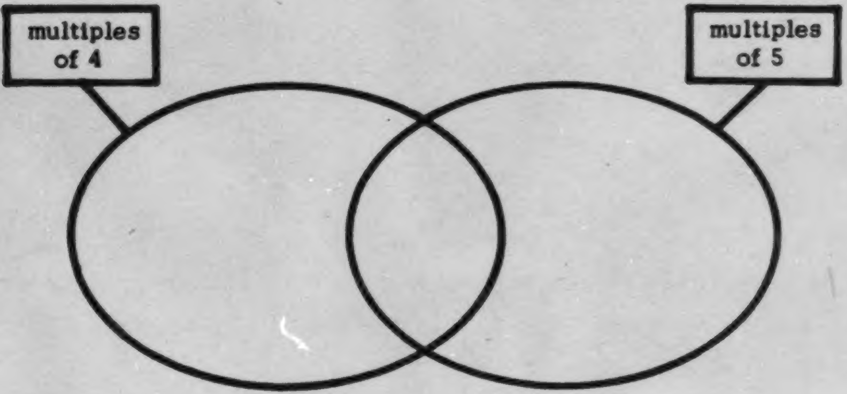
If the lesson is moving quickly, you could ask where the following numbers would be in the picture. Encourage students to locate them without doing the calculations.

3 x 7 3 x 80

4 x 7 4 x 39

Exercise 4

Distribute paper. Ask the students to copy this picture on their papers after you have drawn it on the board.



Instruct students to put any numbers they wish into the picture.

L5 Exchanging Names

CAPSULE LESSON SUMMARY

Exchange names among a group of students. Each student gives his or her right hand to the person whose name he or she receives. That person holds it with his or her left hand. Record the situation in an arrow picture. Repeat the situation with another group of students fewer in number. Draw pictures of what could happen if thirteen people played the game.

MATERIALS

Teacher: Colored chalk
Student: Paper; colored pencils

NOTE: Before the class begins, write the names of seven to ten of your students on separate slips of paper for Exercise 1 and the name of five or six other students on separate slips of paper for Exercise 2. For each exercise choose students whose first names begin with different letters.

DESCRIPTION OF LESSON

Exercise 1

In the description of this exercise, we will assume that these nine names are written on slips of paper: "Mike", "Jenny", "William", "Linda", "Derrick", "Pat", "Kim", "Anthony", and "Scott".

T: Today we are going to play a game. I want these people to come to the front of the room.

Read the names slowly. After reading a name, fold that slip of paper in half so that the name is hidden.

Look at the students at the front of the room and say to them:

T: Hold up your right hand. I'm going to give each of you a slip of paper with one of your names on it. Don't look at the name until I tell you.

In the right hand of each of these students put one of the folded slips of paper with one of their nine names on it.

T: Look at the slip of paper I just gave you and see whose name is on it. When I say "Go", give your right hand to the person whose name is on your slip of paper. That person will take your hand with his or her left hand.
Go!

Several things might happen:

- some students might be holding their own hands,
- some students might be in pairs holding each other's hands,
- some students might be in groups of three or more and form circles holding hands.

In the latter case, the students could get tangled up and need your help to form circles.

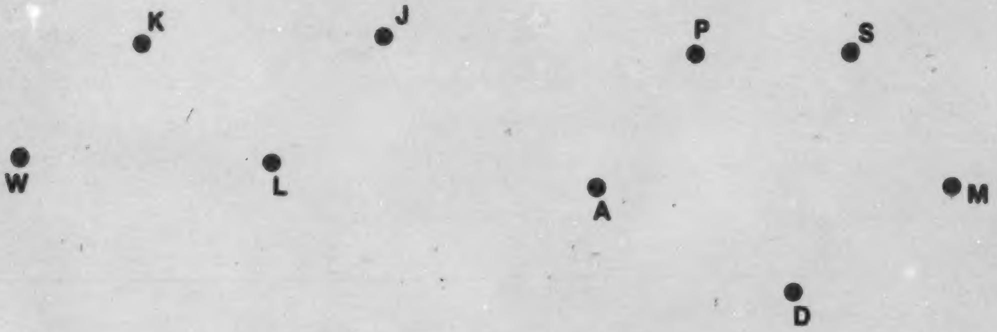
After the students are holding hands correctly, ask them to stay where they are while you and the rest of the class draw a "picture of them" on the board.

T: I want the rest of you (look at the students still seated) to help me describe what happened.

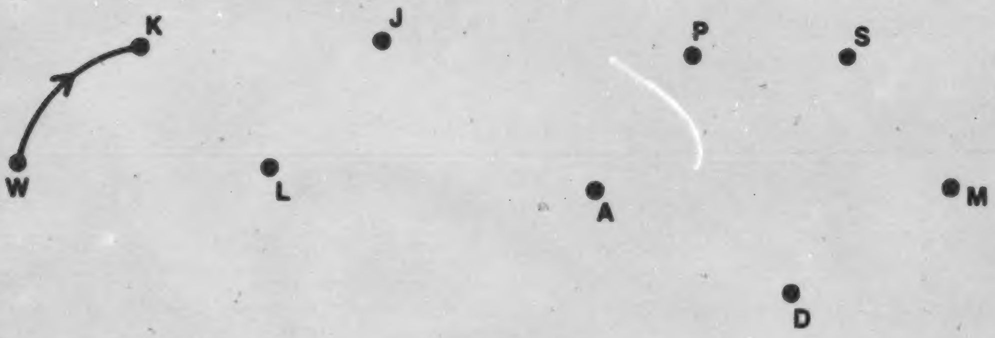
On the board draw dots for the students in the game, labeling the dots with the initial letters of their first names. Draw the dots in a manner suggestive of the students' arrangement. As you label a dot, tell the class who that dot is for.

L5

The pictures and dialogue in the rest of the description of this exercise uses the distribution of names in one of our development classes.

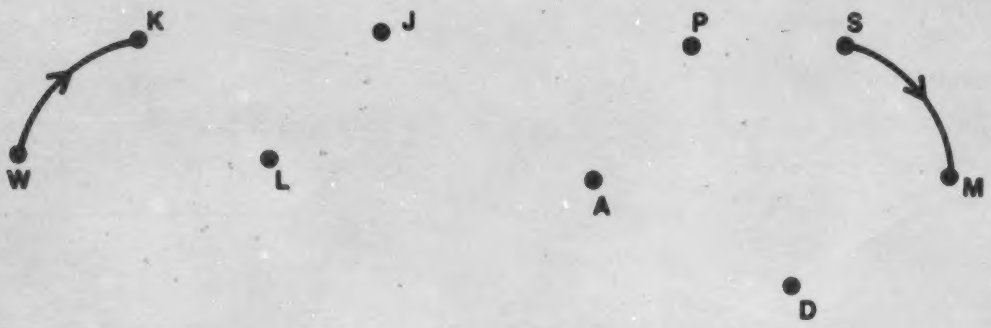


T: William has Kim's name, so William gives his right hand to Kim. I'm going to draw a red arrow to show this in our picture.



T: Who can help me draw some more arrows?

A student might draw an arrow from S (for Scott) to M (for Mike).



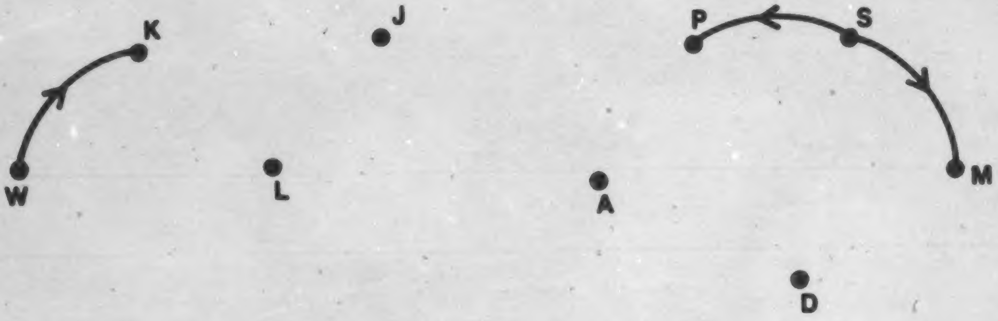
T: Scott, did you get Mike's name?

Scott: Yes.

T: Scott, did you give your right hand to Mike?

Scott: Yes.

Continue letting students draw arrows on the board. It could happen that a student draws an arrow in the wrong direction and needs your help. For example, consider the following dialogue — a student has drawn an arrow from S (for Scott) to P (for Pat).



T (tracing the arrow from S to M): Scott, do you have Mike's name?

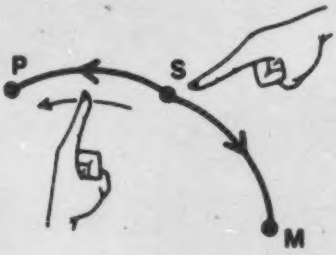
Scott: Yes.

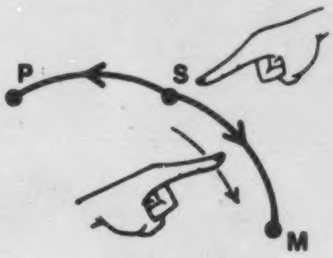
T (tracing the arrow from S to P): Scott, do you have Pat's name?

Scott: No.

L 5

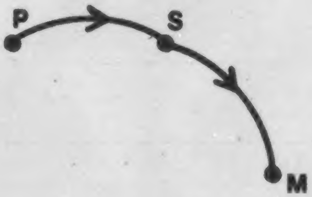
T: We know that this situation (put one finger on S and trace each arrow from S) . . .



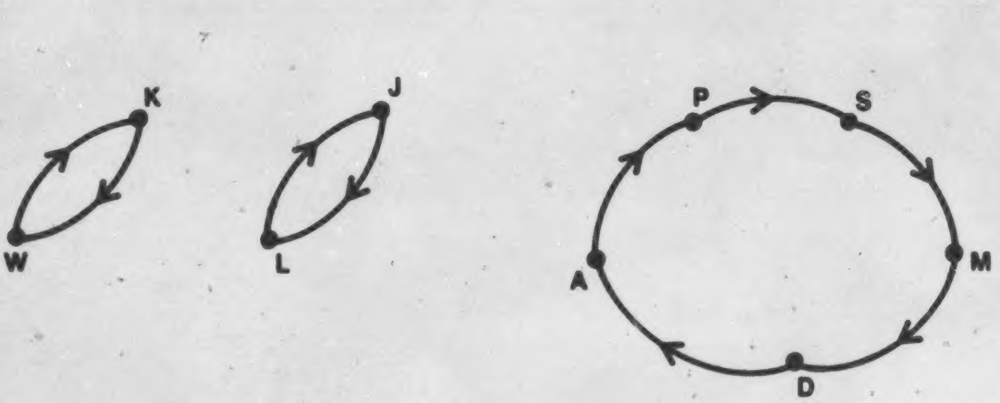


. . . cannot happen because no person in the game was given two names.

Encourage the student who drew the arrow to correct its direction.



A completed picture could look like this one.



Ask the students to return to their places in the classroom.

Exercise 2

Students will need unlined paper and red pencils for this exercise.

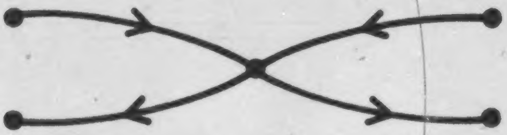
Play the game again as in Exercise 1 with a new group of five or six students. This time ask each student still seated to draw an arrow picture describing what happened. Ask the students in the game to stay in the front of the room so the other students in the class can draw a "picture of them." Students will need unlined paper and red pencils.

Exercise 3

Students need unlined paper and red pencils for this exercise also.

T: I want you to imagine thirteen people are playing the game. On your paper draw an arrow picture of what might happen.

If while observing students' work you notice someone has more than one arrow starting or arriving at a dot, reproduce that part of the student's picture on the board and call the class' attention to it. A possible picture and dialogue follows.



T: This is part of someone's picture. What do you think of it?

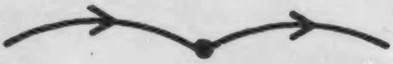
S: It's wrong!

T: Why?

S: Two people are giving their right hands to the person in the middle, but that person has only one left hand to receive them with. Only one person could have gotten the name of the person in the middle.

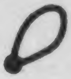
S: Also, the person in the middle is giving his or her right hand to two people, but that person could have gotten only one name.

Replace the arrow picture with this one.



T: In this situation, exactly one arrow should start at each dot and exactly one arrow should end at each dot.

Draw a dot and a loop on the board.



T (pointing to the loop): Does exactly one arrow start at this dot?

S: Yes.

T: Does exactly one arrow end at this dot?

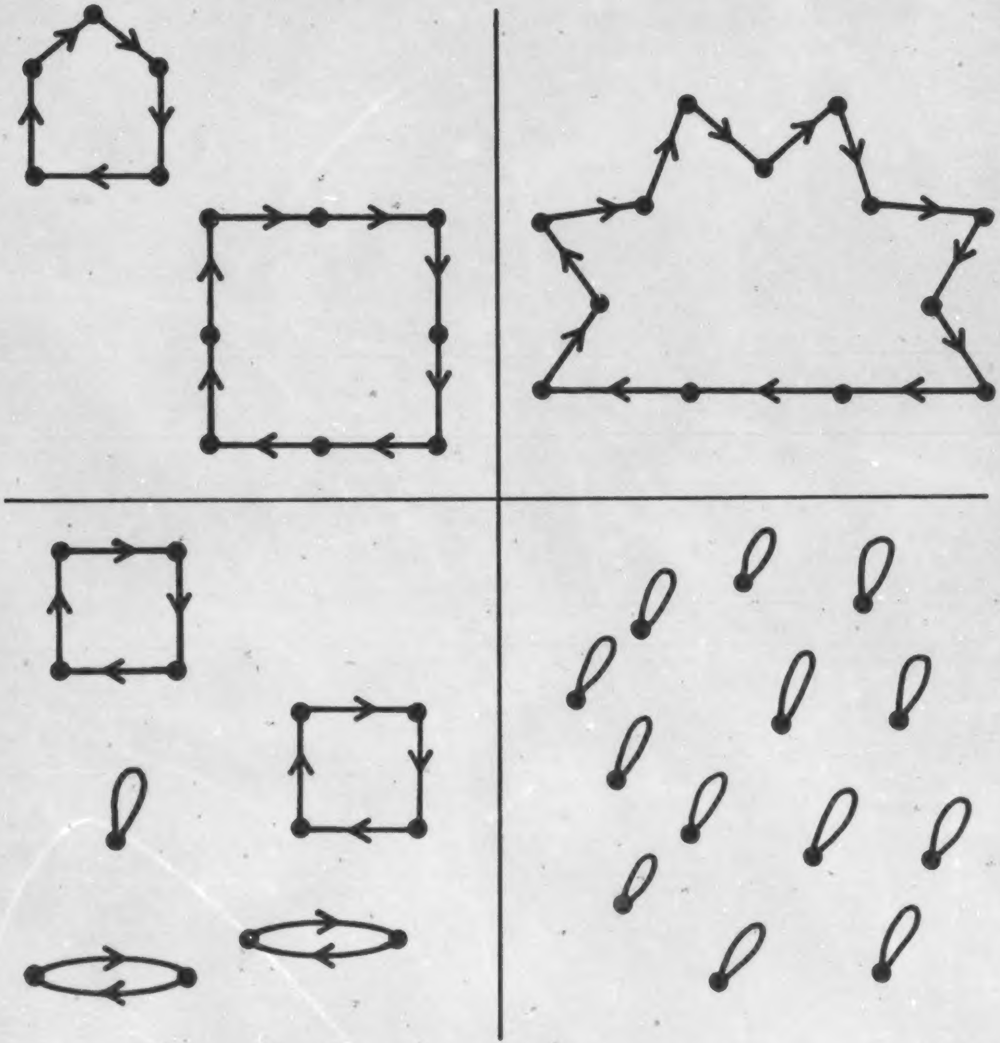
S: Yes.

T: So a loop can be part of your picture. When would there be loops in a picture?

S: When some people get their own names.

As the students start completing their pictures, ask several of them with significantly different pictures to put them on the board.

These are some possibilities.



L5

If very few of your students have drawn loops in their pictures, draw the picture with thirteen loops (see the preceding group of pictures) on the board yourself.

With each picture on the board, discuss with the class how the thirteen students would look in the game.

L6 The String Game #2

CAPSULE LESSON SUMMARY

Play the String Game with two strings and sixteen attributes. Discuss a two-string picture where one string is for multiples of 10 and the other is for numbers greater than 50. Introduce and play a game placing numbers in this string picture.

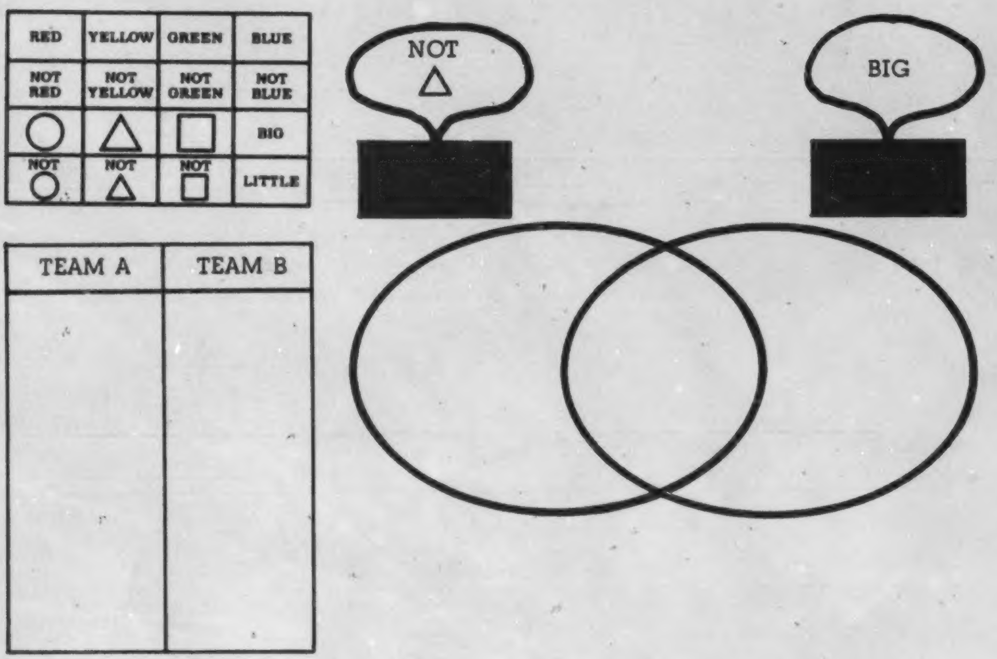
MATERIALS

Teacher: The String Game with A-Blocks kit; colored chalk
Student: None

DESCRIPTION OF LESSON

Exercise 1

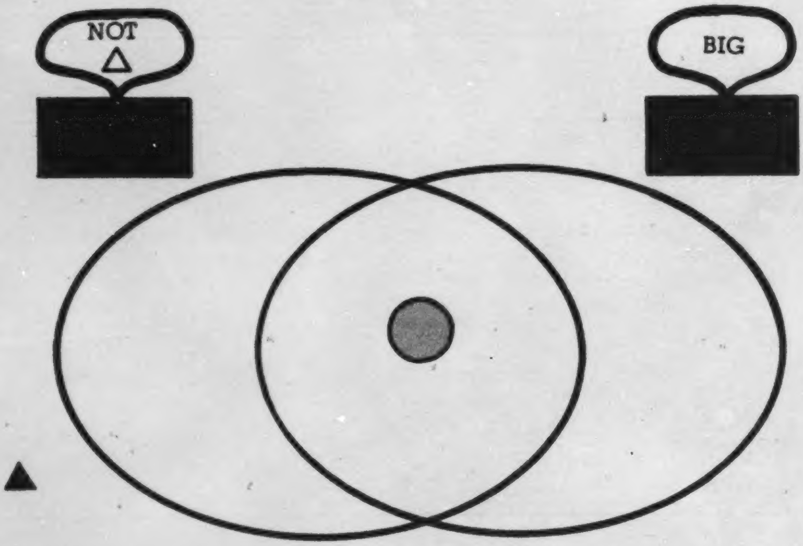
Before the class begins, put the A-blocks for the String Game in a small box. Prepare your board for the String Game as shown below.


RED
YELLOW
GREEN
BLUE
NOT RED
NOT YELLOW
NOT GREEN
NOT BLUE
BIG
NOT
NOT
NOT
LITTLE
TEAM A
TEAM B
NOT △
BIG

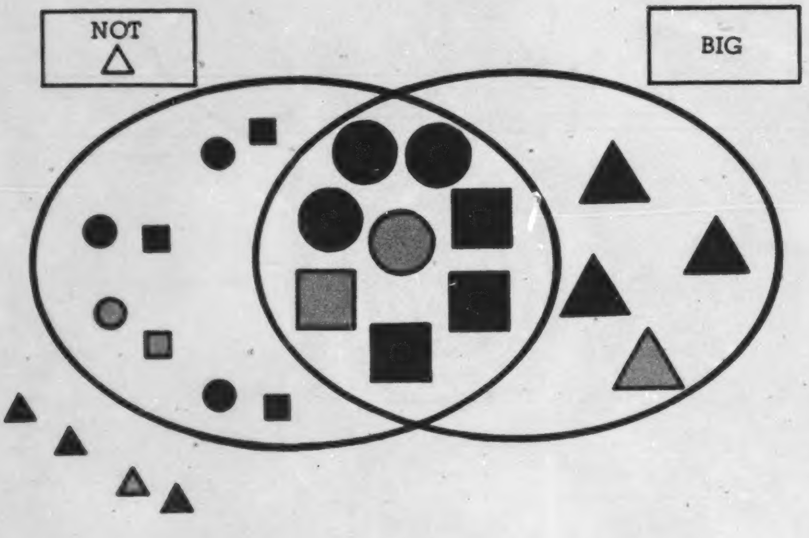
Indicate to your class which students are on Team A and which are on Team B. Ask a student from Team A to describe a game piece and then put it under "Team A" on the board. Ask a student from Team B to describe a game piece and then put it under "Team B" on the board. Students should give complete descriptions of pieces — color, shape, and size. Continue in this manner alternating teams until all the game pieces have been placed.

T: Now we will play the String Game. Each string is for one of these labels (point to the list on the board). I'll give you two clues.

Put the large yellow circle and the small red triangle in the picture.

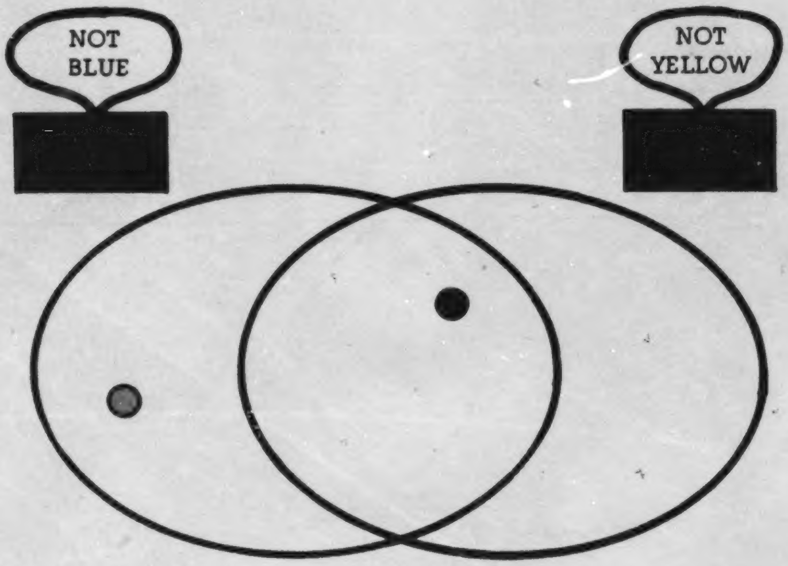


Play the game in the usual way. The next illustration shows the correct placement of all the game pieces.

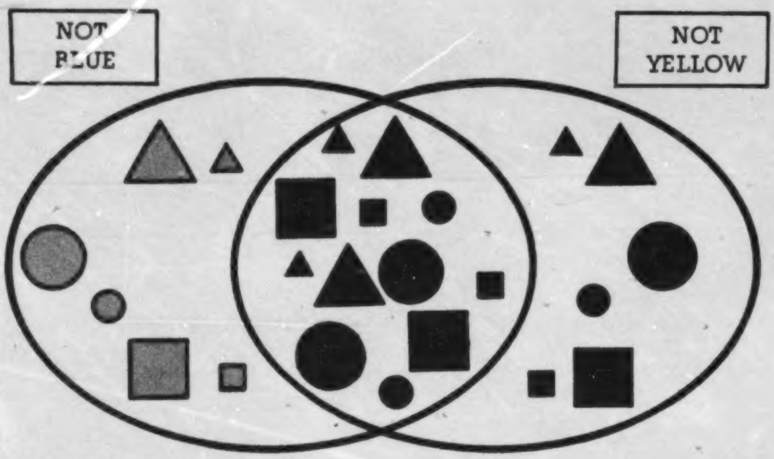


Exercise 2

Play the String Game again, using not-cards for both strings. A game with starting clues is suggested below.

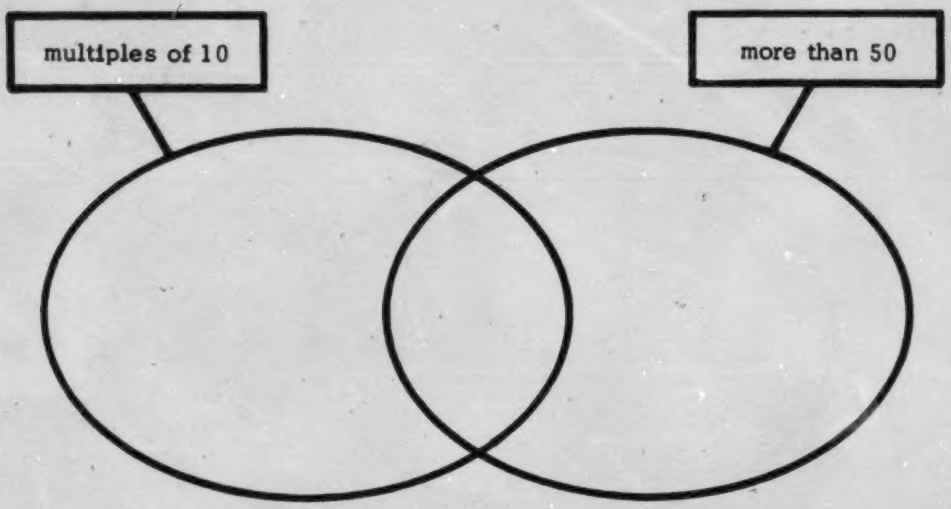

NOT
BLUE
NOT
YELLOW

This picture shows the correct placement of all the game pieces.


NOT
BLUE
NOT
YELLOW

Exercise 3

Take the list of string labels off the board and put the A-Blocks away. Label the red string "multiples of 10" and the blue string "more than 50".



T (tracing the red string): The red string is for multiples of 10. What are some numbers which would be in the red string?

S: 10, 20, and 30.

S: 70.

S: 120 and 200.

T (tracing the blue string): The blue string is for numbers more than 50. What are some numbers which would be in the blue string?

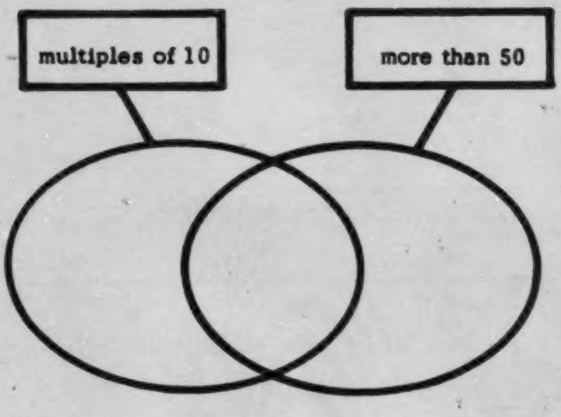
S: 51.

S: 100.

S: 67.

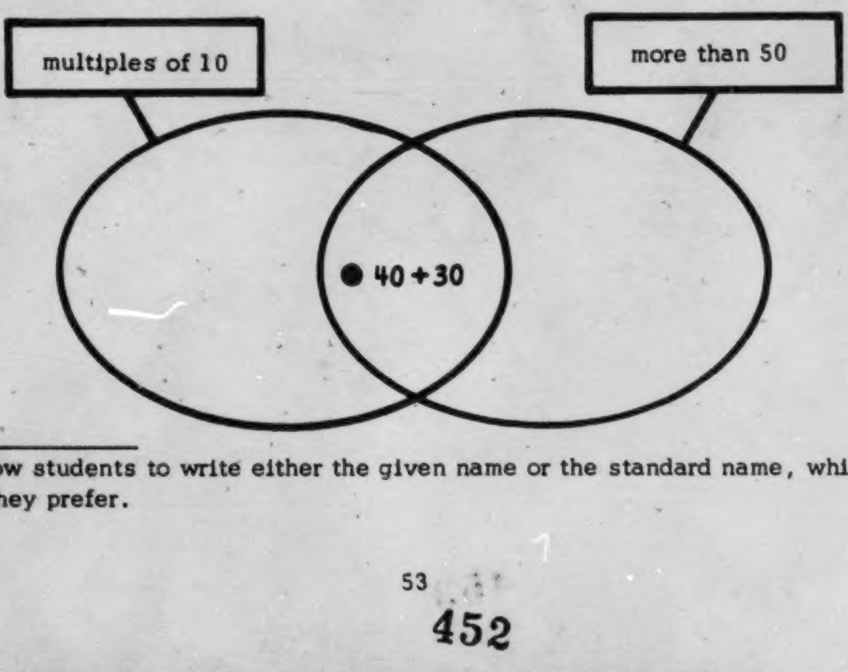
List these numbers under "Team A" and "Team B" on the board near the strings.

TEAM A		TEAM B	
50	$\widehat{13}$	40+30	9×11
48+8	36+$\widehat{6}$	83	0
5	128	60−15	$\widehat{50}$
6×10	10	20	24
72−24	110+2	$\widehat{14}$+4	96+2



T: We're going to play a game with numbers. During a turn, you will have only one chance to put a number in the string picture. The first team to put all the numbers in its list where they belong in the picture wins.

Alternating teams, let students locate numbers in the picture, writing them with chalk. Discuss correct responses; for example, suppose a student puts 40 + 30[#] here (see the next illustration).



[#] Allow students to write either the given name or the standard name, whichever they prefer.

T: That's right. Why does 40 + 30 belong there?

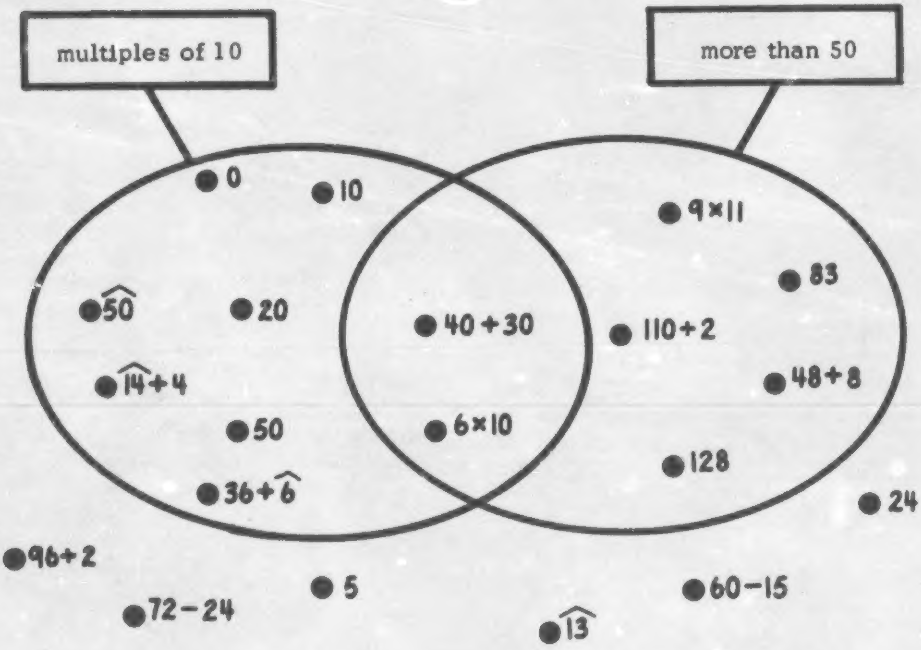
S: 40 + 30 = 70; 70 is a multiple of 10 and is more than 50.

S: 40 + 30 is more than 50 and since 40 and 30 are both multiples of 10, 40 + 30 is a multiple of 10.

Erase "40 + 30" from the list and leave it in the string picture.

If a student locates a number incorrectly, simply erase it from the picture and say, "No, that number does not belong there." Some students might object if you were to discuss why a number does not belong in a certain region because you would give information to the team with that number.

This picture shows the correct location of all the numbers in the game.


multiples of 10
more than 50
0
10
$\widehat{50}$
20
$\widehat{14}+4$
50
$36+\widehat{6}$
40 + 30
6×10
9×11
83
110 + 2
48 + 8
128
24
96 + 2
72 − 24
5
$\widehat{13}$
60 − 15

L7 Composition Games #2

CAPSULE LESSON SUMMARY

Play a composition game with the rule: yellow followed by blue is red. Reverse the order in which the colors of the arrows are considered to introduce a new rule: blue followed by yellow is green. Play a game using both rules.

MATERIALS

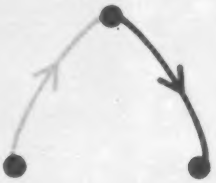
Teacher: Colored chalk

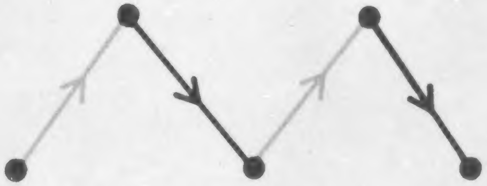
Student: Worksheets L7*, **, and ***; colored pencils

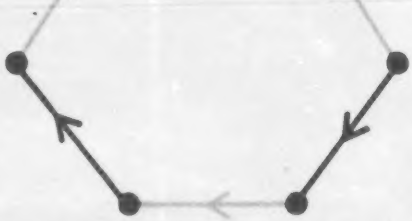
DESCRIPTION OF LESSON

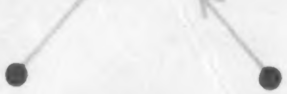
Exercise 1

Before the lesson begins, draw this arrow picture on the board.







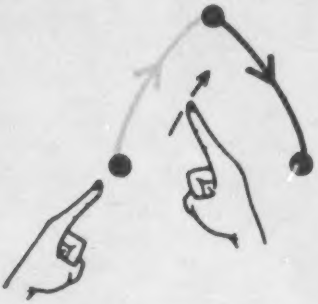


T: Do you remember the game in which if you follow a yellow arrow and then a blue arrow, you can draw a red arrow?

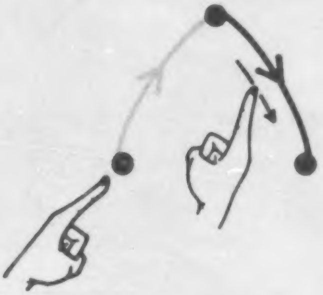
L7

Use the upper left corner of the arrow picture to review the rule of the game.

T: Yellow (trace the arrow) . . .

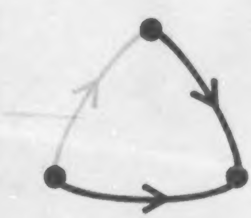


followed by blue (trace the arrow) . . .



gives red.

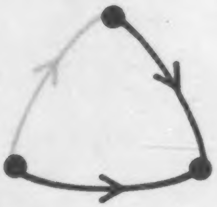
Trace and then draw the red arrow.

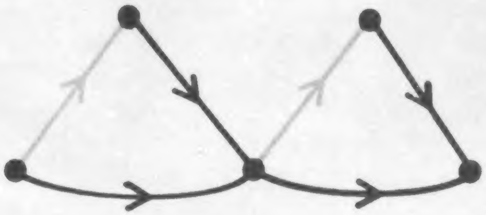


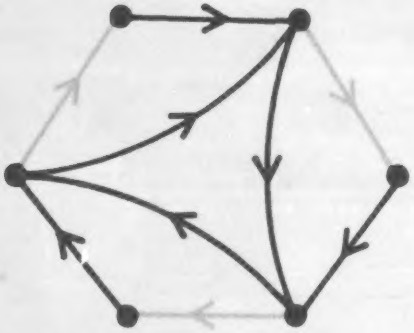
T: Who can find some more red arrows we may draw?

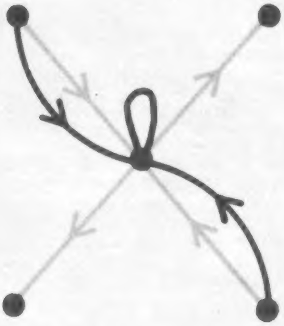
Let students one by one trace arrows on the picture. Draw the red arrows yourself after they have been correctly traced. Help a student who has difficulty with this by holding the students' left forefinger at a dot where a yellow arrow starts. Then guide the student's right forefinger to trace the yellow arrow followed by a blue arrow.

The completed picture should look like this.



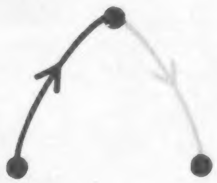






T: Since there are no more red arrows, let's introduce another rule.

In the upper left part of your picture draw this configuration of arrows.

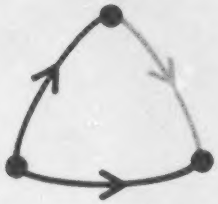


L 7

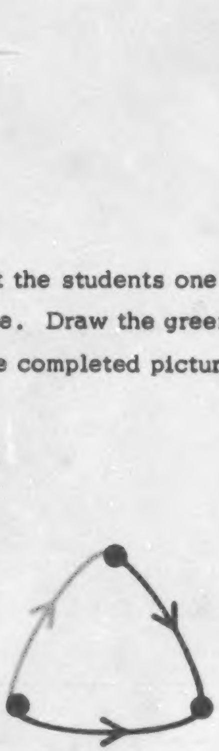
Refer to the arrows you just drew.

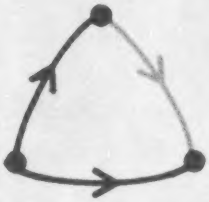
T: Blue (trace the blue arrow) followed by yellow (trace the yellow arrow) gives green.

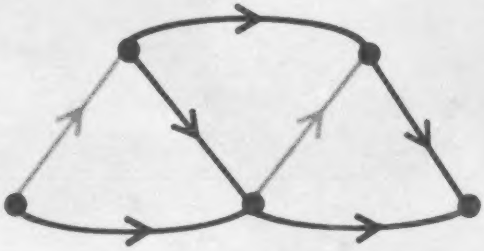
Trace and then draw a green arrow.

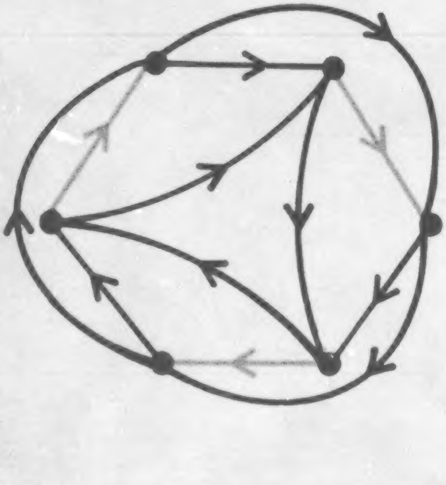


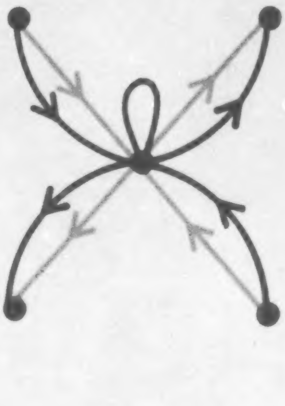
Let the students one by one trace where green arrows may be drawn in the picture. Draw the green arrows yourself after they have been correctly traced. The completed picture should look like this.





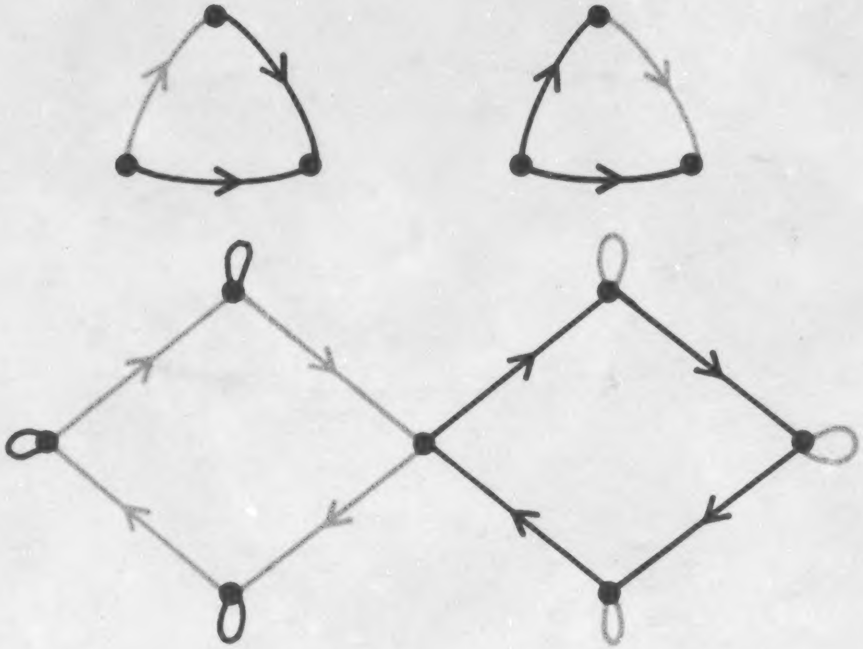






Exercise 2

Erase the board except for the upper left part of the picture and then draw this configuration of arrows.

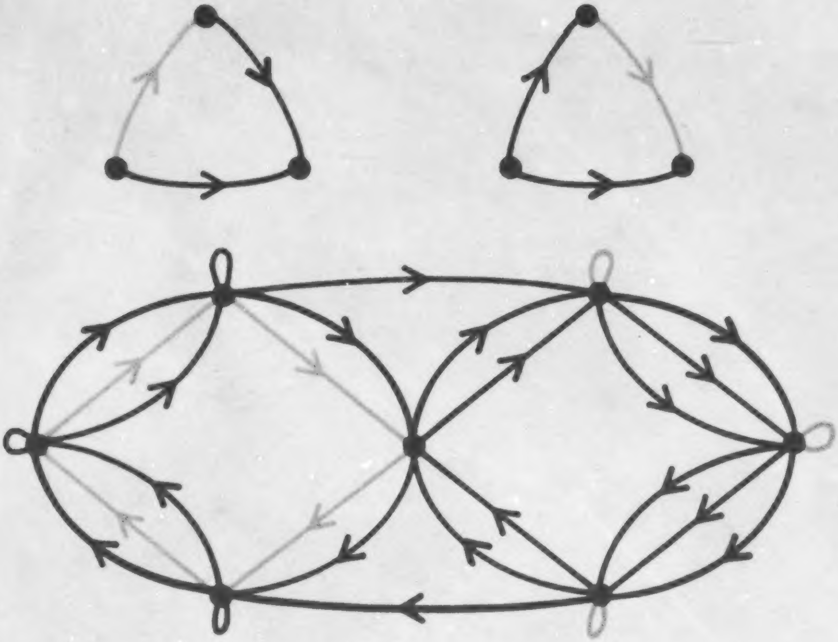


T: Who can find where some red or some green arrows may be drawn?

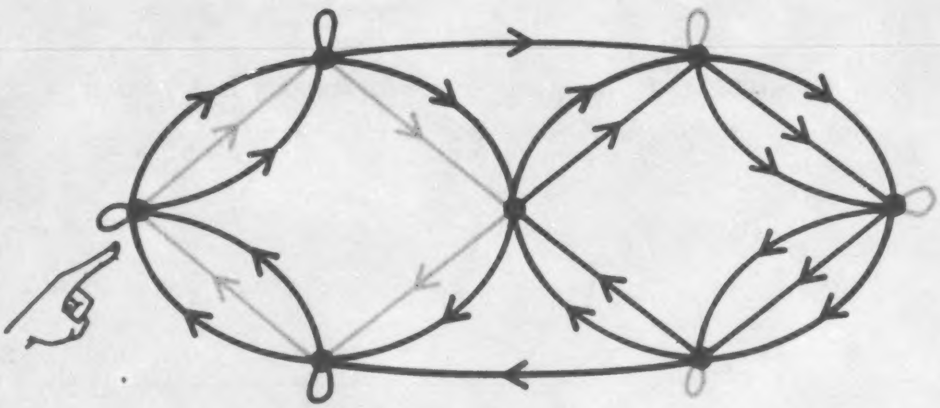
Let students trace arrows on the picture. Draw green arrows and red arrows after they have been correctly traced. Occasionally ask a student, "Are you going to show us where to draw a red arrow or a green arrow?"

L 7

The completed picture should look similar to this one.



Point to the dot indicated in the next illustration.



T: If we start here (tap the dot) can we follow green arrows and get back to here (tap the dot again)?

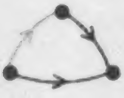
Let a student trace the cycle. Then repeat the question referring to the same dot only ask for a cycle using red arrows. Do the same for yellow arrows and then for blue arrows.

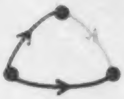
NOTE: Students may have difficulty finding a cycle with blue arrows since the cycle is simply the blue loop at the indicated dot.

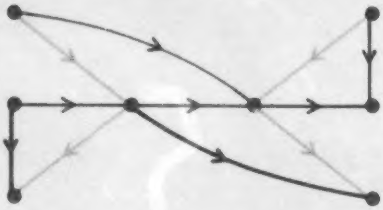
Worksheets L7*, **, and *** are available for individual work.

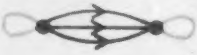
Name ______________________ **L 7 ***

Draw the missing red arrows and green arrows.



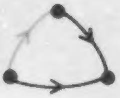


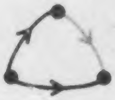


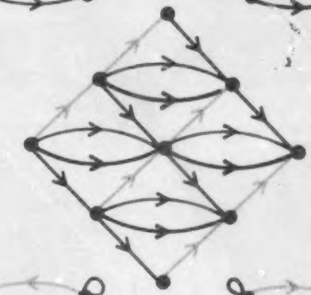


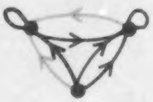
Name ______________________ **L 7 ****

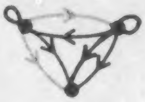
Draw the missing red arrows and loops. Draw the missing green arrows and loops.





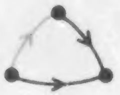


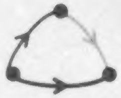


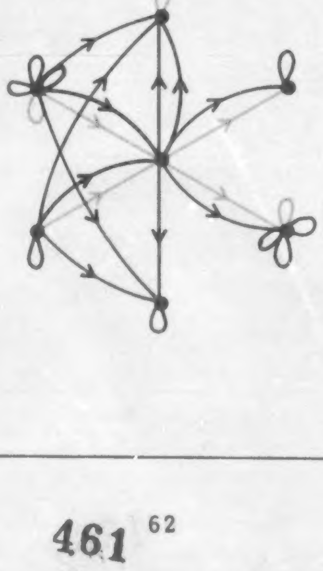


Name ______________________ **L 7 *****

Draw the missing red arrows and loops. Draw the missing green arrows and loops.







L8 Marble Game #1

CAPSULE LESSON SUMMARY

Do some mental arithmetic involving $\frac{1}{2}\times$ and $\frac{1}{3}\times$. Examine an experiment of choosing one marble from some collection of marbles of two colors and discuss the chances of getting one color marble versus that of getting the other color. Introduce and play a marble game involving prediction.

MATERIALS

Teacher: Marbles; colored chalk
Student: None

DESCRIPTION OF LESSON

Exercise 1

Spend about five minutes on mental arithmetic involving $\frac{1}{2}\times$ and $\frac{1}{3}\times$.

T:	$\frac{1}{2} \times 12 =$[*] . . . ? How do you know?	S:	6. 6 + 6 = 12.
T:	$\frac{1}{3} \times 12 =$ . . . ? How do you know?	S:	4. 4 + 4 + 4 = 12.
T:	$\frac{1}{2} \times 6 =$ . . . ? How do you know?	S:	3. 3 + 3 = 6.
T:	$\frac{1}{3} \times 6 =$ . . . ? How do you know?	S:	2. 2 + 2 + 2 = 6.
T:	$\frac{1}{2} \times 18 =$ . . . ? How do you know?	S:	9. 9 + 9 = 18.

---

[*] Read this as "One-half of twelve is . . . ?"

T:	$\frac{1}{3} \times 18 = \ldots$?	S:	6.
	How do you know?		$6 + 6 + 6 = 18$.
T:	$\frac{1}{3} \times 9 = \ldots$?	S:	3.
	How do you know?		$3 + 3 + 3 = 9$.
T:	$\frac{1}{3} \times 15 = \ldots$?	S:	5.
	How do you know?		$5 + 5 + 5 = 15$.
T:	$\frac{1}{3} \times 24 = \ldots$?	S:	8.
	How do you know?		$8 + 8 + 8 = 24$.

Exercise 2

For this exercise, you need four red and four blue marbles.

T: I have two marbles in my hand, one red and one blue. (Show them to the students.)

Shake the marbles behind your back while you explain what you are doing to the class.

T: I hold the marbles with both hands behind my back and shake them; then I put one of the marbles in my right hand.

Hold out your closed right hand.

T: Which marble could I have in my right hand?

S: The red marble.

S: The blue marble.

Open your hand and show the marble to the class.

T: I have two marbles (hold them in an outstretched open hand) and one of them is red (hold up the red marble). We say the chances of getting the red marble are 1 out of 2.

Draw two marbles and write this on the board.

● Red 1 out of 2
● Blue

T: What are the chances of getting the blue marble?

S: The same — 1 out of 2.

Record this information on the board.

● Red 1 out of 2
● Blue 1 out of 2

T: With one red and one blue marble, the chances of getting red is the same as the chances of getting blue. If I put another red marble and another blue marble in my hand, will I have a better chance of getting red?

S: I don't think so.

S: The chances for red and for blue will still be the same.

Draw the marbles on the board. Write "Red" and "Blue" next to them.

● Red 1 out of 2
● Blue 1 out of 2

● ● Red
● ● Blue

T: I have four marbles (hold the four marbles in an outstretched open hand) and two of them are red (hold up the two red marbles). What are the chances of getting red?

S: 2 out of 4.

T: . . . of getting blue?

S: The same — 2 out of 4.

Write this information in the chart forming on the board.

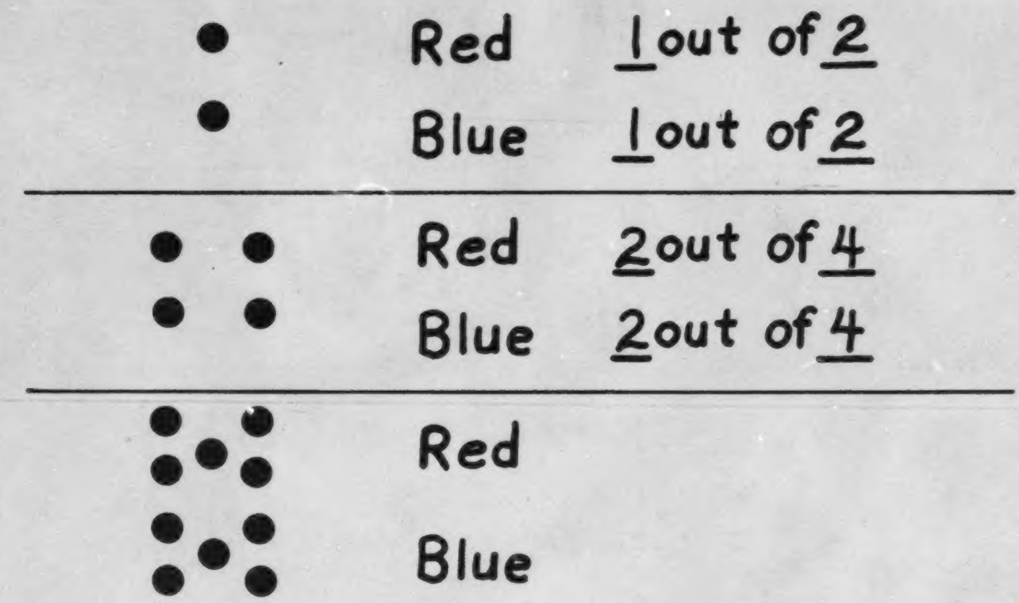
●	Red	1 out of 2
●	Blue	1 out of 2
● ●	Red	2 out of 4
● ●	Blue	2 out of 4

T: Here (point to the record of the first situation) the chances of getting red and the chances of getting blue are both 1 out of 2.
And here (point to the record of the second situation) the chances of getting red and the chances of getting blue are both 2 out of 4.

Who can give us another situation where the chances of getting red would be the same as the chances of getting blue?

Suppose a student suggests five red and five blue marbles. Draw another section to the chart on the board and draw the marbles.

T: How many marbles are red (point to the red marbles)?



Red 1 out of 2

Blue 1 out of 2

Red 2 out of 4

Blue 2 out of 4

Red

Blue

S: Five.

T: And how many marbles are there altogether?

S: Ten.

T: What are the chances of getting red?

S: 5 out of 10.

Write "5 out of 10" in the chart appropriately.

Repeat these questions for blue, after which your chart should look like this.

●	Red	1 out of 2
●	Blue	1 out of 2
● ●	Red	2 out of 4
● ●	Blue	2 out of 4
● ● ● ● ●	Red	5 out of 10
● ● ● ● ●	Blue	5 out of 10

T: What are some other situations where the chances of getting red are the same as the chances of getting blue?

There are many answers to this question; some are given in the dialogue.

S: Three red and three blue marbles.

S: Four red and four blue marbles.

S: Twenty red and twenty blue marbles.

T: In all these situations the chances for red and for blue are the same. 1 chance out of 2 is the same as 2 chances out of 4 or as 5 chances out of 10 or as 3 chances out of 6, and so on.

This time hold out two red marbles and one blue marble.

T: What if I put two red marbles and only one blue marble behind my back — are the chances of getting red the same as the chances of getting blue?

S: No, the chances of getting red are greater.

T: What are the chances of getting a red marble (show the students the three marbles)?

S: 2 out of 3.

T: What are the chances of getting a blue marble (show the students the three marbles)?

S: 1 out of 3.

Write this information on the board to the side of the first chart — not in it!

● ● Red $\underline{2}$ out of $\underline{3}$

● Blue $\underline{1}$ out of $\underline{3}$

T: If I add two more red marbles to those in my hand, how many blue marbles should I add to keep a similar situation with the chances of getting red twice as great as the chances of getting blue?

S: One blue marble.

T: That's right and I will have four red and two blue marbles.

Draw them on the board.

● ● Red $\underline{2}$ out of $\underline{3}$

● Blue $\underline{1}$ out of $\underline{3}$

● ●

● ●

●

●

T: With these six marbles (point to the picture of them), what are chances of getting a red marble?

S: 4 out of 6.

T: . . . of getting a blue marble?

S: 2 out of 6.

Write this information next to the picture of the six marbles. Your chart will look like this.

● ●	Red	2 out of 3
●	Blue	1 out of 3
● ● ● ●	Red	4 out of 6
● ●	Blue	2 out of 6

Exercise 3

Draw these charts on the board.

Red Team

Prediction	# of Reds	Points

Blue Team

Prediction	# of Blues	Points

Tell the students that they are going to play a game. Divide the class into two teams and choose a scorekeeper for each team. The scorekeepers should come to the front of the room near the charts.

T: The game we will play involves one red and one blue marble. For each round of the game I'll shake the marbles six times. When I shake the marbles, I will put one in my right hand and will hold it out in front of me so we can see what color it is. Let's try a round to show you how the game is played.

The following is a sample dialogue; we cannot know how your students will respond in the game.

T (speaking to the first person on the Red Team): Predict how many times out of the six I'll get a red marble.

S: I predict four out of the six times you'll get a red marble.

Ask the Red scorekeeper to record "4" in the column labeled "Prediction" in their chart.

T (speaking to the first person on the Blue Team): Predict how many times out of six I'll get a blue marble.

S: I predict three out of the six times you'll get a blue marble.

Ask the Blue scorekeeper to record "3" in the column labeled "Prediction" in their chart.

Red Team

Prediction	# of Reds	Points
4		

Blue Team

Prediction	# of Blues	Points
3		

L8

T: Now I'll shake the marbles.

Blue - red - red - blue - blue - blue.

As each marble is revealed, ask the scorekeeper for each team to keep a tally in the chart.

Red Team

Prediction	# of Reds	Points
4	II	

Blue Team

Prediction	# of Blues	Points
3	IIII	

T: This is how we keep score. Each team starts with ten points. (Write "10" above the last column in each chart.) The number of points you lose depends upon how closely you predict the results from the six times I shake and select a marble.

In this round, the Red Team predicted four red; we had only two — so the Red Team loses two points since 4 - 2 = 2. What is their score now?

S: 8.

Record "8" in the Red Team's chart.

Red Team 10

Prediction	# of Reds	Points
4	II	8

T: In this round, the Blue Team predicted three blue; we had four — so the Blue Team loses a point since 4 - 3 = 1. What is their score now?

S: 9.

Record "9" in the Blue Team's chart.

Blue Team 10

Prediction	#of Blues	Points
3	IIII	9

T: Now that you see how the game is played, let's begin again. We'll play several rounds each time making predictions and shaking the marbles. When one team has lost all of its points, the other team will be the winner.

The charts for a game played by one of our development classes is included below. The Red Team won this game.

Red Team 10

Prediction	#of Reds	Points
3	III	10
4	II	8
3	卌	6
3	III	6
3	I	4
3	III	4
3	卌	2

Blue Team 10

Prediction	#of Blues	Points
4	III	9
2	IIII	7
3	I	5
2	III	4
3	卌	2
3	III	2
4	I	↑

L9 Three Strings

CAPSULE LESSON SUMMARY

Explore several situations involving A-Blocks in three-string pictures. In those situations where there are empty regions, observe this fact and hatch such regions. Explore a situation involving numbers in a three-string picture where the strings are for even numbers, multiples of 3, and numbers less than 100.

MATERIALS

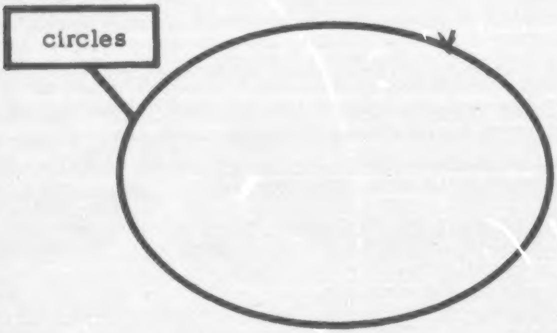
Teacher: Colored chalk; A-Blocks

Student: Worksheets L9*, **, ***, and ****; pencil

DESCRIPTION OF LESSON

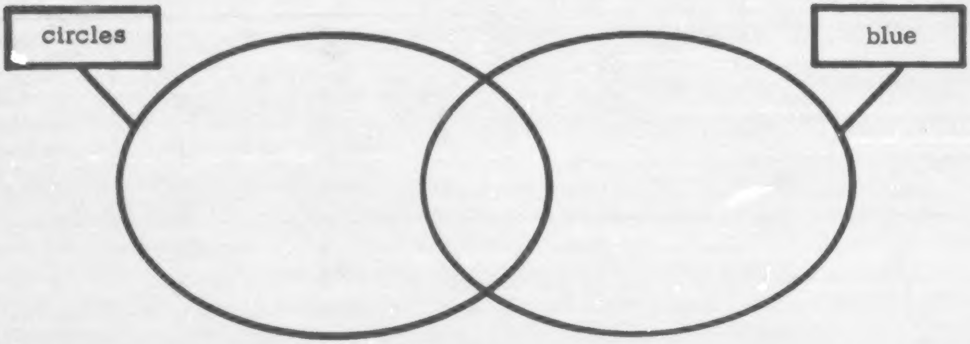
Exercise 1

Display the A-Blocks. Draw a large red string on the board and label it "circles".



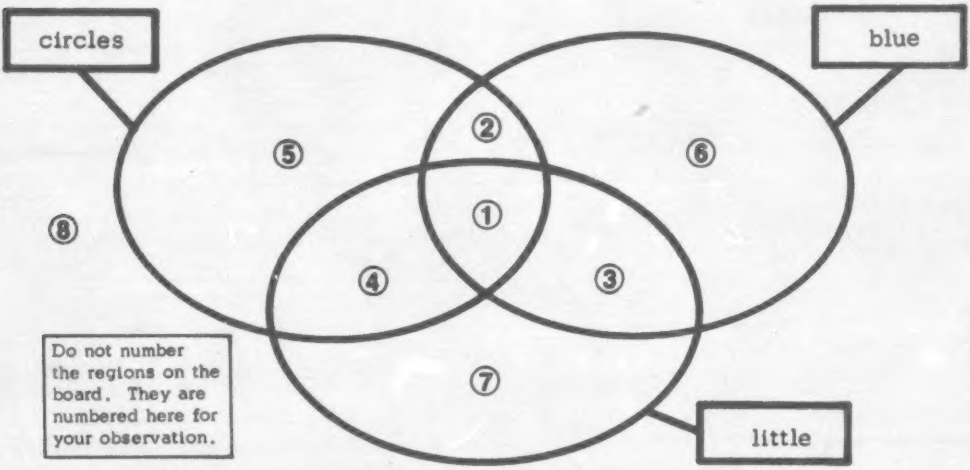
T: The red string is for circles. I would like to draw another string for blue pieces. Who can trace where I should draw it?

When a student traces a string overlapping the red string, draw and label it.



T: I would like to draw another string for pieces which are little. Who can trace where I should draw it?

Draw and label a string after someone traces it correctly overlapping the other two strings. Your picture should have eight regions as indicated below.

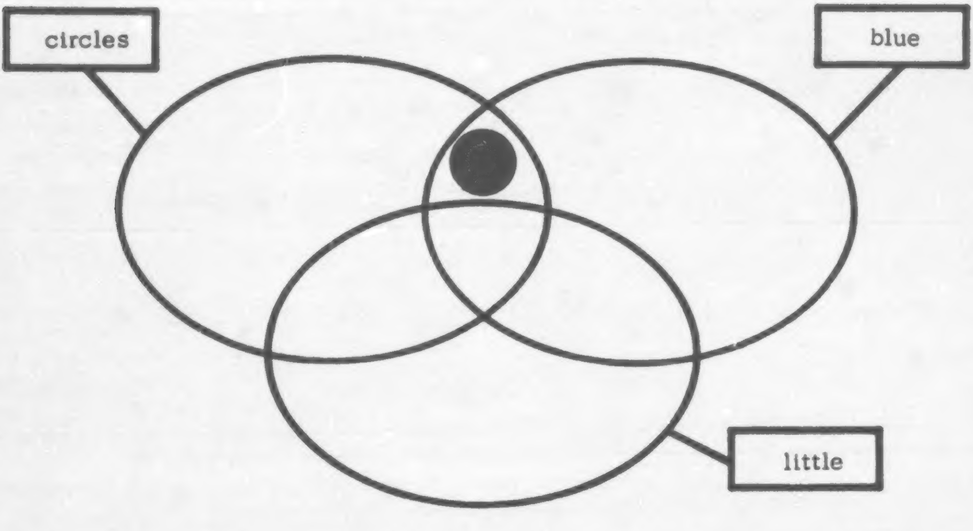


Do not number the regions on the board. They are numbered here for your observation.

Point to one of the eight regions in the picture as you ask:

T: Who knows a piece which belongs here?

Check a student's choice by considering each label. For example, suppose a student places the large blue circle here (see the illustration) . . .

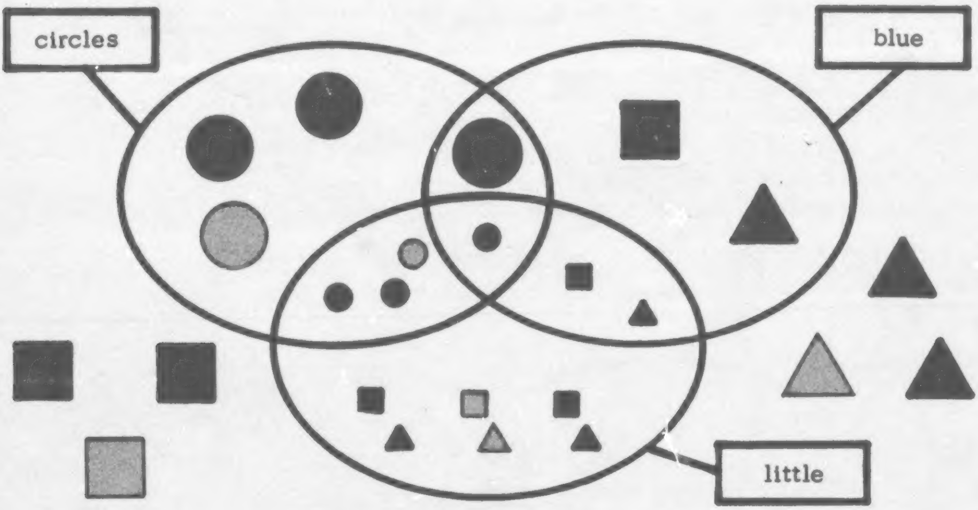


Ask the class as you point to each of the labels: "Is it a circle?" (Yes) "Is it blue?" (Yes) "Is it little?" (No) "So it is in the red and the blue strings but not in the green string."

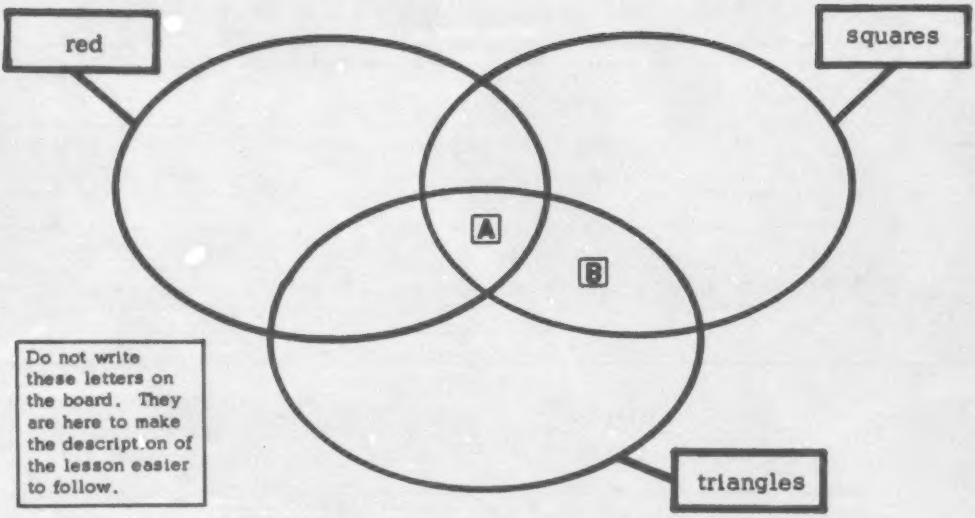
Repeat this activity pointing to three or four different regions. If someone chooses a piece which does not belong in the region being considered at the time, the class should notice the error. Lift the piece from the picture and ask where the piece belongs in the picture.

Change the type of question by holding up some pieces which are not yet in the picture. Ask where the pieces belong in the picture.

Although your picture might have about six to nine pieces in it, this picture shows the correct placement of all the pieces.



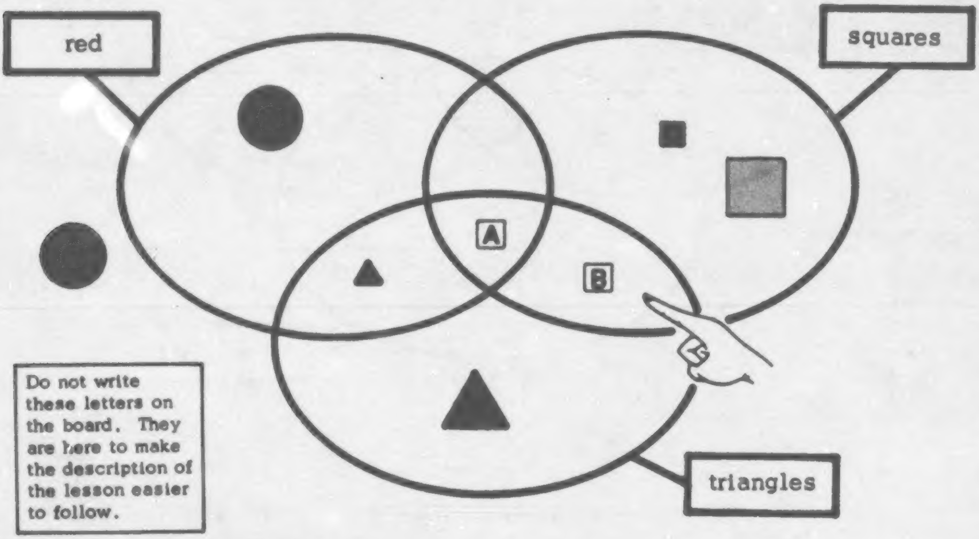
Remove any A-Blocks from the string picture and relabel the strings "red", "squares", and "triangles".



Do not write these letters on the board. They are here to make the description of the lesson easier to follow.

Again point to three or four regions one at a time and ask students for pieces which belong in those regions. After several pieces have been placed correctly, hold up some pieces which are not in the picture and let students place them in the picture.

Suppose your picture looks like the one in the next illustration. Point to Region B.



Do not write these letters on the board. They are here to make the description of the lesson easier to follow.

L9

T: Which piece belongs here?

S: None, because no piece is both a square and a triangle.

T: What can we do to the picture to show that there are no pieces in this region (B)?

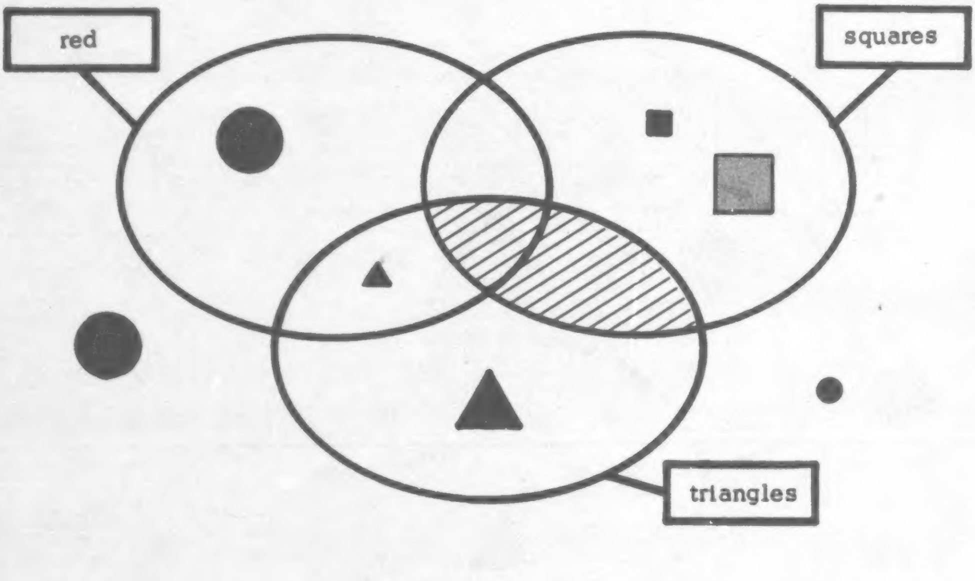
S: Hatch the region.

Hatch Region B yourself.

T: Are there any other regions we could hatch?

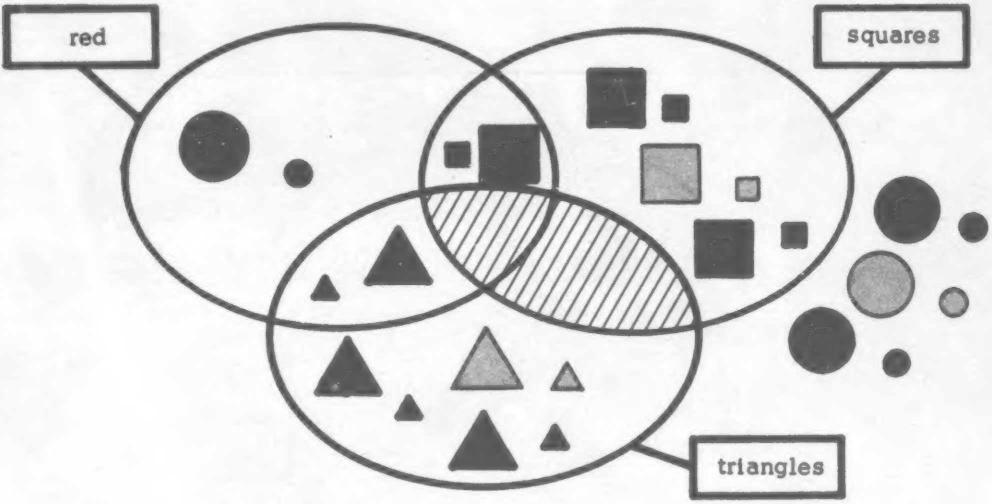
S: We can hatch the middle region (A), because there are no pieces which are squares and triangles and red.

Hatch Region A.

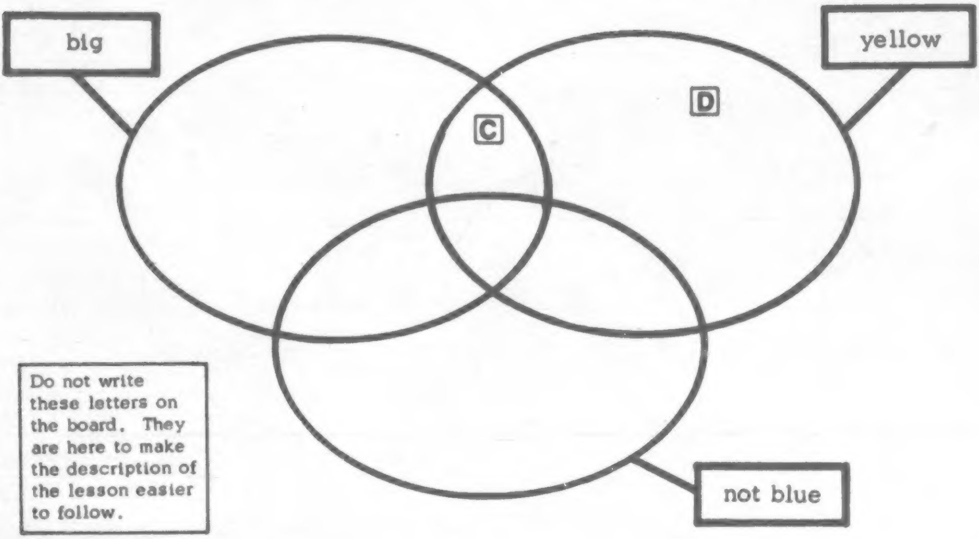


Consider any regions which are not hatched and do not have any pieces placed in them. Ask the students if these regions are empty or if not to give some pieces which belong in them.

Although your picture will have about six to nine pieces in it, the next illustration shows the correct placement of all the pieces.


red
squares
triangles

Repeat this activity with this situation.

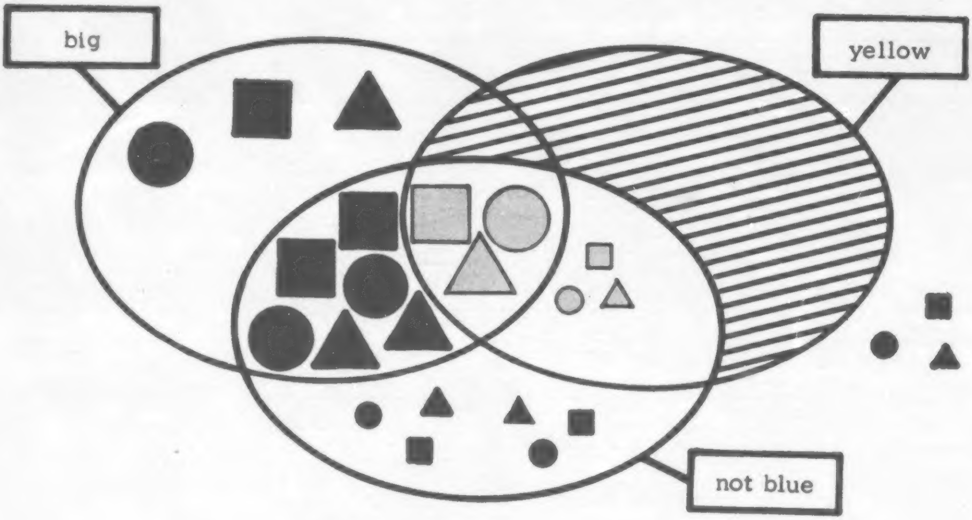

big
yellow
C
D
not blue

Do not write these letters on the board. They are here to make the description of the lesson easier to follow.

In this case, regions C and D are empty and class discussion should lead to the observation of this fact.

L 9

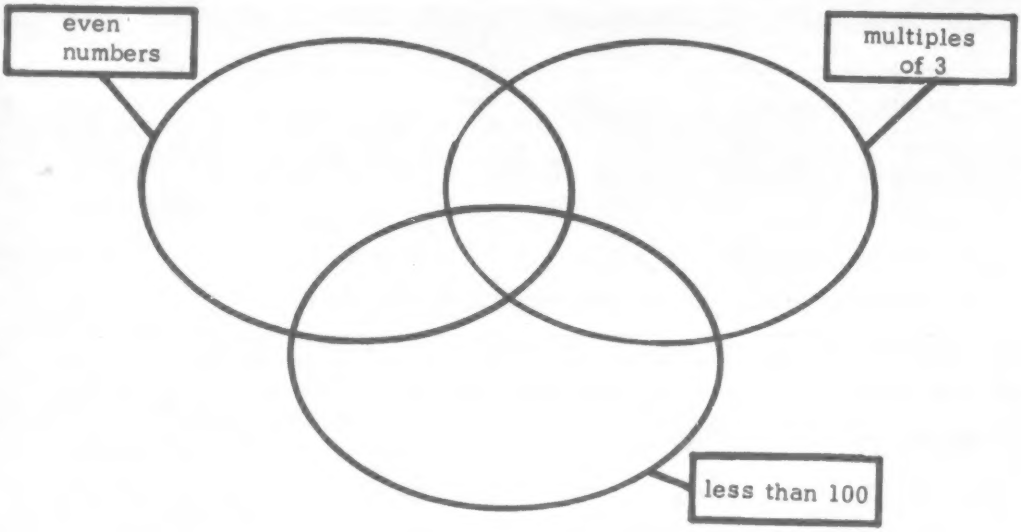
For your information, correct placement of all pieces is shown below.



big
yellow
not blue

Exercise 2

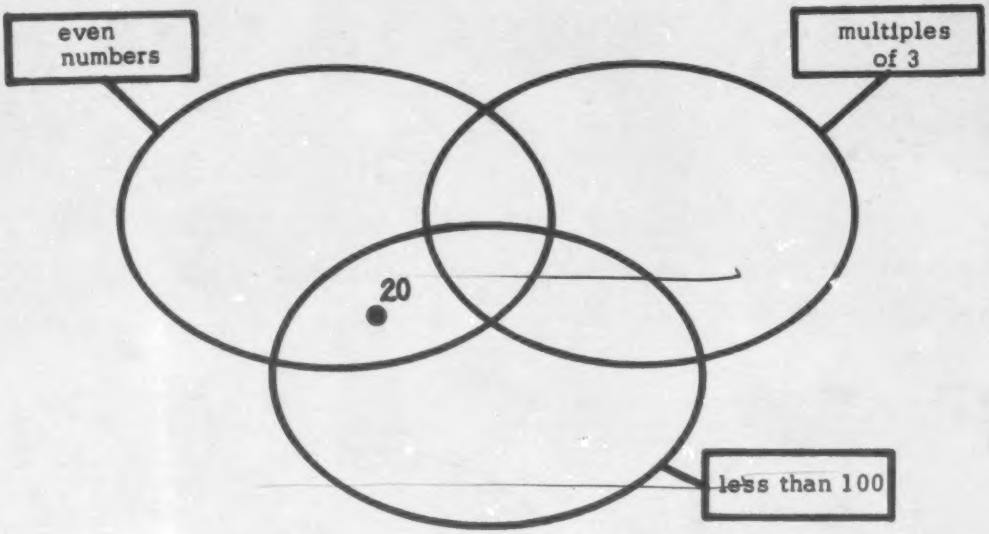
Remove any A-Blocks from the string picture and relabel the strings as in the next illustration.



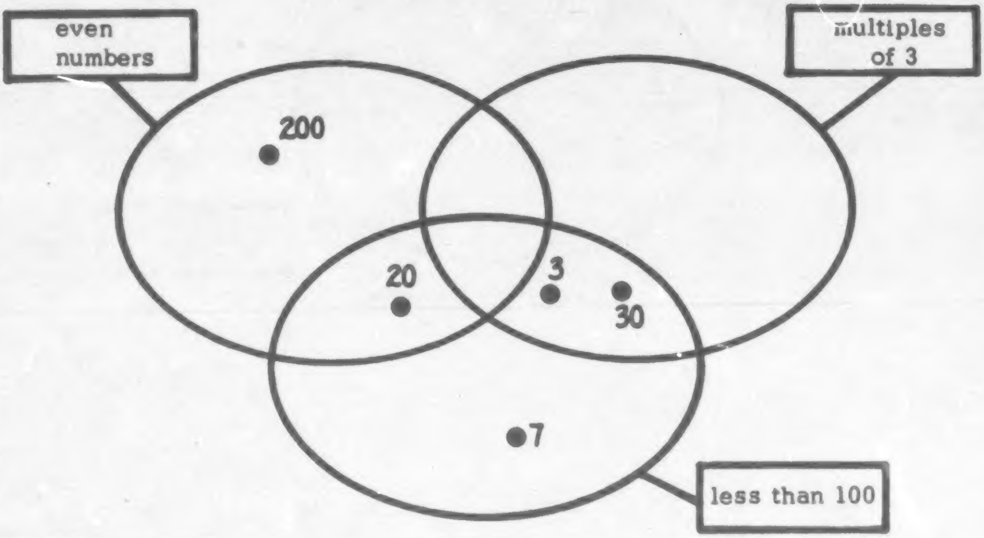
even numbers
multiples of 3
less than 100

T: This time we are going to put numbers in the picture. The red string is for even numbers, the blue string is for multiples of 3, and the green string is for numbers less than 100. Who can point to where 20 belongs?

After someone gives the region correctly, draw a dot and put 20 in that region.


even numbers
multiples of 3
20
less than 100

Similarly ask where these numbers belong in the picture: 200, 3, 30, 7.


even numbers
multiples of 3
200
20
3
30
7
less than 100

L 9

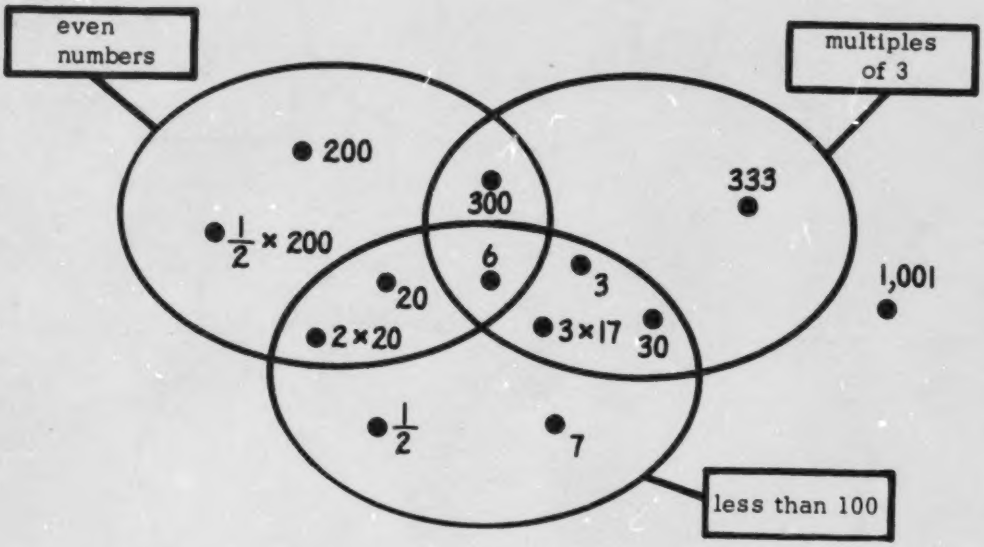
Point to each of the regions in which a number has not been located and for each one ask, "Who can give me a number which belongs here?"

Afterward, consider with the class where four or five of the following numbers belong in the picture.

$\frac{1}{2}$ 333 $\hat{6}$ 2×20

3×17 $\frac{1}{2} \times 200$ 1,001 300

Since we cannot predict which numbers your students will suggest, we give here only the position of the numbers specifically asked for in the lesson description.

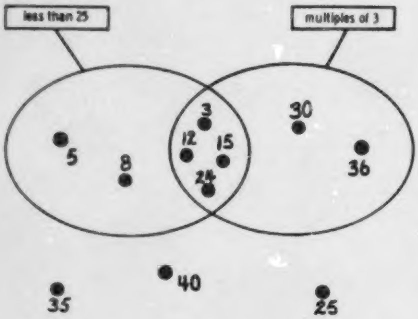

even numbers
multiples of 3
less than 100
200
300
333
1/2 × 200
6
3
20
2×20
3×17
30
1,001
1/2
7

Worksheets L9*, **, *** and **** are available for individual work.

Name ____________ **L 9 ***

Put these numbers in the string picture.

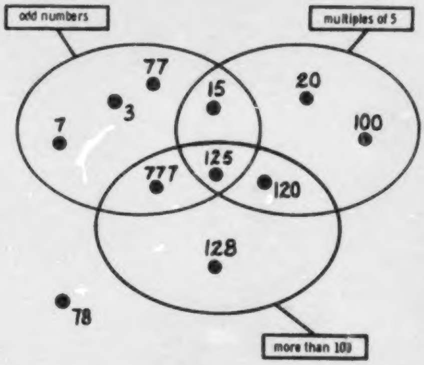
3 8 12 15 24 25 30 35


less than 25
multiples of 3
5
8
3
12
15
24
30
36
35
40
25

Name ____________ **L 9 ****

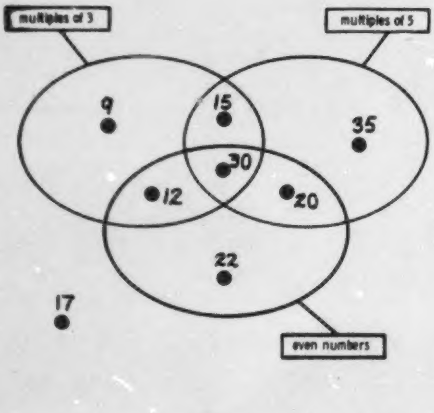
Put these numbers in the string picture.

15 125 128 7 20 100 77 777


odd numbers
multiples of 5
more than 100
77
3
7
15
20
100
125
777
120
128
78

Name ____________ **L 9 *****

Label the dots with your choice of numbers.


multiples of 3
multiples of 5
even numbers
9
15
35
30
12
20
22
17

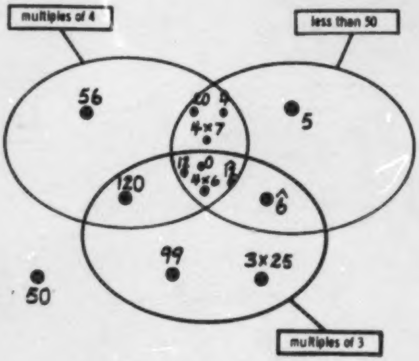
Many solutions are possible.

Name ____________ **L 9 ******

Put these numbers in the string picture.

20 0 12 120 $\hat{12}$ $\hat{6}$ $\hat{4}$ 99 50 56

3 × 25 4 × 6 4 × 7


multiples of 4
less than 50
multiples of 3
56
20
4
4×7
5
12
0
4×6
12
120
6
99
3×25
50

L10 A Detective Story

CAPSULE LESSON SUMMARY

Discover a secret number from three clues, the first involves putting numbers on the Minicomputer using exactly one positive checker and one negative checker; the second involves building roads with exactly two 2x arrows and two +3 arrows; and the third presents the secret number's place in a three-string picture.

MATERIALS

Teacher:	Minicomputer kit; colored chalk
Student:	Paper; colored pencils

DESCRIPTION OF LESSON

Before the lesson begins, write a large "36" on a slip of paper and fold it over so the "36" is hidden.

T: Have you ever read a detective story?
What do detectives do?

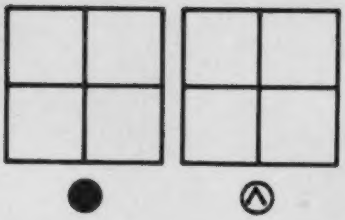
S: Find clues and use them to solve mysteries.

T: I have a secret number on this paper. You are going to be detectives. I will give you some clues to help you figure out the secret number. If you are good detectives, you will not need to do any guessing.

First clue

Display two Minicomputer boards.

T: The secret number can be put on these Minicomputer boards using exactly two checkers. It can be put on with one regular checker on this board (draw a regular checker below the tens' board) and with one negative checker on this board (draw a negative checker below the ones' board).



T: What are some numbers that could be my secret number?

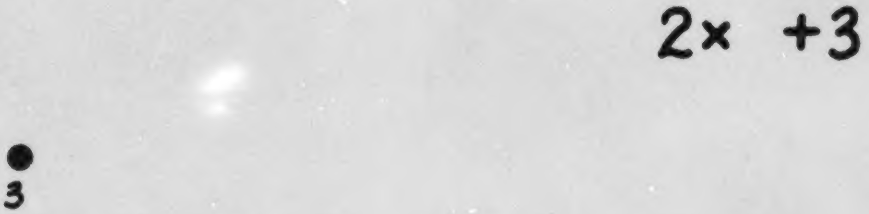
When a number is suggested, ask the student to show that number on the Minicomputer using one regular checker on the tens' board and one negative checker on the ones' board. If correct, record it on the board. As numbers are being suggested, you will be forming a list on the board. Try to organize the list so that when all the possibilities are found your list delineates some patterns. In fact, the patterns here may help students find some of the solutions.

2	6	8	9
12	16	18	19
32	36	38	39
72	76	78	79

Take away the Minicomputer boards but leave the list on the chalkboard.

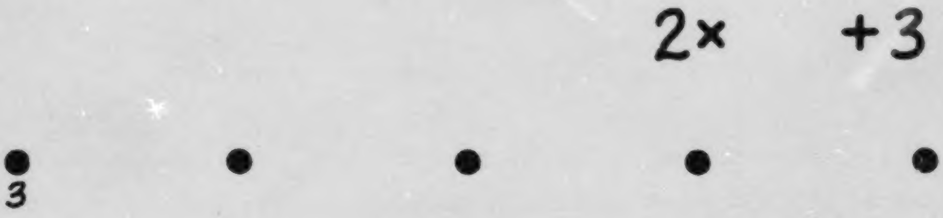
Second clue

Draw this picture on the board to the side of the list of numbers.



T: This is my next clue. My secret number is the ending number of a red-blue road that starts at 3 (point to "3") and that has exactly two red arrows (point to "2 x") and two blue arrows (point to "+ 3").

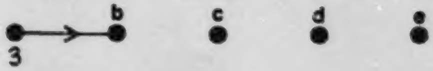
Draw these dots for the arrow road.



Let a student suggest such a red-blue road. There are six possibilities; one is described in the following sample dialogue.

Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

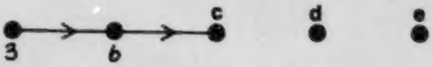
S: Red.



T: What number is here (at b)?

S: 6.

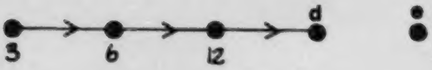
S: Red.



T: What number is here (at c)?

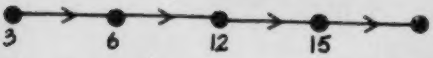
S: 12.

S: Blue



T: What number is here (at d)?

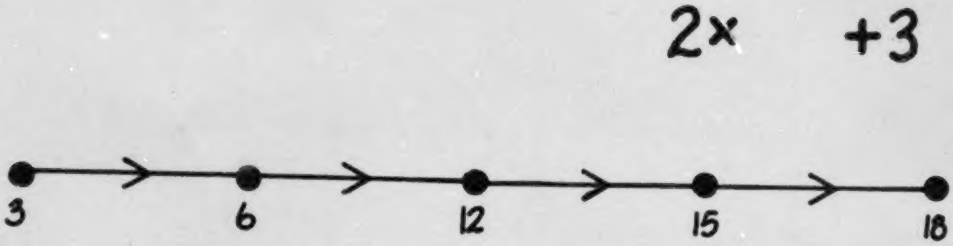
S: 15.



S: Blue.

T: What number is here (at e)?

S: 18.



T: This road ends at 18. Is 18 on our list?

S: Yes.

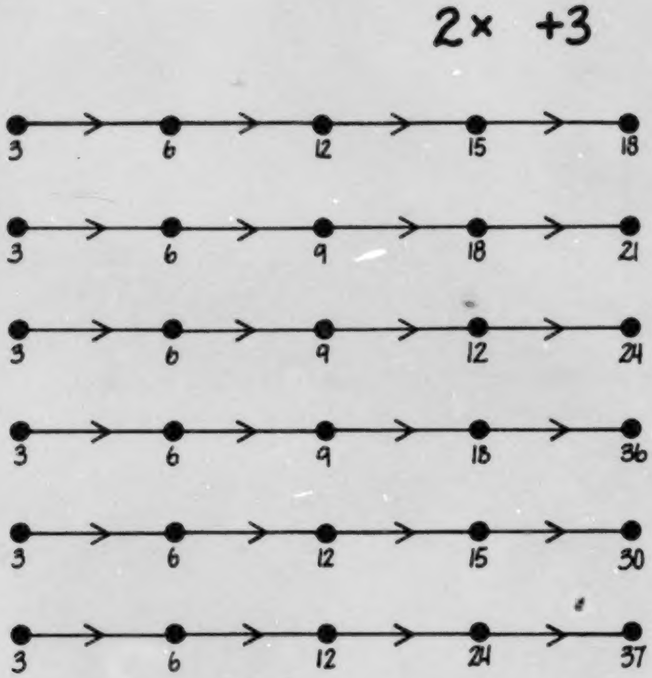
T: So my secret number could be 18. I will circle it in the list.

2	6	8	9
12	16	⑱	19
32	36	38	39
72	76	78	79

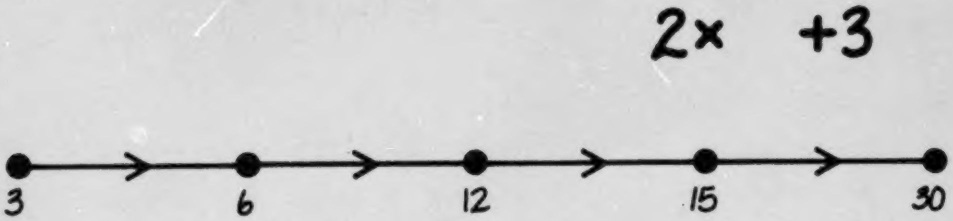
Distribute unlined paper and colored pencils.

T: On your paper draw all the different roads which have exactly two red arrows and two blue arrows. All of your roads should start at 3.

As students are working encourage them to be thinking about how many different roads there are. Invite students to put the six different roads on the board.


2× +3
3 6 12 15 18
3 6 9 18 21
3 6 9 12 24
3 6 9 18 36
3 6 12 15 30
3 6 12 24 37

After all six roads have been drawn on the board, draw the class' attention to the board and consider each road, asking if its ending number could be the secret number. Circle any entry in the list which is also the ending number of one of the red-blue roads. For example, suppose this road is being discussed.


2× +3
3
6
12
15
30

T: This road is blue-red-blue-red and ends at 30. Could 30 be my secret number?

S: No, 30 is not in the list.

NOTE: If there is confusion that any number in the arrow pictures could be the secret number, erase the number labels from all but the ending dots.

After comparing all the ending numbers of the red-blue roads with the numbers in the list from the first clue, you should have only "18" and "36" circled. Erase all others.

⑱

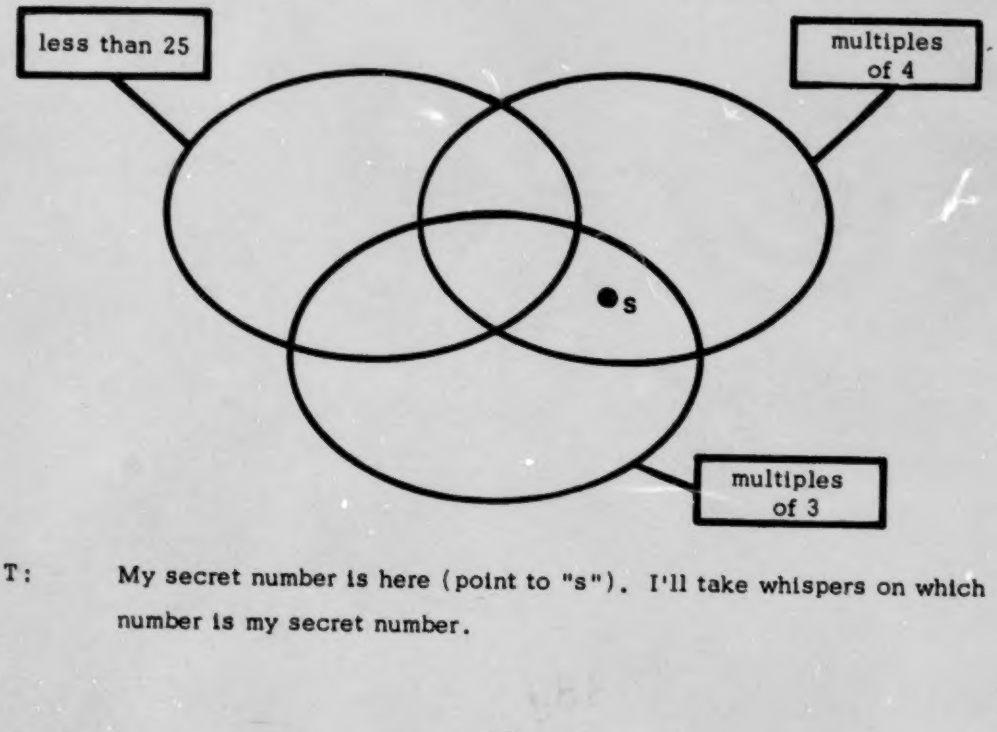
㊱

T: What do you need to help you decide which of these numbers (18 or 36) is my secret number?

S: Another clue.

Third clue

Erase the arrow roads from the board and draw this string picture.



T: My secret number is here (point to "s"). I'll take whispers on which number is my secret number.

Allow several students to whisper their answers to you and then ask someone to answer aloud. Conclude that the secret number is 36 and show the class that it is written on the slip of paper.

T: Who can explain why 36 is here (point to "s")?

S: 36 is a multiple of 3 and a multiple of 4.

T: Is 36 less than 25?

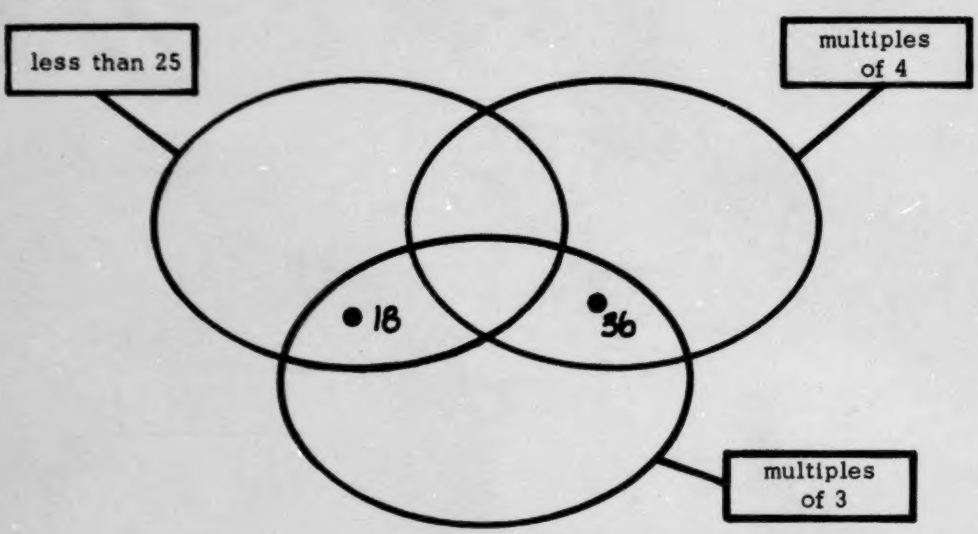
S: No.

T (tracing the red string): So 36 must be outside the red string.

Erase the "s" and label the dot 36.

T: Who can show us where 18 belongs in this picture?

When a student indicates the correct region, draw a dot in that region and label it "18".



T (tracing the appropriate strings in turn): Is 18 less than 25? (Yes)

Is 18 a multiple of 3? (Yes)

Is 18 a multiple of 4? (No)

Which are multiples of 4 and are close to 18? (16 and 20)

L11 THE LITTLE DONKEY

CAPSULE LESSON SUMMARY

Discuss an arrow picture as it develops in the Storybook THE LITTLE DONKEY. The arrows are for "You are my mother" and "You are my father."

MATERIALS

Teacher: Colored chalk; one copy of the Storybook THE LITTLE DONKEY

Student: One copy of the Storybook THE LITTLE DONKEY for every two students; colored pencils; Worksheets L11* and **

DESCRIPTION OF LESSON

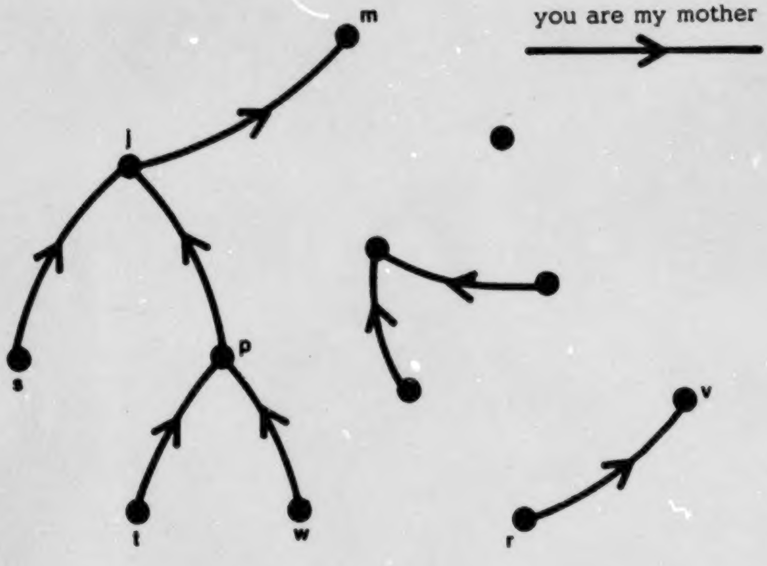
Distribute copies of the Storybook THE LITTLE DONKEY. Ask your students not to read ahead of the class during the lesson.

Pages 1-4

Read these pages with your class.

Pages 5 and 6

After reading page 5, draw the arrow picture from that page on the board. Add the key arrow to your picture, indicating that the blue arrows are for "you are my mother."



Do not label the dots. They are labeled here to make the description of the lesson easier to follow.

Read page 6 with your class and label dots r and v accordingly. Remind the students not to turn the page.

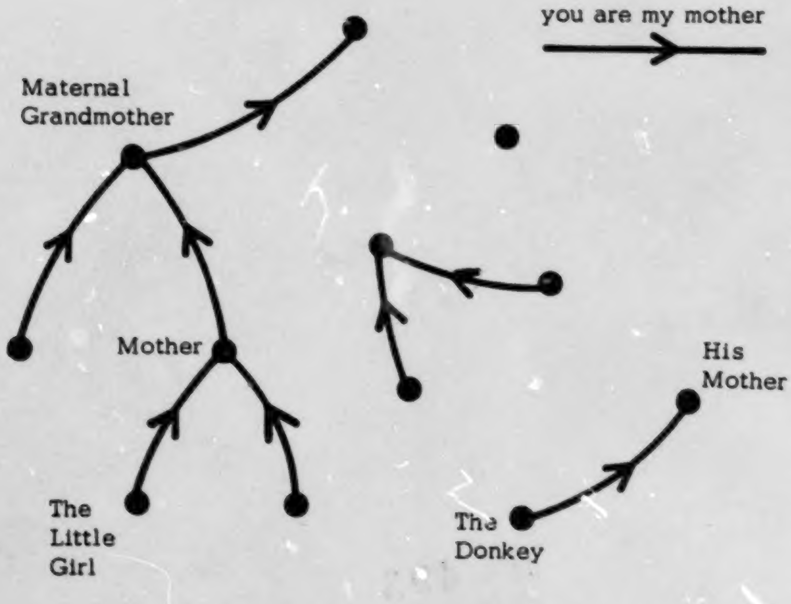
T: I'll tell you where the little girl is in this picture.

Label the dot (t) for the little girl.

T: Now can you tell for sure who some of the other dots are for?

Your class should conclude who the dots p, j, and m are for. Take the opportunity to discuss how to distinguish a person's two sets of grandparents and then introduce the terminology "maternal" and "paternal." Label p, j, and m appropriately.

Perhaps some of your students will comment that w must be for either a brother or a sister of the little girl and that s must be for either an aunt or an uncle. If so, when page 7 is read, note which of the choices is made for each dot.


you are my mother
Maternal Grandmother
Mother
The Little Girl
His Mother
The Donkey

L 11

Pages 7 and 8

Read these pages with your class. Label the dots in the picture on the board for the girl's brother and for their aunt.

Pages 9 and 10

Read these pages with your class. Add the red arrows to your picture as shown on page 9 and add a key arrow indicating that the red arrows are for "you are my father." Let a student locate the dot for the little girl's father and then label the dot yourself.

Pages 11 and 12

Read these pages with your class. Invite students to locate the dots for the little girl's paternal grandparents and then label them yourself.

There is still one dot which is unlabeled (see page 14). Ask what it could be for. Your students should decide that it is for either a brother or a sister of the girl's father, i.e. an aunt or an uncle of the girl.

Pages 13, 14, 15

Finish reading the storybook with your class.

Worksheets L11* and ** are available for individual work.

Name ______________________ **L 11** *

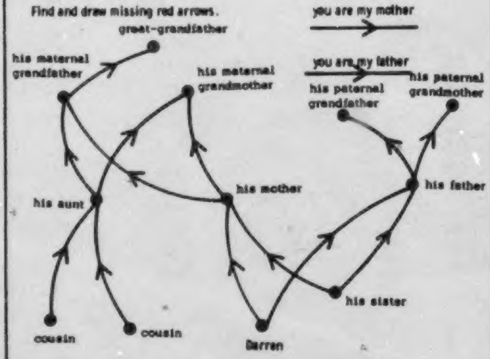
The dots in this arrow picture are for some of Darren's relatives:

his mother
his father
his sister
his aunt
two cousins
(two dots)

his paternal grandfather
his paternal grandmother
his maternal grandfather
his maternal grandmother
one of his great-grandfathers

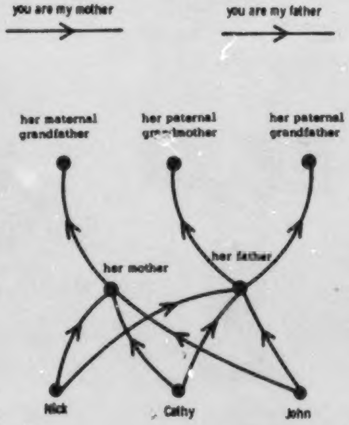
Can you label the dots?

Find and draw missing red arrows.


you are my mother
great-grandfather
his maternal grandfather
his maternal grandmother
you are my father
his paternal grandmother
his paternal grandfather
his mother
his father
his aunt
his sister
cousin
cousin
Darren

Name ______________________ **L 11** **

Cathy is the sister of Nick and John. Both of Cathy's paternal grandparents and her maternal grandfather are still alive, but the mother of Cathy's mother died last year. Draw an arrow picture showing Cathy's family.


you are my mother
you are my father
her maternal grandfather
her paternal grandmother
her paternal grandfather
her father
her mother
Nick
Cathy
John

Other family members might be added to the picture; however, these are just those mentioned in the description of Cathy's family.

L 12

L12 Negative Numbers Game with Two Cubes

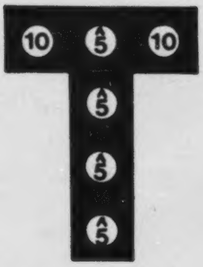
CAPSULE LESSON SUMMARY

Examine two numeral cubes noting that a cube has six faces and what is on each face of the two cubes. Find all the possible scores one can get by tossing one of the cubes three times and adding the outcomes. Play a game with the blue cube against the red cube and observe which cube gets the higher score most often.

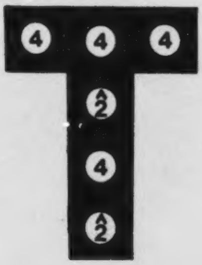
MATERIALS

Teacher: Two numeral cubes; cube cutout
Student: Paper; pencil

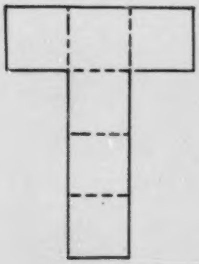
NOTE: Before the lesson begins, put labels on the blue cube and the red cube as shown below. Using sturdy paper, an index card for example, prepare a cube cutout with unlabeled faces. Such a cutout (right picture below) can be folded to form a cube. Dotted lines indicate fold lines.



blue cube



red cube



cube cutout

DESCRIPTION OF LESSON

T (holding up a cube): What shape is this?

S: A box.

T: Yes, but it is a special kind of box shape.

Perhaps students will say "square," in which case you should point out that one side (or face) is a square.

S: A cube.

If necessary, give this name to the shape yourself.

T (indicating a side of the cube): A side of a cube is called a "face." How many faces does a cube have?

S: Six.

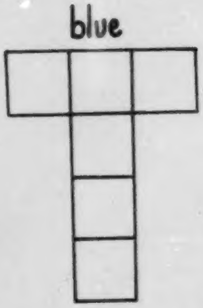
Hold up your cube cutout.

T: Does this look like a cube?

S: It's like a cube flattened out.

S: You can fold it into a cube.

Demonstrate how the paper cutout folds to form a cube. Then draw a similar map for the blue cube on the board.



Give a student the blue cube to examine.

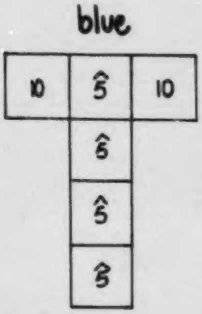
T: What numbers are on the faces of the blue cube?

S: $\hat{5}$ is on four of the faces and 10 is on two of the faces.

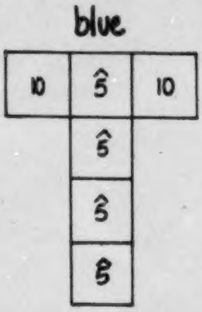
T: How can we show this in our picture (point to the map of the blue cube on the board)?

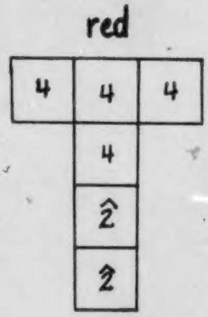
S: By writing "$\hat{5}$" in four of the squares and "10" in the other two squares.

Invite a student to label the faces in the picture; one way is shown below.


blue
10
$\hat{5}$
10
$\hat{5}$
$\hat{5}$
$\hat{5}$

Give a student the red numeral cube to examine and draw a map of the red cube on the board. Ask the student to announce which numbers are on the faces while another student labels the faces in the map of the red cube.


blue
10
$\hat{5}$
10
$\hat{5}$
$\hat{5}$
$\hat{5}$


red
4
4
4
4
$\hat{2}$
$\hat{2}$

T: Suppose I toss the red cube three times and add the numbers which appear (on top); what score could I get?

S: 12.

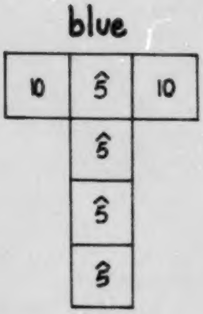
T: How could we get a score of 12 with the red cube?

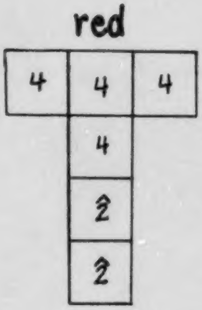
S: Roll 4, 4, and 4.

Write this number sentence on the board.

$$4 + 4 + 4 = 12$$

Keep a record of the possible scores near the map of the red cube.


blue
10
$\hat{5}$
10
$\hat{5}$
$\hat{5}$
$\hat{5}$


red
4
4
4
4
$\hat{2}$
$\hat{2}$

red scores

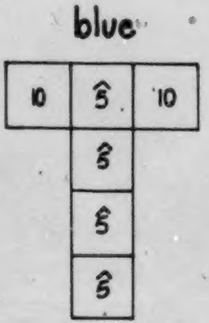
12

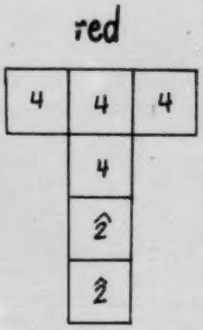
Continue in this manner until all four possible scores are found. You should have these number stories recorded in some order . . .

red

$$4 + 4 + 4 = 12$$
$$4 + 4 + \hat{2} = 6$$
$$4 + \hat{2} + \hat{2} = 0$$
$$\hat{2} + \hat{2} + \hat{2} = \hat{6}$$

. . . and a list of the possible scores.


blue
10
$\hat{5}$
10
$\hat{5}$
$\hat{5}$
$\hat{5}$


red
4
4
4
4
$\hat{2}$
$\hat{2}$

red scores

12

6

0

$\hat{6}$

L 12

Similarly ask what are the possible scores if you toss the blue cube three times. You should have these number stories recorded in some order . . .

blue

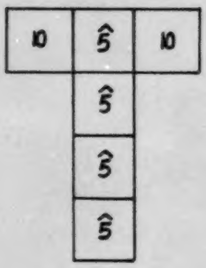
$$\begin{aligned} 10 + 10 + 10 &= 30 \\ 10 + 10 + \hat{5} &= 15 \\ 10 + \hat{5} + \hat{5} &= 0 \\ \hat{5} + \hat{5} + \hat{5} &= \widehat{15} \end{aligned}$$

. . . and a list of the possible scores.

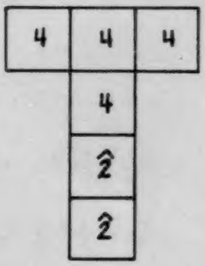
blue scores

30
15
0
$\widehat{15}$

blue



red



red scores

12
6
0
$\hat{6}$

Erase the number stories.

T: Suppose we play a game with these cubes, rolling each three times. High score wins. Would you rather play with the red or the blue cube?

Perhaps some students will have an intuitive feel for the cubes and choose the red. Others might look only at the possible scores and choose the blue.

T: The red cube has more positive numbers on its faces, but the blue cube has the possibility of a higher score.

Draw this chart on the board.

blue	red	winner

T: The player with the blue cube always goes first. Who would like to throw that cube?

Give a student who volunteers the blue cube.

T: Who would like to play with the red cube?

Give a student who volunteers the red cube. Ask the two students to sit on the floor at the front of the room to throw the cubes.

After the first person has thrown the blue cube three times, ask, "Can the second person (use actual name) win? Can the second person lose?" For example, if . . .

blue throws 10 — 10 — 10 ⟶ red cannot win

blue throws Ŝ — Ŝ — Ŝ ⟶ red must win

In other cases, it is impossible to decide until the red cube is thrown.

As the throws are being made, record the information as a number story on the board. If a student rolls Ŝ, 10, and then 10, write "Ŝ + 10 + 10 = 15". Write the scores for each student in the chart and then erase the number stories. Repeat this activity several times choosing different students each time to throw the cubes.

Call on two more students to toss the cubes. After the student tossing the blue cube throws twice, stop the game and note the intermediate score. For example, suppose the student has gotten 10 and $\hat{5}$.

T: After two throws, what is the blue score?

S: 5, because $10 + \hat{5} = 5$.

T: Do you think the blue score could be 30 after the third throw?

S: No; the third throw could be 10 or $\hat{5}$, so the total blue score could be 15 ($5 + 10 = 15$) or 0 ($5 + \hat{5} = 0$).

T: Which score, 15 or 0, do you think has a better chance of occurring?

S: 0, because there are more faces with $\hat{5}$ than with 10. Let the student continue by making the third toss and record the score. Proceed in this way also for the red cube.

Repeat this activity (i.e. stopping the game after two tosses and asking some questions about the intermediate score and the possible final score) several times until most students have had a turn.

The outcome of the experiment with your class cannot be predicted, but the chart from the lesson with one of our development classes is included here. (About one-half of the students in the class had a chance to play.)

blue	red	winner
0	0	tie
30	0	blue
0	12	red
$\widehat{15}$	6	red
$\widehat{15}$	6	red
15	6	blue

At this point you might discuss which cube students prefer to use in the game. Direct the discussion toward choosing a cube with which you have the better chance to win and avoid preferences based on favorite colors or numbers.

If there is sufficient time, you might ask the class to find the possible scores when each cube is thrown four times or five times.

L13 The String Game #3

CAPSULE LESSON SUMMARY

Play two games of the String Game with A-Blocks using sixteen attributes as possibilities for the strings. Play a type of string game with numbers where the strings are for multiples of 3 and multiples of 5.

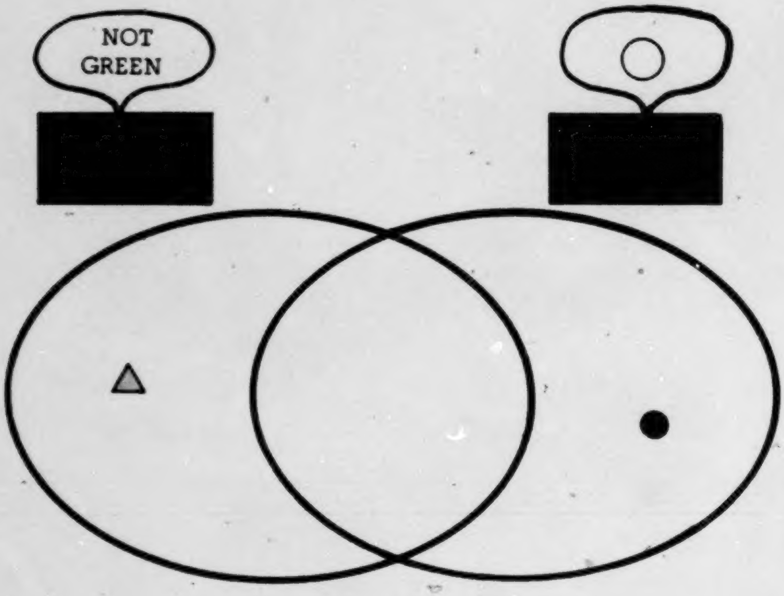
MATERIALS

Teacher: String Game with A-Blocks kit; colored chalk
Student: None

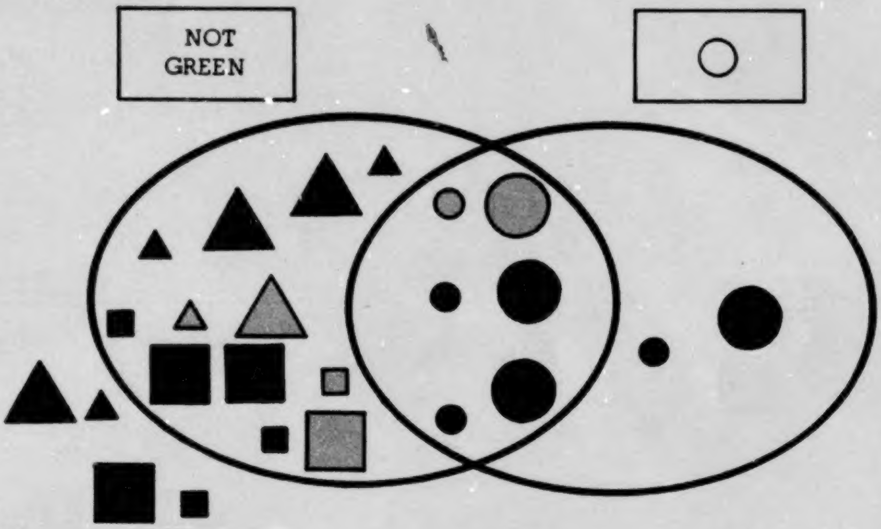
DESCRIPTION OF LESSON

Exercise 1

Prepare to play the String Game with A-Blocks in the usual way. Tape a copy of the list of sixteen labels above the team board. Draw two overlapping strings using different colors. Next to the strings attach the string label cards face down (bubbles in the next illustration indicate what is hidden on the string cards) and give two starting clues as shown here.

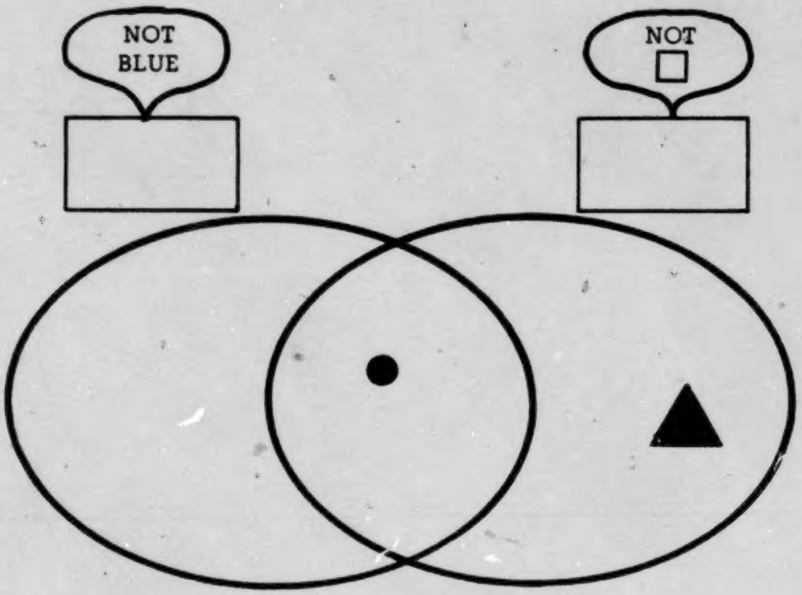


The following picture shows the correct placement of all the game pieces.



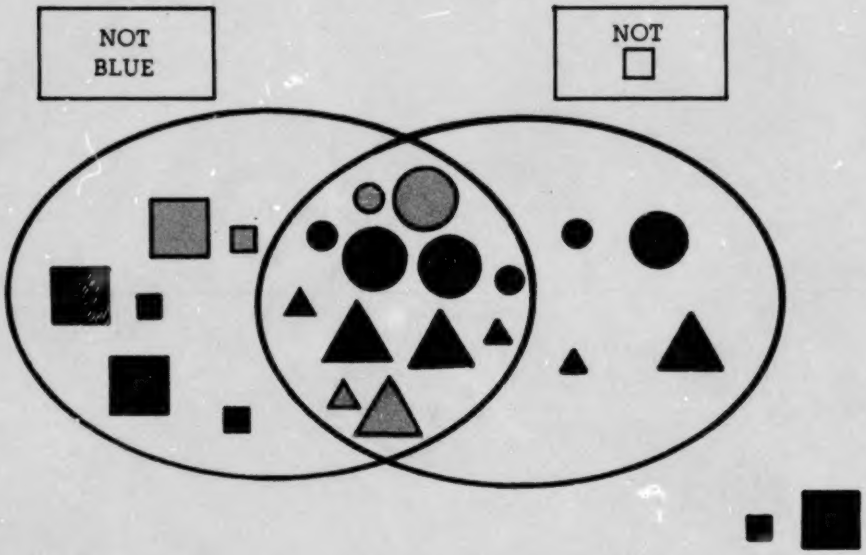
Exercise 2

Play the String Game again using not-cards as labels for both strings. A game with two starting clues is suggested below.



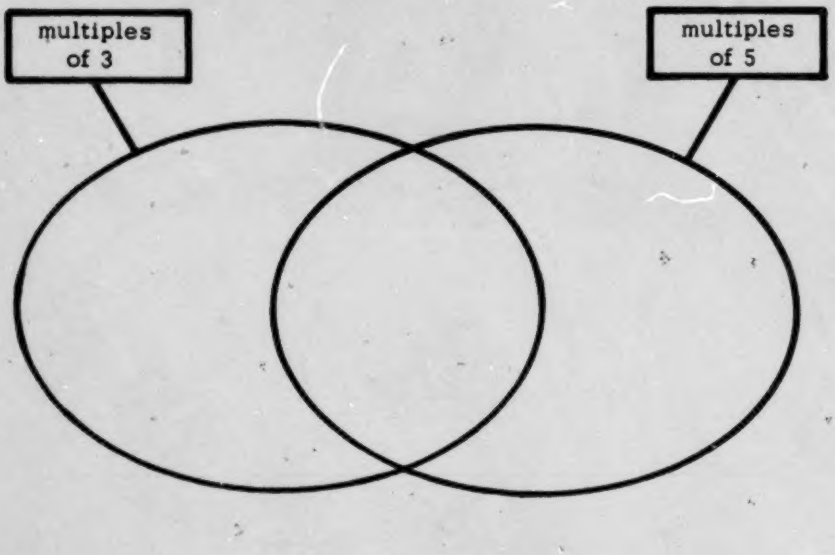
L 13

The following picture shows the correct placement of all the game pieces.


NOT BLUE
NOT □

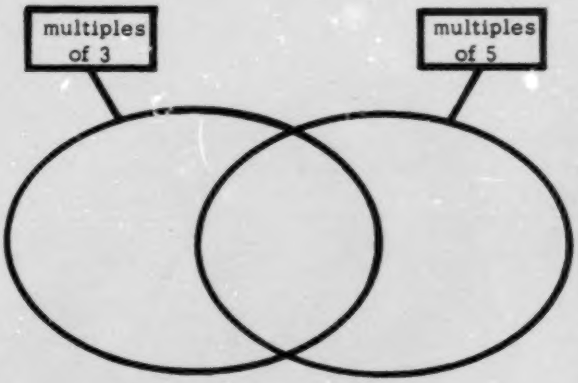
Exercise 3

Take down the list of string labels and put the A-Blocks away. Label the red string "multiples of 3" and the blue string "multiples of 5".


multiples of 3
multiples of 5

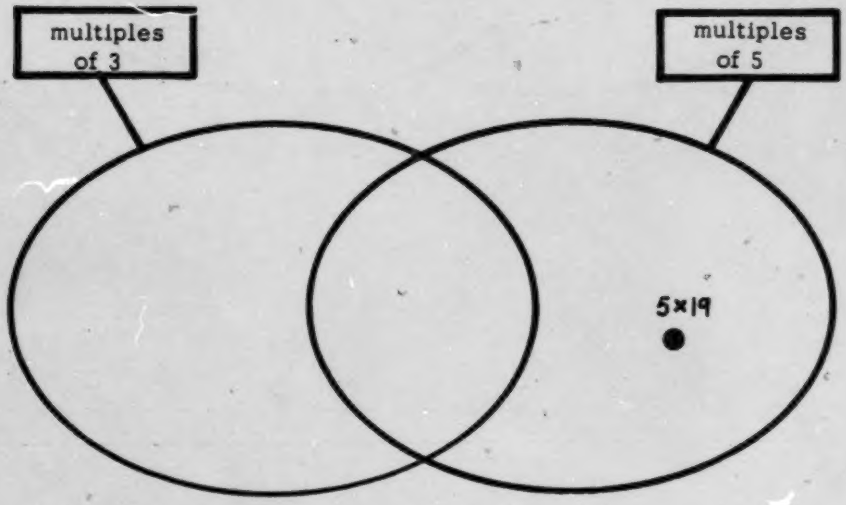
Draw a table near the string picture and put the illustrated entries under "TEAM A" and "TEAM B" respectively.

TEAM A		TEAM B	
30	5×30	15	3×13
3×7	17	3×35	60
45	2×60	19	$\hat{9}$
5×17	26	5×19	50
100	40	5	80
$\hat{5}$	3	2×45	32



T: This time we'll play a game with numbers. During a turn, you will have only one chance to put a number in the string picture. The first team to locate correctly in the picture all the numbers from its list wins.

Alternating teams, let students locate numbers in the picture. Discuss correct responses; for example, suppose a student from Team B locates 5 × 19 in the string picture.



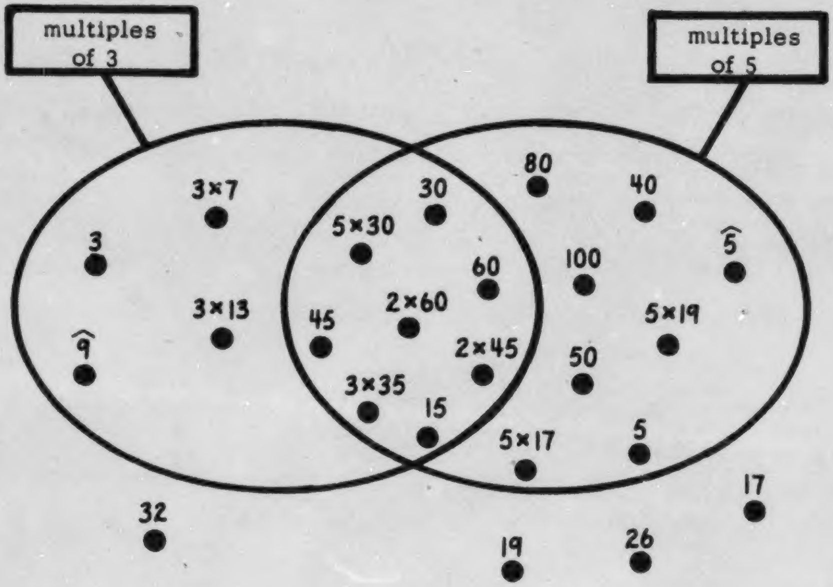
T: That's correct. Why does 5 x 19 belong there?

S: 5 x 19 is a multiple of 5, but it is not a multiple of 3.

Erase "5 x 19" from Team B's entries in the table.

If a student locates a number incorrectly, simply erase it from the picture and say, "No, that number does not belong there."

This picture shows the correct location of all the numbers in both lists.


multiples of 3
multiples of 5
3×7
3
3×13
9̂
30
5×30
60
45
2×60
2×45
3×35
15
80
40
100
5̂
5×19
50
5×17
5
17
32
19
26

L14 Basketball Bar Graphs/Marble Game #2

CAPSULE LESSON SUMMARY

Introduce a long term project of keeping performance records of two basketball teams during the basketball season. Play the Marble Game as described in Lesson L8 but use a different collection of marbles.

MATERIALS

Teacher: Two copies of Language Poster #1 for UPG-III/IV;
Marbles
Student: None

NOTE: Before the lesson begins, cut out the chart from both copies of Language Poster #1. Post these charts, one under the other, on a bulletin board or some other convenient place in the classroom. Save the extension pieces (19 to 36 and $\widehat{19}$ to $\widehat{36}$) for the chart in case they are needed.

DESCRIPTION OF LESSON

Exercise 1: Basketball Bar Graphs

The project described in this exercise involves keeping a record with your class of the performances of two basketball teams during the entire basketball season. You could either choose two teams yourself or let the class choose from a list of six or eight teams. Make the list of teams with local appeal and for which it will be easy to find scores for all their games. Label the two charts with the names of the chosen teams and do this exercise after one or both teams have played at least one game.

Announce the result(s) of the game (or games) and fill in the chart appropriately with your class. The following sample dialogue assumes both teams have played one game and not against each other. Always use the names of the teams involved although here they are referred to as Team A, Team B, and the Opposing Team.

L 14

T: Team A played its first game of the season.

Tell the class the scores as you write them on the board.

TEAM A	OPPOSING TEAM
113	107

T: Did Team A win or lose?

S: They won.

T: By how many points . . . ?

S: Six.

Write this information on the board.

TEAM A	OPPOSING TEAM	(TEAM A)
113	107	won by 6 points

Let a student shade the square above "6" in Team A's chart. See page 112 for an illustration of the chart.

T: Team B played its first game of the season.

Tell the class the scores as you write them on the board.

TEAM B	OPPOSING TEAM
85	96

T: Did Team B win or lose?

S: They lost.

T: By how many points . . . ?

S: Eleven.

Write this information on the board.

TEAM B	OPPOSING TEAM	(TEAM B)
85	96	lost by 11 points

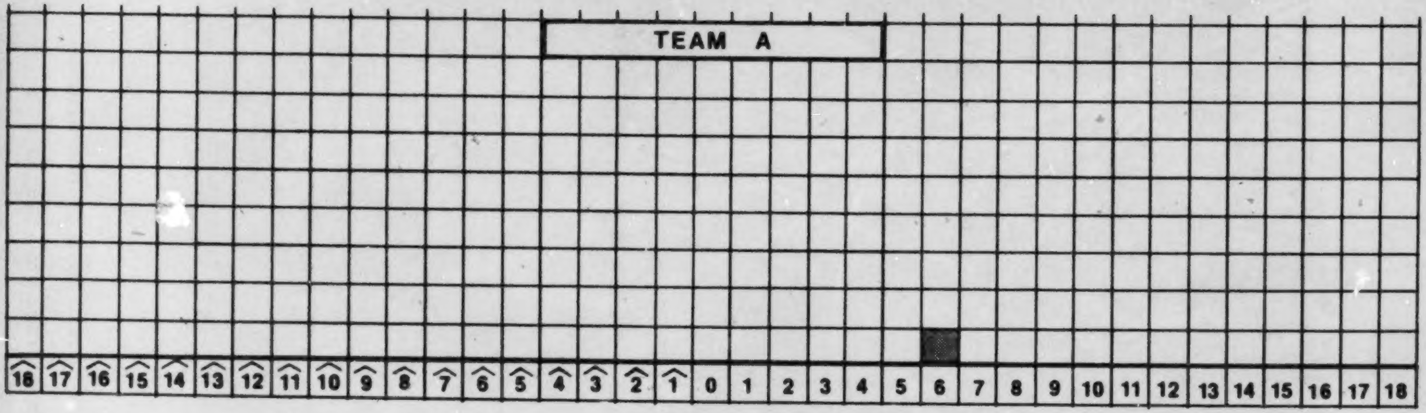
T: Which square should we shade in Team B's chart?

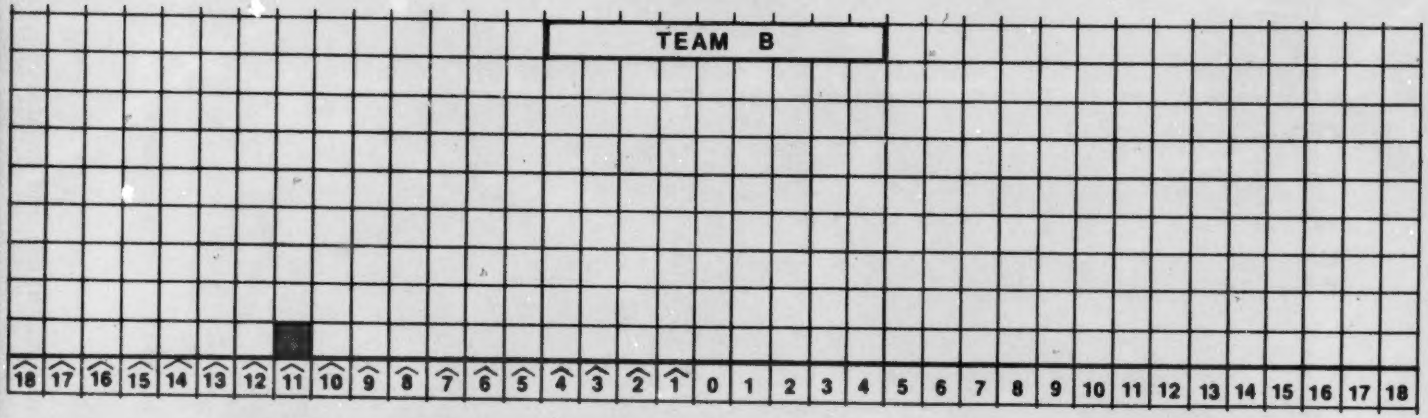
S: A square above "$\widehat{11}$".

Let a student shade the appropriate square. See the next page for an illustration of the chart.

During the entire basketball season, collect and record the results of the games played by the two teams.

NOTE: At the end of the basketball season, you should have graphs similar to those at the end of Lesson L8 of UPG-IV. Of course, your graphs will reflect the teams that your class is following.


TEAM A
18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18


TEAM B
18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Exercise 2: Marble Game

For the remainder of the lesson play the Marble Game as in Lesson L8, only this time play with four red and two blue marbles.

4 REDS
2 BLUES

RED TEAM 10

Prediction	#of Reds	Points

BLUE TEAM 10

Prediction	#of Reds	Points

L15 Changing Seats

CAPSULE LESSON SUMMARY

Associate the dots in an arrow picture with the seats in a row. Change seats by following the arrows. Play this game of changing seats several times in succession and after each round note who are in their original seats. Predict how many rounds need to be played before everyone is back in his or her seat.

MATERIALS

Teacher: Colored chalk
Student: Paper; colored pencils

DESCRIPTION OF LESSON

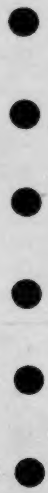
Organize your class into rows of six or seven students. The chart below suggests a way of doing this for class sizes between 18 and 31.

18 3 rows of 6	19 3 rows of 6 with one recorder	20 3 rows of 6 with two recorders	21 3 rows of 6 with three recorders	22 3 rows of 7 with one recorder	23 3 rows of 7 with two recorders	24 3 rows of 7 with three recorders
25 4 rows of 6 with one recorder	26 4 rows of 6 with two recorders	27 4 rows of 6 with three recorders	28 4 rows of 6 with four recorders	29 4 rows of 7 with one recorder	30 4 rows of 7 with two recorders	31 4 rows of 7 with three recorders

The lesson description assumes there are four rows of six students and also some recorders. The dialogue here includes names to facilitate the description; you, of course, will use the names of your students.

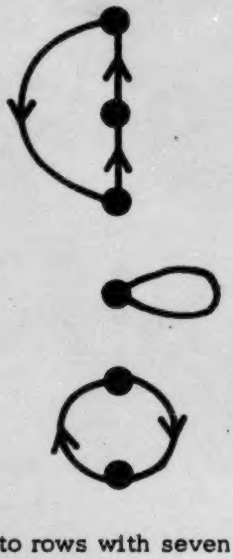
Ask any recorders to wait at the board. If possible, assign a recorder to each row.

Draw six (or seven) dots in a column on the board.



As you explain the column of dots to the class, indicate that the dot at the bottom is for the first seat in a row and the dot at the top is for the last seat in a row.

T: These dots are for the seats in any one of the rows. I will draw some arrows to show you how we are going to change seats.*



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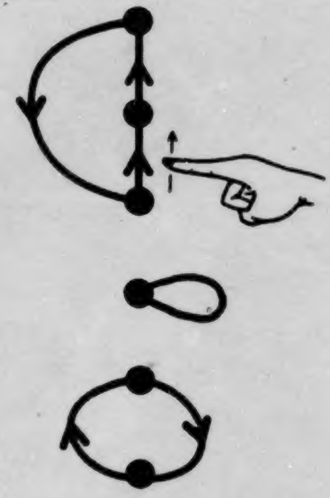
* If your class is organized into rows with seven students, draw a loop at each of two dots.

L 15

T: Look at the picture on the board. Mike, where will you go when we change seats?

S: To Pat's seat.

Trace the appropriate arrow. If Mike were sitting in the fourth seat in his row, you would trace the arrow from the fourth dot to the fifth dot.



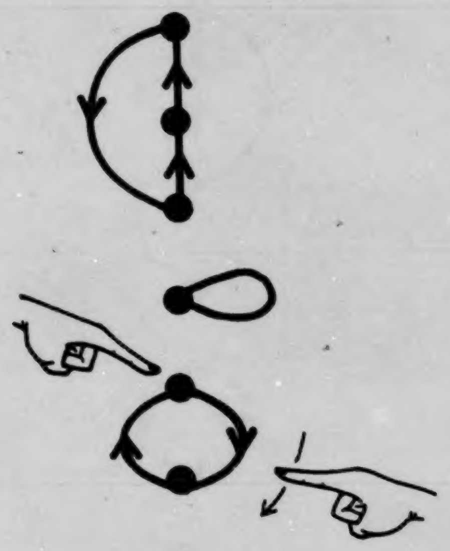
T: William, where will you go when we change chairs?

S: I'm not sure.

T (pointing successively to desks in William's row): First, second; you are sitting in the second seat in your row, William.

T (pointing successively to the dots on the board): First, second; this dot is for your chair. Now do you know where you will go when we change seats?

Keep one finger on the second dot and trace the arrow starting at that dot.



S: To Cameron's seat.

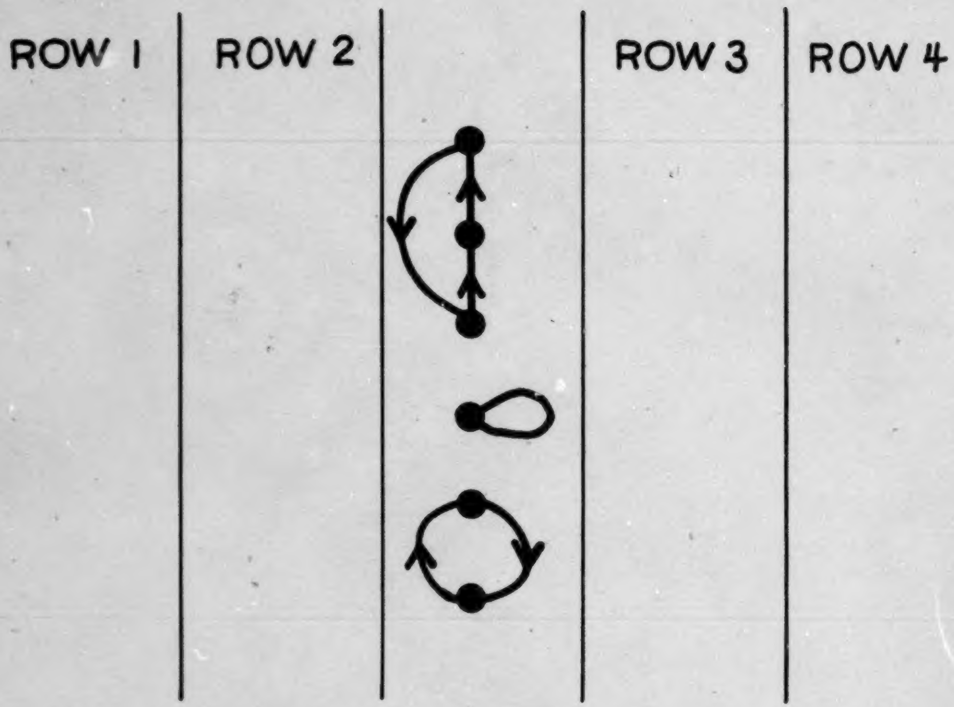
T: Mindy, where will you go?

S: To Lisa's seat.

T: The recorders and I are going to keep records on the board of what happens when you change seats.

Provide a labeled column for each row. (See the next illustration.)

L 15



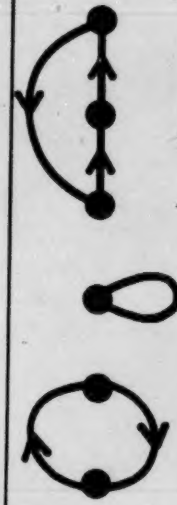
T: Each row moves in the same way following the arrows in this picture.
Does everyone know where to go?
Get ready; get set; go!

Let the students make their moves and then let them quiet down. Children may get excited and noisy during this activity, so you might want to preface your introduction of this game with some remarks about moving as quietly as possible.

T: Raise your hand if you are now in your own chair.

You and the recorders should record the names of the students whose hands are raised. (See Round 1 in the next illustration.)

Repeat this activity for a second round and record the names of the students who are in their own chairs after the second round.

ROW 1	ROW 2		ROW 3	ROW 4
Round 1	Round 1		Round 1	Round 1
Patrick	Derrick		Linda	Ki Jon
Round 2	Round 2		Round 2	Round 2
Patrick	Derrick		Linda	Ki Jon
Sheila	William		Terrell	Gary
Anthony	Cameron		David	Scott

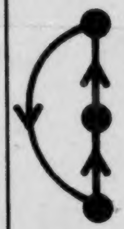
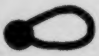
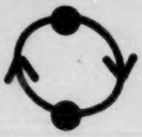
Before playing a third round of the game, ask:

T: If we change seats again, can you predict who will be back in their own seats?

Let a person from each row make a prediction for that row. Repeat each response for the benefit of the class. Then let the students play the third round.

T: Get ready; get set; go!

When the class has quieted down, ask which students are in their own chairs. You and the recorders' should record the names of those students under "Round 3" on the board.

ROW 1	ROW 2		ROW 3	ROW 4
ROUND 1	ROUND 1		ROUND 1	ROUND 1
PATRICK	DERRICK		LINDA	KI JONG
ROUND 2	ROUND 2		ROUND 2	ROUND 2
PATRICK	DERRICK		LINDA	KI JONG
SHEILA	WILLIAM		TERRELL	GARY
ANTHONY	CAMERON		DAVID	SCOTT
ROUND 3	ROUND 3		ROUND 3	ROUND 3
PATRICK	DERRICK		LINDA	KI JONG
ANDREA	ANDY		SHARON	MARY
MIKE	STANLEY		LISA	PATTY
PAT	LORI		MINDY	JASON

T: How many times do you think we need to play this game before you are all back in your own seats? (Six)

S: One more time.

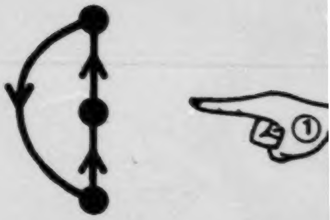
T: Raise your hand if you will be in your own seat if we play one more time.

Only some of your students should raise their hands.

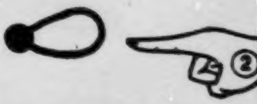
T: Some of you would not return to your own seats after one more round. Who else has an idea?

If no one gives the correct answer with a clear explanation, the following dialogue suggests how to help students arrive at the solution. The next illustration focuses only on the arrow picture.

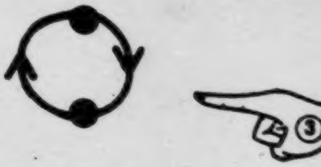
T: The people who start here ① return to their own chairs during the third round. When would the next time be? (During the sixth round) . . . the next time? (During the ninth round) So every third round they return to their own seats.



During every round this person ② is in the same chair.



The people who start here ③ return to their own chairs during the second round. During which other rounds would they return to their own chairs? (During the fourth, the sixth, the eighth, and so on)



When is the first time (during which round) that you all return to your own seats?

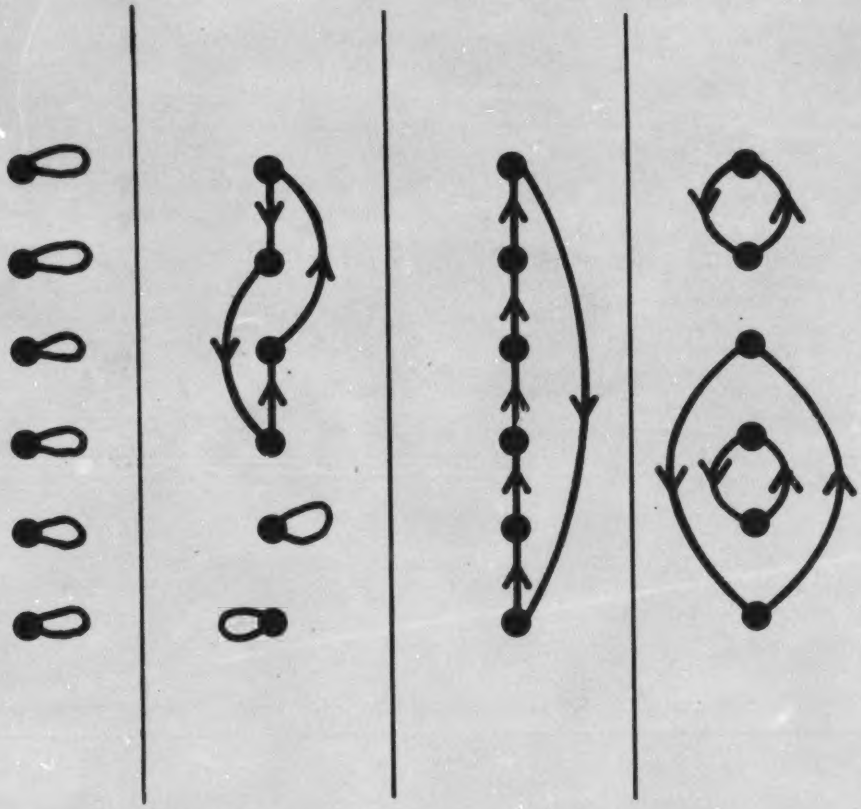
S: During the sixth round.

Quickly play three more rounds of the game without recording names on the board. All students should then be in their own seats. Distribute paper and red pencils.

T: Draw six (seven) dots on your paper and draw red arrows for another way we could play the game of changing seats.

L 15

After the students begin find ways, ask some of them to put their pictures on the board. Include a wide variety of pictures; for example,



If time allows, consider each picture on the board and ask when is the first time (during which round) that all the students return to their own seats.

In the preceding illustration (starting from the left),

- First picture: all students are always in the same seats.
- Second picture: all students return to their own seats during the fourth round but not before.
- Third picture: all students return to their own seats during the sixth round but not before.
- Fourth picture: all students return to their own seats during the second round but not before.

L16 Counting Problems

CAPSULE LESSON SUMMARY

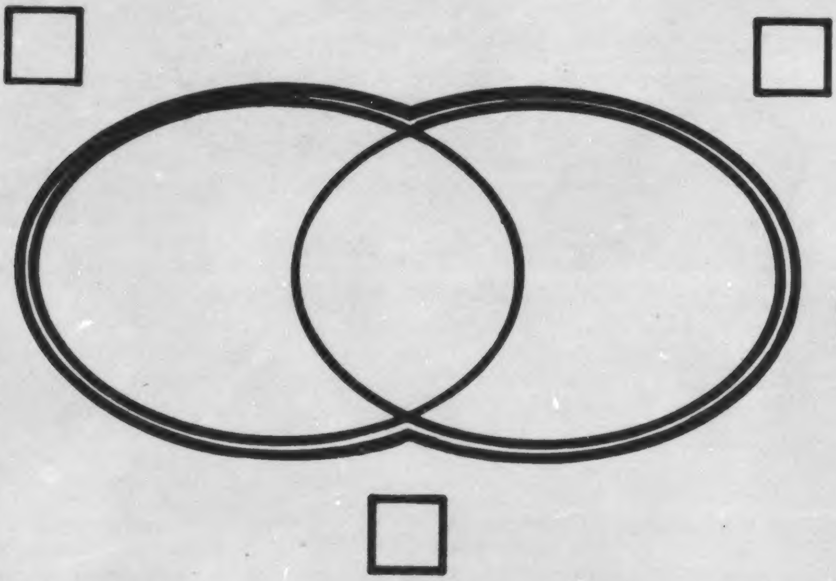
Solve several problems to show how there can be a given number of objects inside one string and a given number of objects inside a second string and the total number of objects less than the sum of the given numbers. Given the total number of objects, the number of objects inside one string, and the number of objects common to two strings, solve a problem of finding how many objects are in the second string.

MATERIALS

Teacher: Colored chalk
Student: Worksheets L16*, **, ***, and ****

DESCRIPTION OF LESSON

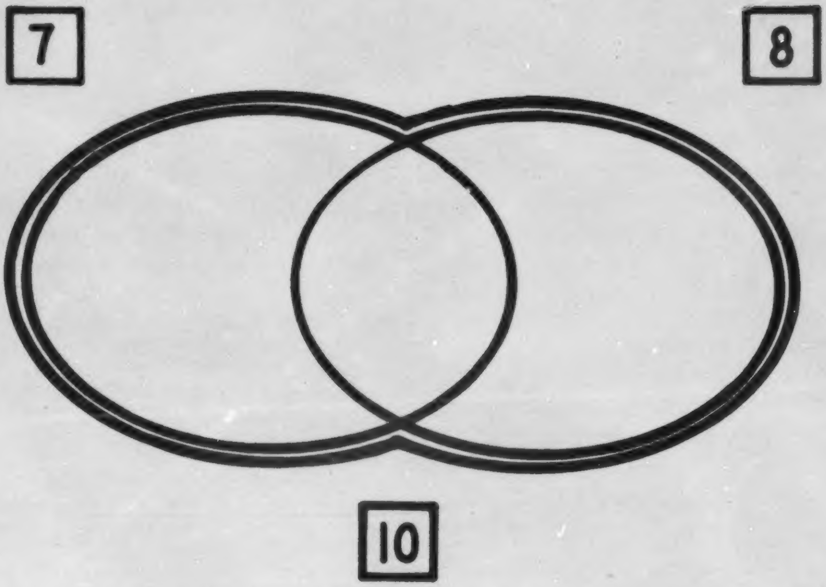
Distribute paper and colored pencils. Draw this string picture on the board.



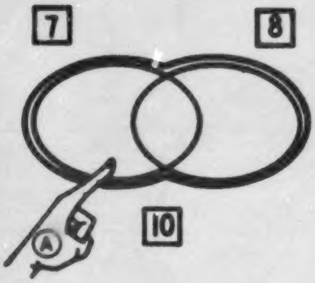
Record the appropriate number in the boxes as you say:

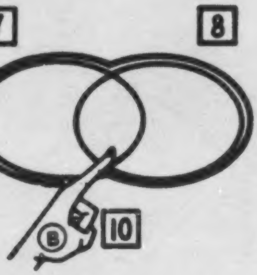
T: I want to put eight dots inside the red string and seven dots inside the blue string, and I want there to be exactly ten dots altogether.

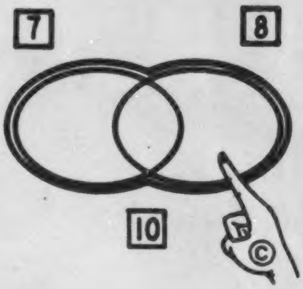
L 16



Point to these three regions as you say:





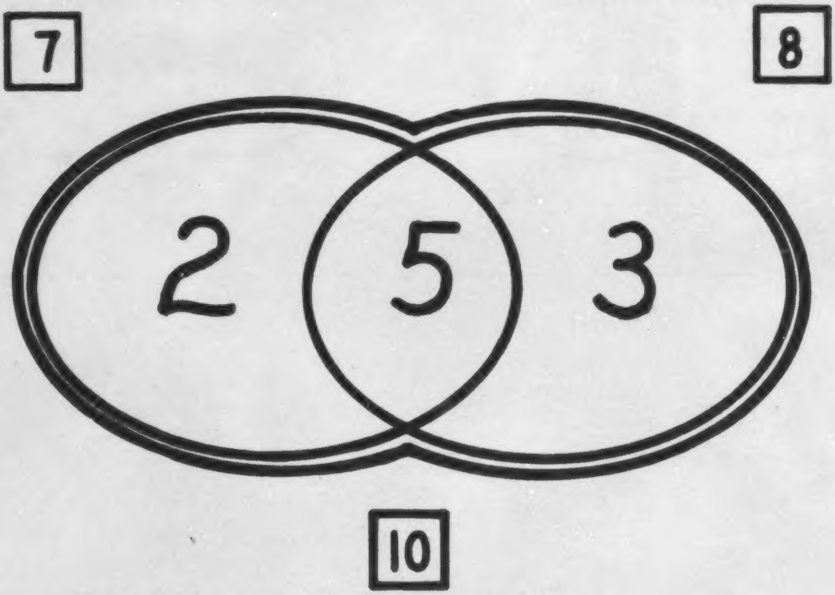


T: How many dots should I put here Ⓐ ? . . . here Ⓑ ? . . . here Ⓒ ? Draw them in a string picture on your paper.

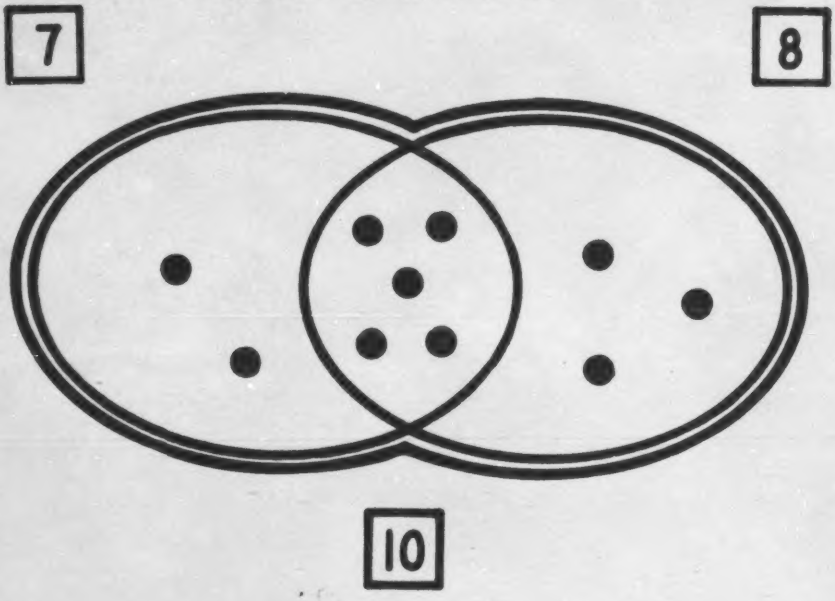
Let the students experiment for several minutes. When a student wants to offer a possible solution, record the number of dots in each region instead of drawing the dots as the student suggests them. Check the picture with the class by asking these questions.

T: Are there seven dots in this string (trace the blue string)?
Are there eight dots in this string (trace the red string)?
Are there ten dots altogether (trace the outer string)?

If one or two of the conditions are not met, erase the numerals inside the regions and ask for another way until a student gives the correct solution.

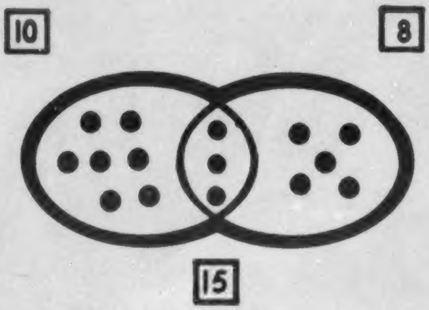


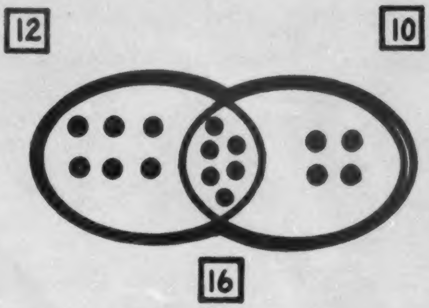
Erase the numerals in the regions and draw the dots.



L 16

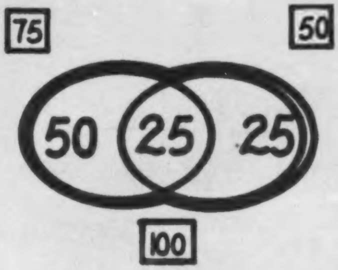
Erase the numerals in the boxes and the dots in the regions. Keep the string picture on the board for the entire lesson. Repeat this activity with these string totals. The illustrations also show the correct solutions.



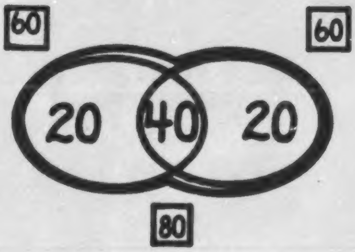


Vary the activity by adding story lines. Two examples are given below with correct solutions given on the right.

T: Pretend I have a large box which has one hundred blocks inside it. There are seventy-five green blocks (blue string) and fifty blocks which are cubes (red string).

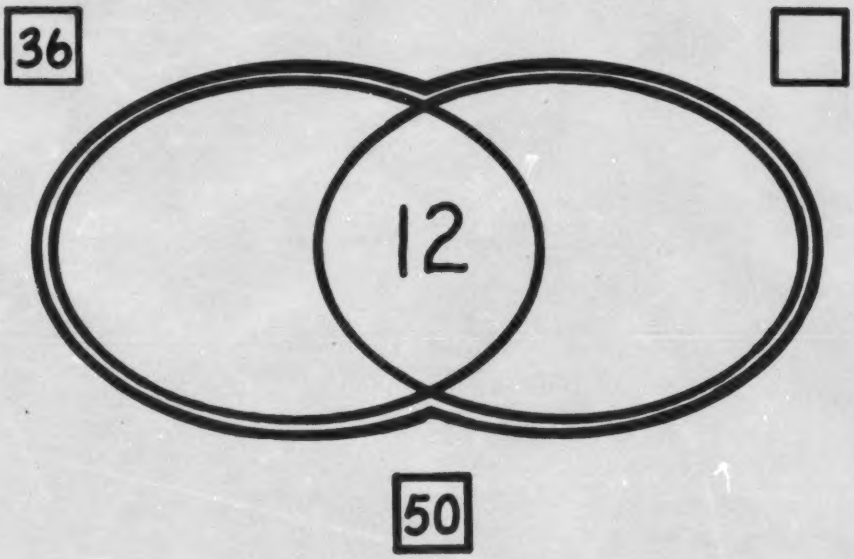


T: Pretend I sold eighty sandwiches at a football game. Sixty were hotdogs (blue string) and sixty had mustard on them (red string).



Vary the activity by changing which information is given. An example is presented in the remainder of the lesson description.

T: Pretend I made a bouquet with fifty flowers (record "50" in the black box). In this bouquet there are thirty-six roses (record "36" in the blue box). Some of the flowers are yellow (trace the red string) but I won't tell you how many. I will tell you this (write "12" in the center region).



T: What does that tell you (point to "12")?

S: There are twelve yellow roses in your bouquet.

T: Who can tell me the other numbers in the picture?

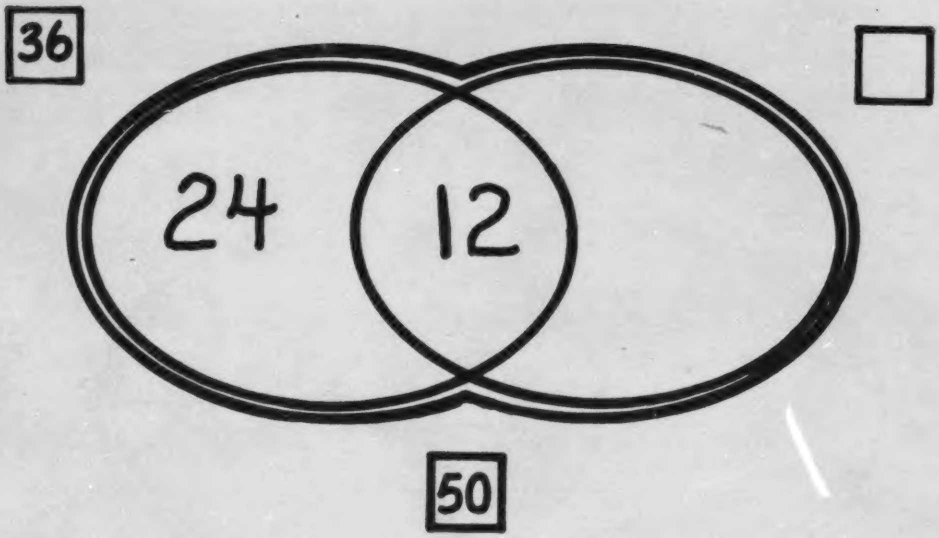
Although students may suggest some numbers which do not belong in the picture and which will be ruled out as possibilities, the class might proceed as follows.

S (pointing to the region in the blue string on the left): Twenty-four dots belong here.

T: How do you know?

S: Because, 24 + 12 = 36.

Let the student record "24" in that region.

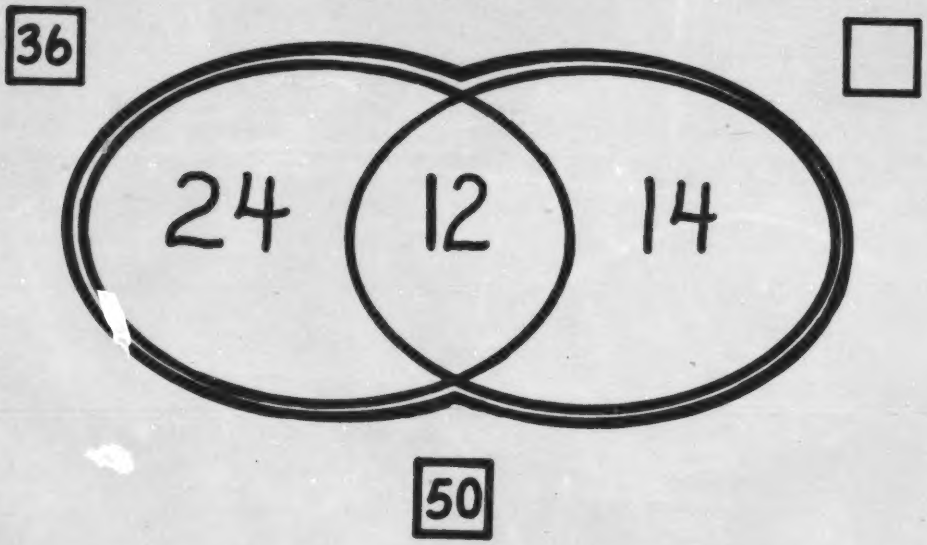


S (pointing to the region in the red string on the right): Fourteen dots belong here.

T: How do you know?

S: Because 24 + 12 + 14 = 50.

Let the student record "14" in that region.



T: Let's check to see if we have fifty flowers altogether.

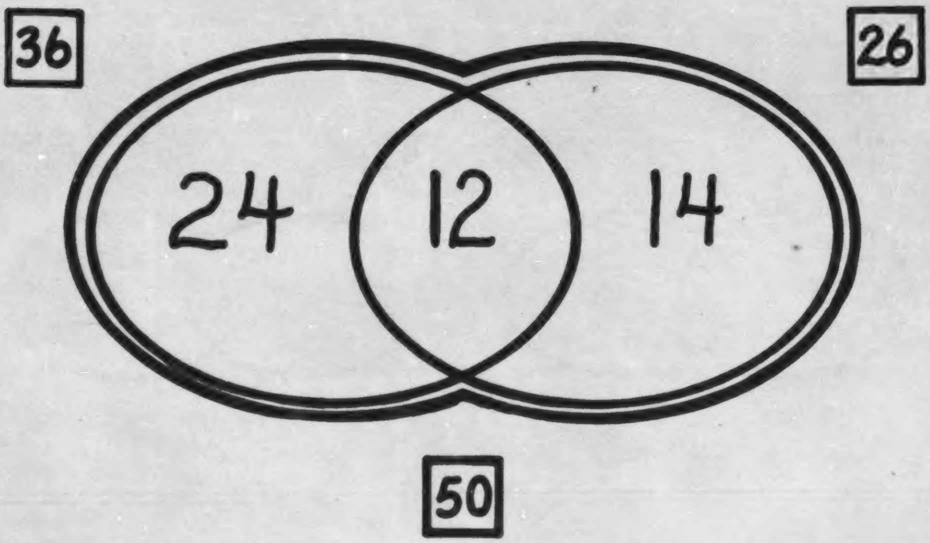
Write this addition problem on the board and invite a student to do the calculation at the board.

$$\begin{array}{r} \overset{1}{2}4 \\ 12 \\ +14 \\ \hline 50 \end{array}$$

T (pointing to the red box): How many yellow flowers are in my bouquet?
How do you know?

S: 26, because 12 + 14 = 26.

Record "26" in the red box.

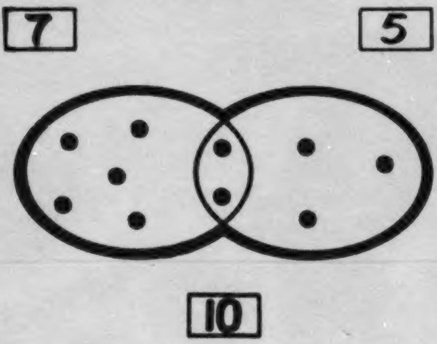


Worksheets L16*, **, ***, and **** are available for individual work.

Name ______________ | L 16 *

Draw dots in this string picture so that there are:

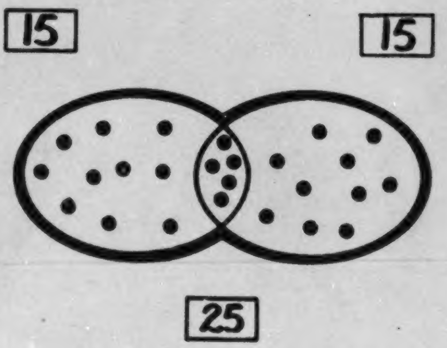
7 dots in the red string;
5 dots in the blue string;
10 dots altogether in the two strings.



Name ______________ | L 16 **

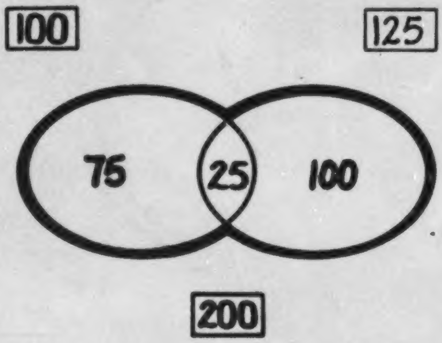
Draw dots in this string picture so that there are:

15 dots in the red string;
15 dots in the blue string;
25 dots altogether in the two strings.



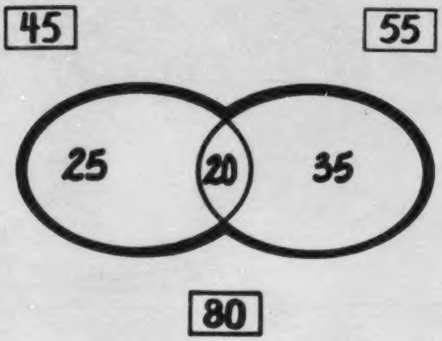
Name ______________ | L 16 ***

Label the parts of this picture to show how many dots are in each part.
Fill in the blue box to show how many dots are in the blue string.



Name ______________ | L 16 ****

Label the parts of this picture to show how many dots are in each part.



L17 SINGING FRIENDS

CAPSULE LESSON SUMMARY

Read and discuss the storybook SINGING FRIENDS. Find the numbers which can be named by inserting parentheses in expressions such as "3 x 5 + 2". Explore the possibilities which result when " x" and "+ " are interchanged or when "3" and "2" are interchanged.

MATERIALS

Teacher: Storybook SINGING FRIENDS; colored chalk

Student: Storybook SINGING FRIENDS (one copy for every two students)

DESCRIPTION OF LESSON

Distribute copies of the storybook SINGING FRIENDS and let the students look at them until you are ready to begin.

Pages 2-3

As you read page 3, stop and ask a student to read the numbers:

1 3 5 7 10 14

T: Do you know a number smaller than any of these?

S: 0.

T: . . . smaller than 0?

S: $\hat{5}$.

S: $\widehat{48}$.

T: Who can read the larger numbers on this page?

After someone reads those numbers correctly, ask:

T: Does anyone know a number larger than 65,375?

Let several students respond and for each correct suggestion, ask if anyone can write that number on the board.

Pages 4-5

Read these pages aloud or let a student do so.

T: What is your favorite number?

Pages 6-9

Read these pages aloud (or let a student do so) and then ask the students to complete the +1 snake dance on page 9.

T: What is the largest number in this picture? (38)
If we drew one more arrow, what would the largest number be? (39)
If we drew ten more arrows starting from 38, what would the largest number be? (48)
If we drew twenty more arrows . . . ? (58)
If we drew thirty more arrows . . . ? (68)

What is the smallest number in this picture? (0)
If we go back ten arrows, what number would we be at? ($\widehat{10}$)
If we go back twenty arrows from 0, . . . ? ($\widehat{20}$)
If we go back thirty five arrows . . . ? ($\widehat{35}$)

Pages 10-11

Ask the students to compare their +1 snake pictures on page 9 to the one on page 10. Then read page 11 aloud or let a student do so.

T: Why is 38 unhappy?

Page 12-13

Read page 12 aloud or let a student do so.

T: This + 2 dance starts at 0. What do you notice about these numbers?

S: They are even numbers.

T: What kind of numbers are in a + 2 dance which starts at 1?

S: Odd numbers.

T: Turn the page and you'll see the + 2 dance starting at 1.

Pages 14-15

Read these pages aloud or let a student do so.

T: Name some of the numbers in the yard who are dancing in this + 2 dance.

Let several students respond. Any odd number larger than 203 is dancing in the yard.

T: Name some of the numbers in the yard who are dancing in the + 2 dance shown on pages 12 and 13.

Students may respond that any even number larger than 240 is dancing in the yard

Pages 16-17

Read page 16 aloud.

Write this expression on the board.

$$3 + 5 \times 2$$

L 17

T: What number is this? How do you know?

S: 16, because 3 + 5 = 8 and 8 x 2 = 16.

S: 13, because 5 x 2 = 10 and 3 + 10 = 13.

Read page 17.

T: Like you, the numbers 3 and 2 cannot decide whether this (point to "3 + 5 x 2") is a name for 13 or for 16.

Pages 18-19

As you read page 18, write these expressions on the board. (Be sure to emphasize the spacing.)

$$3 + 5 \quad \times 2$$

$$3 + \quad 5 \times 2$$

T (reading page 19): I smiled and drew two parentheses. I put them like this.

Draw parentheses around "3 + 5" in the first expression.

$$(3 + 5) \times 2$$

$$3 + \quad 5 \times 2$$

T: How does that help?

S: It shows that we calculate "3 + 5" first.

T: 3 + 5 = . . . ? (8) And 8 x 2 = . . . ? (16)

Record "= 16" to the right of the first expression.

T (reading): "Now we name 16," agreed 3 and 2. I moved the parentheses.

Draw parentheses around "5 x 2" in the second expression.

$$(3+5)\times 2=16$$

$$3+(5\times 2)$$

T: How does that help?

S: It shows that we calculate "5 x 2" first.

T: 5 x 2 = . . . ? (10) 3 + 10 = . . . ? (13)

Record " = 13" to the right of the second expression.

$$(3+5)\times 2=16$$

$$3+(5\times 2)=13$$

Finish reading the page.

Pages 20-21

To the right of the two number stories on the board, write this expression.

$$3+5\times 2$$

T (reading page 20): "And what happens if we switch + and × ?" asked 5.

L 17

Switch the symbols on the board.

$$3 \times 5 + 2$$

T: What number could this be?

S: 17.

T: Can you put in parentheses so we can see how you are looking at this expression?

Let the student put in parentheses, check the solution with the class, and then record "= 17" next to the expression.

$$(3 \times 5) + 2 = 17$$

Write the expression again and let a student who sees it as "21" put parentheses in the expression.

$$(3 \times 5) + 2 = 17$$

$$3 \times (5 + 2) = 21$$

Finish reading pages 20 and 21.

Vary the activity in the storybook by asking what happens if:

a) we switch "3" and "2" (in the original expression);

b) we switch "3" and "2", and "x" and "+" (in the original expression).

Your board should look similar to this.

$(3+5)\times 2=16$ $(3\times 5)+2=17$ $(2+5)\times 3=21$ $(2\times 5)+3=13$

$3+(5\times 2)=13$ $3\times(5+2)=21$ $2+(5\times 3)=17$ $2\times(5+3)=16$

Your students should notice the repetition of answers.

Pages 22-25

Read these pages aloud. Reproduce the picture on pages 24 and 25 on the board. Let students locate $\frac{1}{2}$ and then draw the red arrow from 1 to $\frac{1}{2}$.

Pages 26-27

Read these pages aloud.

T: What do you suggest we call this new dance?

S: $\frac{1}{2}$ x.

S: 2 x.

Discuss why $\frac{1}{2}$ x is correct.

Pages 28-31

Read the final pages aloud or let the students read them silently.

APPENDIX: The String Game

The String Game is used in many versions throughout the CSMP Elementary School Program. It gives students an opportunity, in a game-like atmosphere, to become familiar with the language of strings, while at the same time it involves them in the kind of reasoning that will be developed and reinforced in various contexts throughout the program.

The String Game is first played with A-Blocks (shapes), but later on in the Intermediate Grades variations which make use of numbers appear. In this appendix we provide you with the necessary information and examples to enable you to play the game using A-Blocks, and we suggest some of the various possibilities open to you when you play the game with shapes. Although only a few lessons in UPG Part III are scheduled for the String Game, we hope you will not feel restricted to playing the game only at these times. It has been found that it is most beneficial and enjoyable for students if you make a regular practice of playing the game about once every two weeks, or whenever you have an extra 10-15 minutes during the course of the day.

Equipment

PLAYING BOARDS

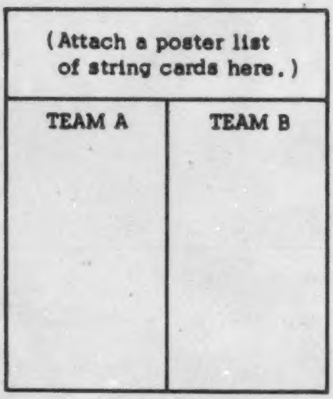
The equipment for this game may be most easily managed if you have a magnetic* chalkboard available. Many of the permanently mounted chalkboards in classrooms are magnetic, and you should test yours using one of the demonstration Minicomputer checkers. If your permanent chalkboards are not magnetic, try any portable chalkboard (grid board, etc.) that the school has available. If you do not have a magnetic chalkboard available, you can use your regular chalkboard.

---

* Strictly speaking the chalkboard itself is not magnetic, but rather magnet-sensitive; i.e., magnets adhere to it. However, for simplicity we call such boards "magnetic".

TEAM BOARD

The Team Board is divided into three regions as illustrated below.



a) Magnetic: If you do locate a magnetic chalkboard, you will also need a magnetic or metallic Team Board. With a large magnetic chalkboard, say at least three chalkboard squares, the Team Board could be drawn on the chalkboard. However, if you have a relatively small (portable) magnetic chalkboard, then you will need to obtain a sheet of metal (minimum size: 60 cm by 80 cm) or locate a convenient metallic surface in the classroom such as the side of a file cabinet on which to put the Team Board. In this case, draw the markings for the three regions of the Team Board on a large sheet of (easel pad) paper and tape this paper to your metallic surface.

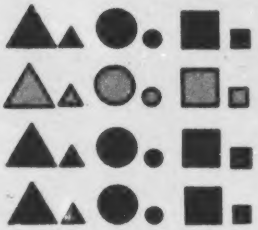
b) Non-magnetic: If you do not have a magnetic chalkboard available for the playing board, then your Team Board could be a large piece of poster board (minimum size: 60 cm by 80 cm). Make the markings for the three regions of the Team Board with a heavy felt-tip pen.

GAME PIECES and STRING CARDS

One set of game pieces and string cards are needed for each version of the game. A list of the string cards (A-Blocks String Game Poster) is to be attached to the upper section of the Team Board - it is a constant reminder during the game of the possible labels for the strings.

String Game
with
A-Blocks

Game Pieces



String Cards

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

Game pieces, string cards, and the poster list of the string cards can be found in the A-Blocks String Game Kit.

a) Magnetic: You can magnetize the game pieces (A-Blocks) by sticking a small piece of magnetic material to the back of each one. (Magnetic material is included in the A-Blocks String Game Kit, or it is available in many hobby and notions departments of stores.) Similarly, you can magnetize string cards by sticking a small piece of magnetic material to the front of each card, taking care not to obscure what is written thereon.

b) Non-magnetic: Game pieces can be attached to the Team Board by means of loops of masking tape stuck to the backs. A string card should have a loop of masking tape stuck to the front so that what is written on the card is not obscured. With this type of equipment, you should prepare yourself for running repairs by having sufficient masking tape available so that if a loop of tape loses its stickiness, it can be replaced on the spot. An alternative is to use a small wad of a plastic caulking compound (Rope Caulk or Mortite, for example) in place of the loop of masking tape. This material will maintain its stickiness over a much longer period of time.

Preparation for the Game

Draw two (or three, depending on which variation you are using) large, overlapping strings on the playing board using two (or three) different colors. Next to each of these strings attach one string card face-down. Locate the Team Board conveniently near to the playing board. Share the game pieces between the two sections of the Team Board labeled "TEAM A" and "TEAM B" by randomly placing one-half of the pieces in one section and the remainder in the other section. Divide the class into two teams using whatever method is acceptable to your class; e.g., boys vs. girls; rows 1, 2, 3 vs. rows 4, 5, 6; etc. Call one team "Team A" and the other "Team B". Before any student takes a turn, correctly place an equal number (at least one) of each team's game pieces in the string picture. This removes from the first players the necessity of operating on the basis of pure guesswork. You can influence how long the game will take by the number of pieces you place in the string picture before the game begins.

Object of the Game

Each team tries to place all of its game pieces correctly (according to the face-down string cards) in the string picture. The winning team is the one that first places all of its game pieces correctly and identifies the face-down cards correctly, after playing according to the rules.

Rules of the Game

1) The game is played in silence. This is to avoid one student telling another where to play a certain piece, or what the strings are for. Each student should have the opportunity to analyze the game alone. Infringement of this rule by anyone is penalized by the talker's team losing its next turn.

2) The teams alternate, and the members take turns within each team. A player comes to the board and selects a piece from his or her team's collection to place in one of the regions of the string picture.

3) You are the judge. If the piece is correctly placed, say "yes". The piece remains in the string picture and the player immediately has a second turn (no player may have more than two consecutive turns). If the piece is incorrectly placed, say "no". The player returns the piece to the team's unplayed collection and play passes to the other team.

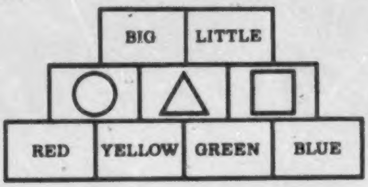
As an aid in judging, prepare a "crib-sheet" showing the correct position of each game piece or at least reminding you of what is on the face-down cards. If at any time you discover that you have made an error, say so immediately and rectify the mistake. Either move an incorrectly placed piece whose position you had approved to its correct region or replace in the string picture a correctly placed piece that had been rejected because you had disapproved its position.

4) When a team has correctly placed all of its pieces, the player who placed the last piece may thereupon attempt to identify each of the string cards. If these are all correct, the team has won. If a mistake is made (even if it is only in the case of one of the string cards), simply indicate that the identification is incorrect and let the game continue.

5) If a team has exhausted its stock of game pieces and the strings have not been identified, that team continues on its turn to attempt to identify the strings, while the other team works to place its game pieces.

Version A: The String Game with A-Blocks

The simplest version of the game uses twenty-four A-Blocks as game pieces and only nine string cards.

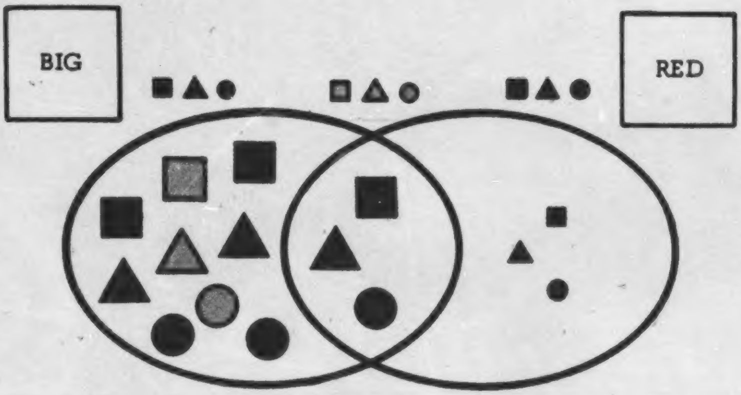


This list of string cards should be posted on the board or attached to the game board.

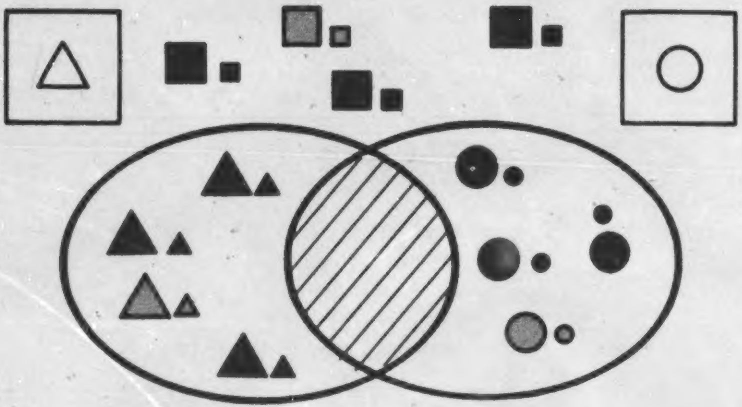
We provide here several example "crib-sheets" for variations of the game with two and with three strings.

Variation #1 : Two Strings

Example 1 : No Empty Regions

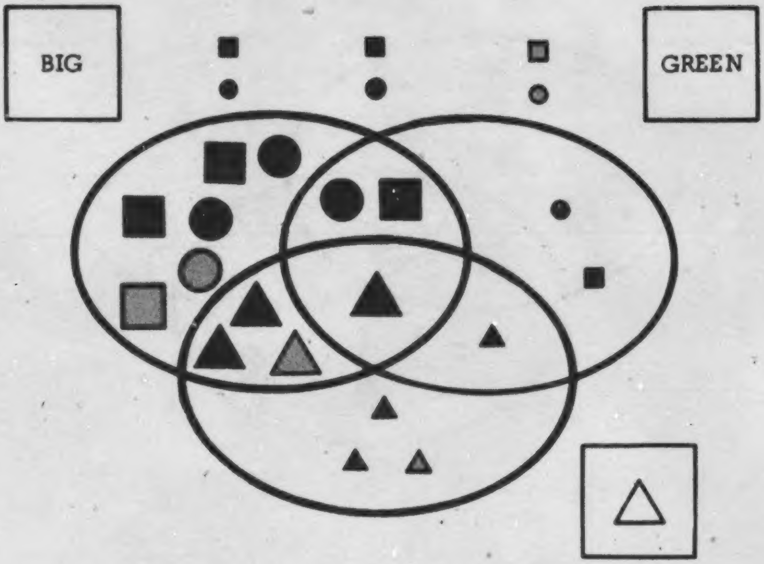


Example 2 : One Empty Region

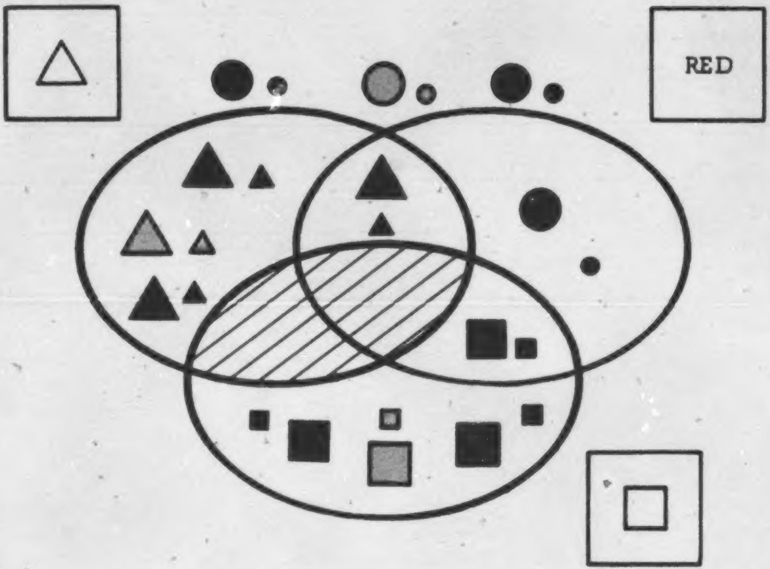


Variation #2: Three Strings

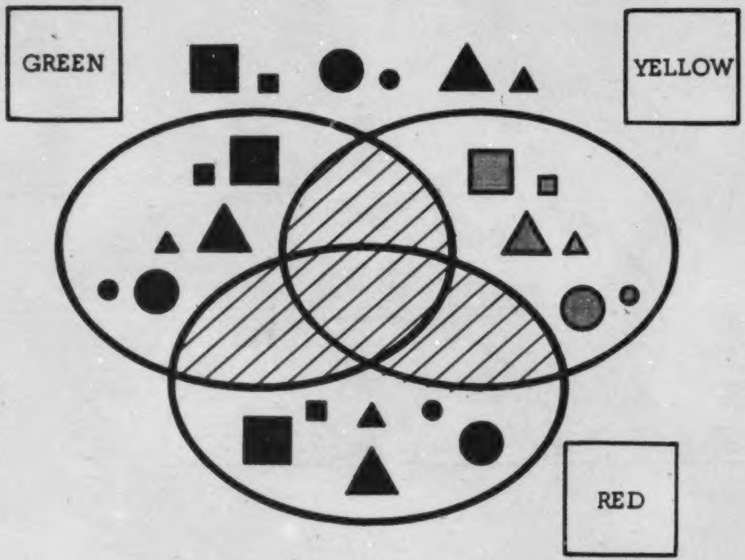
Example 1: No Empty Regions



Example 2: Two Empty Regions



Example 3: Four Empty Regions



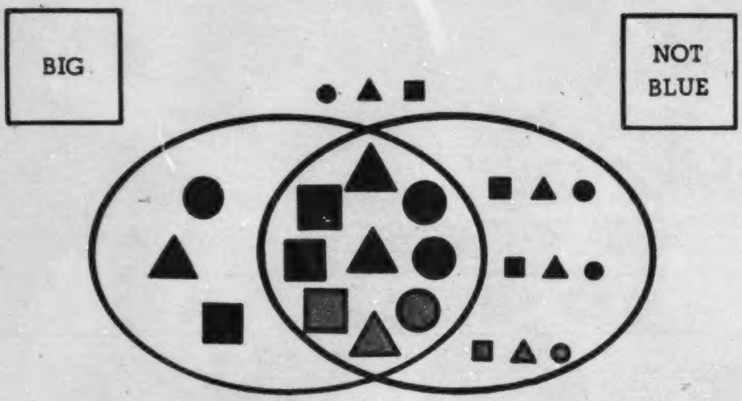
Version B: The String Game with A-Blocks, using "Not" Cards

A more complicated version of the game with A-Blocks uses all sixteen of the string cards. A list of all sixteen string cards should be posted or attached to the Team Board.

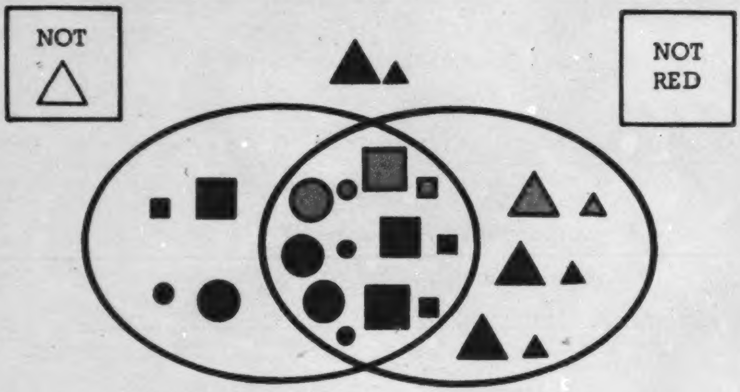
NOTE: Since "NOT BIG" and "LITTLE" describe exactly the same A-Blocks (and similarly for "NOT LITTLE" and "BIG"), such a "not" card is redundant. If a student questions why we do not include these possibilities, let the class discuss why they are unnecessary.

We provide here several example "crib-sheets" for this version of the String Game played with two strings.

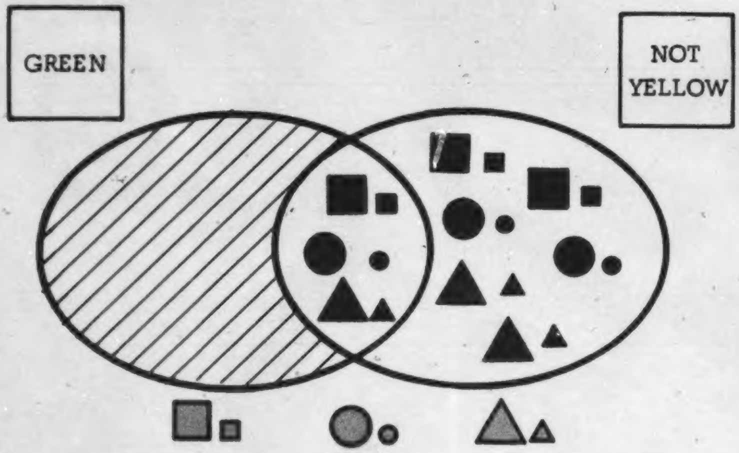
Example 1: No Empty Regions


BIG
NOT BLUE

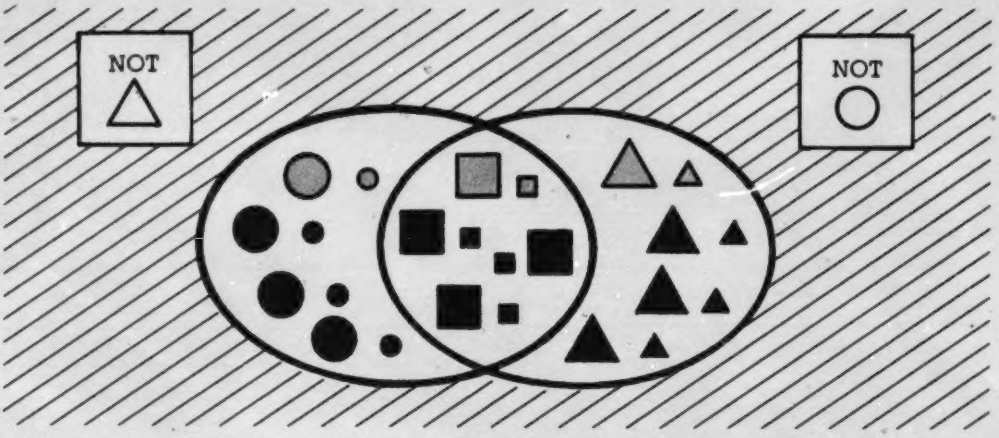
Example 2: No Empty Regions



Example 3: One Empty Region


GREEN
NOT YELLOW

Example 4: One Empty Region



Of course, by hatching the "outside" region of the diagram we mean to indicate that no game pieces can be placed there correctly. Strictly speaking, that region is not empty because, for example, the number 50 is in the outside region.

You should be warned that the "NOT" version of the game played with three strings is almost impossible to judge without a "crib-sheet" and is correspondingly difficult to play. Hence you would be well-advised to avoid it until such time as you think the two-string version is no longer challenging enough for the majority of your students.

Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades
Part III

Geometry and Measurement

CEMREL, Inc.

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GEOMETRY AND MEASUREMENT

INTRODUCTION

Geometry is useful and interesting, an integral part of mathematics; geometric thinking permeates mathematics and relates to most of the applications of mathematics. Geometric concepts provide a vehicle for exploring the rich connections between arithmetic concepts (such as numbers and calculations) and physical concepts (such as length, area, symmetry, and shape). Even as children experience arithmetic notions in their daily lives, they encounter geometric notions. Common opinion recognizes that each child should begin developing a number sense early. There is no sound argument to postpone the development of geometric sense. These are compelling reasons why the systematic development of geometric thinking should be a substantial component of an elementary school curriculum.

A benefit of early development of geometric thinking is that it provides alternative opportunities for "slow learners". Children develop number concepts at individual rates. Relative lateness of a child's numerical development often is signified by the child's trouble remembering number facts. A common response to this problem is assignment of arithmetic drill, which serves only to confirm the diagnosis of slowness. However, numerical development may not be an accurate indication of other development, in particular, of geometric development. Often the labels "slow" and "fast" are incorrect when alternative activities are presented.

We are careful to allow students a great deal of time to explore quantitative relationships among physical objects before studying arithmetic. In the same spirit, students should be allowed to explore geometrical relationships through free play and guided experiences. Before studying geometrical properties of lines and shapes, they should have free play with models and with drawing instruments in order to create new figures and patterns. Before considering parallelism and perpendicularity as relations on geometric lines, they should have free play in creating patterns with parallel and perpendicular lines.

Before learning length and area formulas, they should have informal experiences to establish and sharpen intuitive notions of area and length. These early experiences are aesthetically rewarding and just as stimulating and suggestive of underlying concepts as early experiences leading to arithmetic. Such experiences are a crucial first step in the development of a geometric sense.

As with all educational development occurring through informal experiences, the payoffs come unexpectedly and spasmodically. In the development of geometric thinking, the suggested activities vary widely, reach into so many other facets of the program, and continue for so long a span that it would be nonsense to set a priori milestones of accomplishments.

Thus, we must not rush the students through the free play activities that precede the more substantive lessons. These preliminary activities have implicit significance for geometric thinking and also provide aesthetic opportunities and explicit practice. We must restrain our inclination to view these activities as mere play. On the other hand, some activities appear to have important consequences or clearly to relate geometry to other parts of the program. Here the inclination might be to spend more time on these activities than is suggested. Again this should be resisted, remembering that similar activities will provide further experience later in slightly different contexts.

The key word is "experience" not "mastery".

But what sorts of experiences will lead students to think geometrically? The standard curriculum usually consists of measurement activities, recognition of some special geometric figures, and association of these figures with their technical names; in short, a good deal of vocabulary but little opportunity for geometric thinking.

A glance at the lessons of this strand indicates that the CSMP Upper Primary Grades curriculum includes measurement activities (leading to the use of metric units) as well as topics not usually in traditional programs at this level, for example, networks and taxi-geometry.

Measurement activities eventually lead to notions of distance. Traditionally one waits for a good deal of maturity before studying properties of figures, such as circles, that are defined in terms of distance. The reason is that Euclidean distance (distance "as the crow flies") requires sophisticated notions of "exact" measurement in terms of the real number system in order to be able to measure every line segment. But in the CSMP curriculum very young children (even kindergarteners) begin thinking about distance in the simplified setting of "taxi-geometry". Taxi-distance (as a taxicab travels) between points is measured in terms of whole numbers by an easy counting process. Experiences in taxi geometry lead to some creative situations, yet do not require special technical skills or vocabulary.

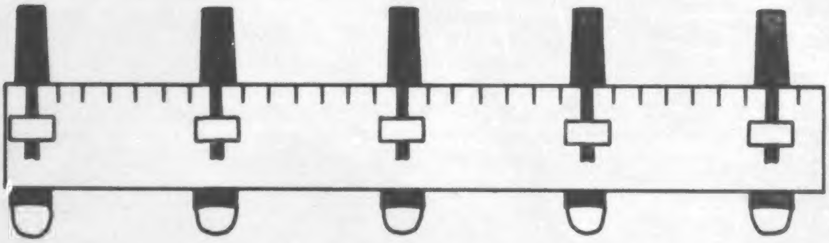
Putting the emphasis on experience rather than on mastery frees us to engage in imaginative activities with children that we ordinarily would not attempt. For example, even when considering such commonplace topics as area and length, we should feel free to devise activities that help students recognize patterns, develop cognitive strategies, and relate the topics to others in the program, while suppressing the urgency to get the "basic" formulas and to master "basic" skills. What is basic here, at this level, is that children are thinking rather than doing tasks on command. The skills and mastery will come eventually, in their own time and place in the curriculum.

NOTE ON GRIDS

A demonstration grid board is needed for the taxi-geometry lessons and for some of the area lessons. This should be a large square grid board at least 12 squares by 12 squares (and grid lines spaced 6 to 8 centimeters apart) on which you can draw with colored chalk or with colored pens. A portable, commercially made, permanent grid chalkboard is ideal. However, if such a grid board is not available to you, try one of these alternatives.

- Printed grids on large sheets of paper are provided with full classroom sets of materials for CSMP Mathematics for the Upper Primary Grades, Part III. Tape one of these sheets to the board, use thick felt-tip pens for drawings, and discard the sheet after use; or mount one of the sheets on cardboard, cover it with clear contact paper to provide a protective surface, and use erasable wax crayons for drawings.

- For each lesson, carefully draw a grid on the chalkboard. One can make a tool for drawing such grids efficiently. Use a music staff drawer with chalk at the first, third, and fifth positions; or proceed as follows: clip five pen caps to a ruler at centimeter intervals and secure them with tape. Pen caps are usually of a diameter to hold a piece of chalk securely.



G1 Area #1

CAPSULE LESSON SUMMARY

Find several ways to color a shape one-half blue and one-half red.

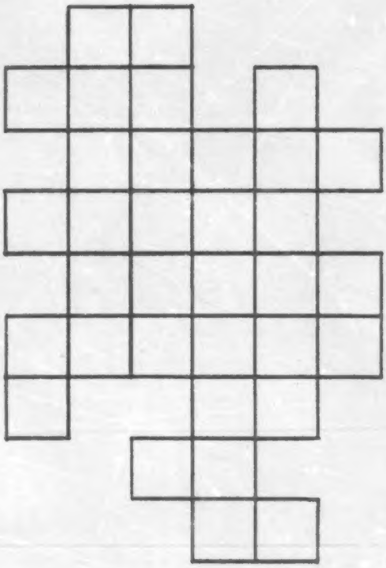
MATERIALS

Teacher: Geometry Posters #1, #2, and #3 for UPG-III; masking tape; a red and a blue marker

Student: Worksheets G1* and **; colored pencils

DESCRIPTION OF LESSON

Tape a copy of shape A from Geometry Poster #1 to the board.



T: I put a shape on the board. I would like to color one-half of this shape red and one-half blue. How can I do it?

S: Put red on the top part and blue on the bottom.

T: Remember I want exactly one-half of the shape for each color.

S: Color one square blue, one red; and one blue, one red; and so on.

S: Color two squares blue, two red; and two blue, two red; and so on.

With the last two methods, you have to be careful that if you start with blue, you end with red (and vice versa). To avoid confusion, choose a certain number of blue with the same number of red simultaneously, e.g. treat one blue-one red as a single choice. Also, when several squares are chosen at a time for one color, half of the total number of squares must be a multiple of that number for the method to continue working; otherwise an adjustment needs to be made at the end. However, such a method is a good one; accept it without comment.

This is another pairing method. The student comes to the board.

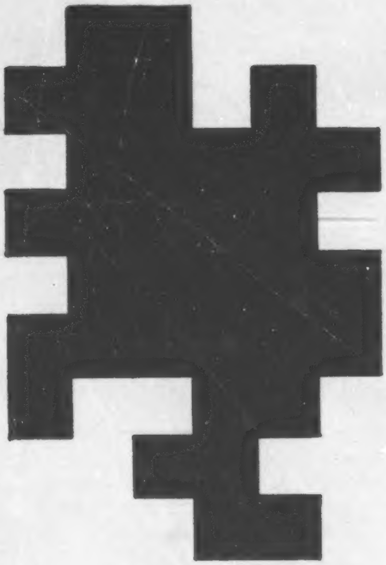
S: Color these three red and those three blue; this one red and that one blue; these five red and those five blue; and so on.

Distribute copies of Worksheet G1* and colored pencils (or crayons). Shape A is drawn on both sides of the worksheet for students needing another chance at the drawing.

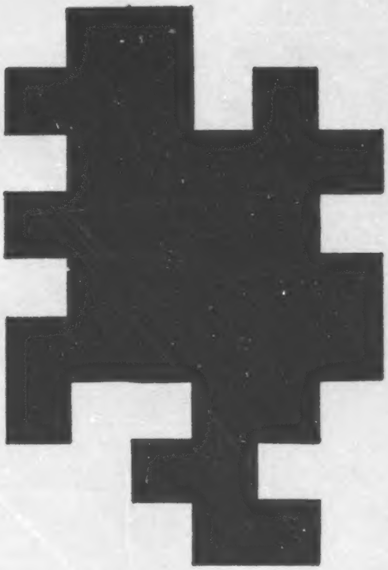
T: This worksheet has the same shape as on the board. Color one-half of it red and one-half of it blue using whichever method you prefer.

Walk around the room and observe students' work. Talk to students individually about which methods they are using. Worksheet G1** could be given at this time to faster students if there is a wide range in the amount of time your students need to color the shape on Worksheet G1*. Otherwise, Worksheet G1** can be used at the end of the lesson.

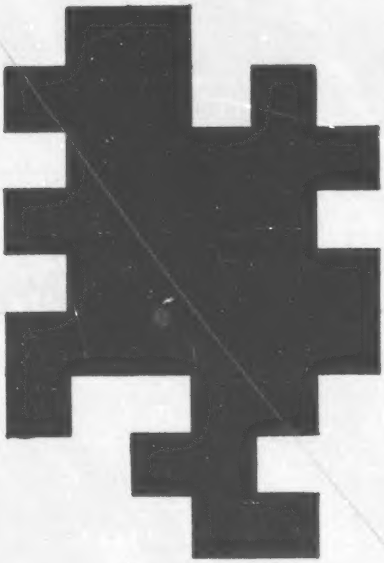
These are some possibilities.



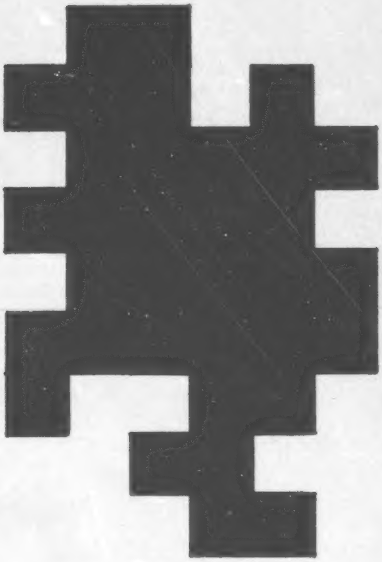
17 blue - 17 red



One-half of each square is red; the other half is blue.



One blue, one red; one blue, one red; and so on.



Three red, three blue; one red, one blue; five red, five blue; and so on.

Tape one or two* more copies of shape A onto the board. Choose two (or three) students who have different methods and who have finished quickly to shade the pictures on the board as on their worksheets. The rest of the class can continue coloring their pictures.

After most of the class have completed a picture or have indicated an approach, draw the class' attention to the colorings on the board.

Illustrate one method of deciding how to color the shape as in the dialogue which follows.

T: Some of you found that there are thirty-four squares in this shape.

Indicate this on the board.

$$34\ \square$$

T: Does anyone know what number is $\frac{1}{2} \times 34$ (read "one-half of thirty-four")? (17)

If no one knows the answer, ask the students what number is $\frac{1}{2} \times 30$? . . . $\frac{1}{2} \times 4$? . . . $\frac{1}{2} \times 34$?

Write the number story on the board.

$$\frac{1}{2} \times 34 = 17$$

T: How many red squares should there be in your picture?

---

* An extra copy of shape A can be found on Geometry Poster #3.

S: Seventeen.

T: How many blue . . . ?

S: Seventeen.

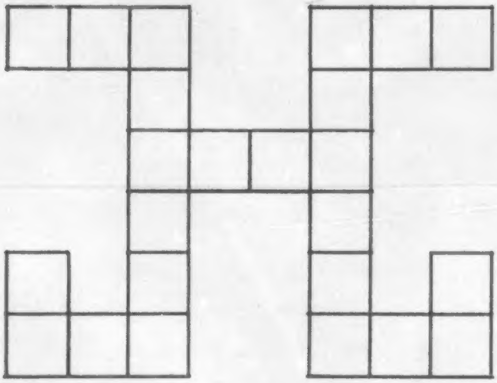
Verify with your class that there are seventeen red squares in one of the colorings on the board.

T: Do we need to count the blue squares?

S: No, there must be seventeen of them if there are thirty-four squares altogether.

Exercise 2

Tape a copy of shape B from Geometry Poster #2 on the board.

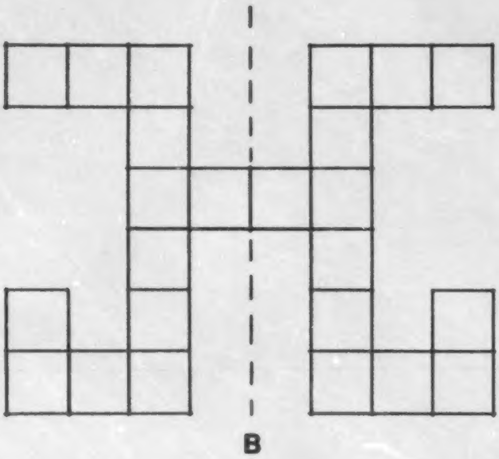


B

T: How can we color one-half of this shape blue and one-half red?

G1

Let a student come to the board to indicate a method. Very likely someone will use the symmetry of the shape.



Do not draw the dotted line. It is here just to make the lesson easier to follow.

A student having traced the dotted, separation line explains:

S: Color this side red (points to the left) and this side blue (points to the right).

T: Why does this method work?

S: There are the same number of squares on each side.

T: How many squares . . . ?

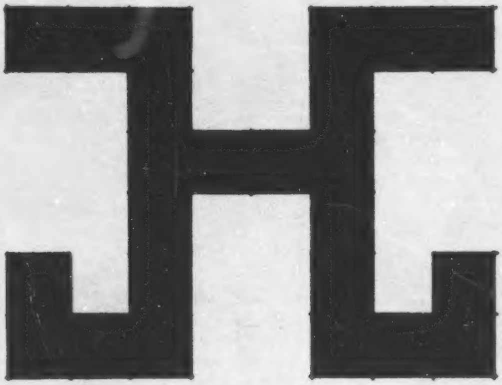
S: Twelve on each side.

T: Can anyone explain this method in a different way?

S: It has the same shape on both sides.

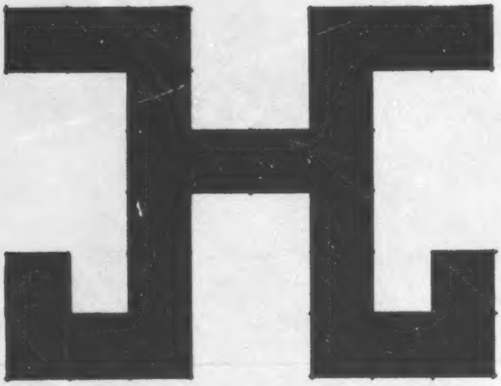
Also, a student might want to come to the board and point to the squares pairing off one red with one blue (or two red with two blue) until every square has been considered.

Quickly color the shape in this manner.

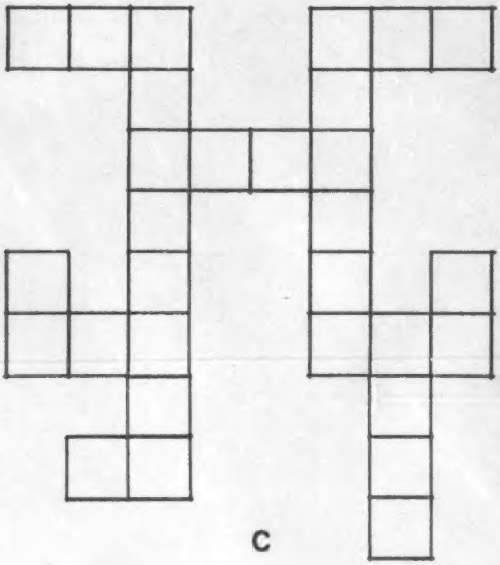


B

Leave shape B on the board and tape a copy of shape C from Geometry Poster #2 next to it.



B

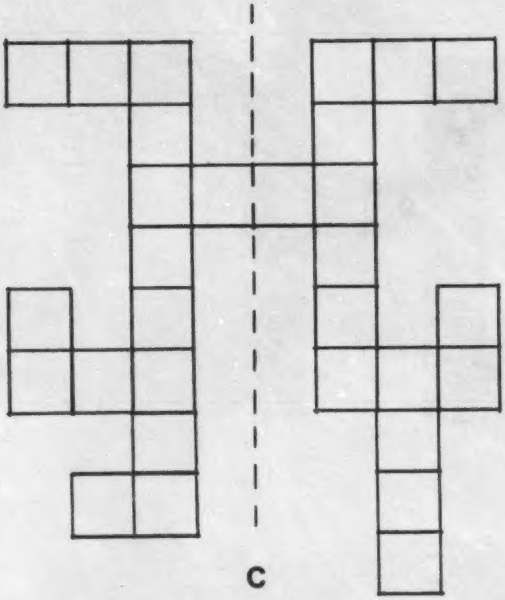


C

T: Once again, how can we color one-half of the shape blue and one-half red?

G1

Let someone indicate a method at the board.



Do not draw the dotted line. It is here just to make the lesson easier to follow.

A student having traced the dotted, separation line explains:

S: Color this side red (points to the left) and this side blue (points to the right).

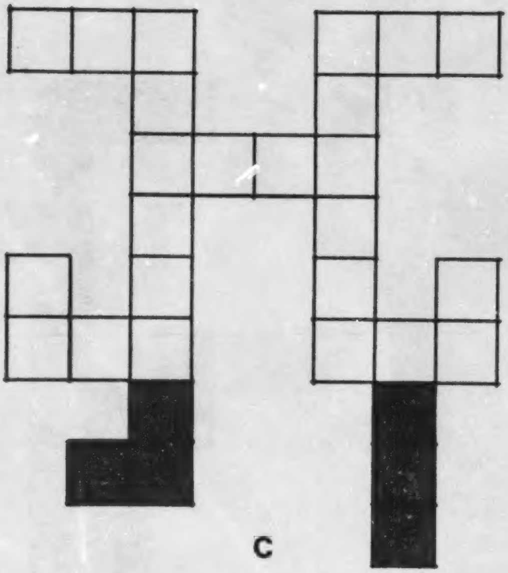
T: How do you know that we would have exactly one-half red and one-half blue?

S: The sides are the same.

T: . . . exactly the same?

S: No. Part of this picture is the same as our last shape, but there are three extra squares on each side.

Invite someone to point to these particular squares. They are shaded in the diagram below.



T: How many red squares were in the last shape (B)?

S: Twelve.

T: How many more red squares should there be in this shape (C)?

S: Three.

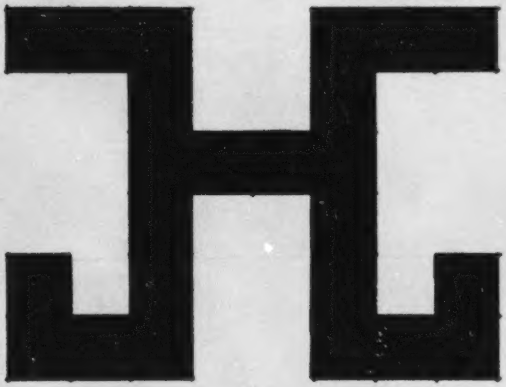
T: So how many red squares should there be in this shape (C)?

S: Fifteen, 12 + 3 = 15.

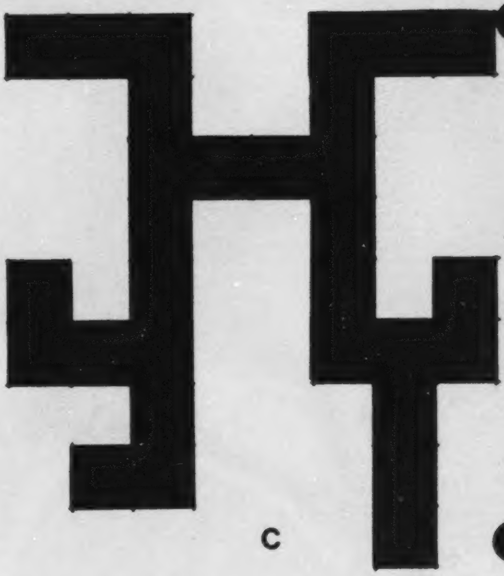
T: How many blue . . . ?

S: Fifteen also.

Color the shape as in the diagram on the right.



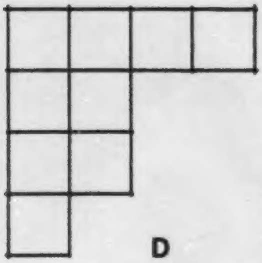
B



C

Exercise 3

Take shapes B and C off the board and tape a copy of shape D from Geometry Poster #3 to the board.



D

T: Here is another shape. I would like to color one-half blue and one-half red.

Let students come to the board and point to squares that they would color red and to squares that they would color blue. Students might suggest these colorings or similar ones.

If so, count the squares with the students to show that these colorings are not correct. Ask for another method.

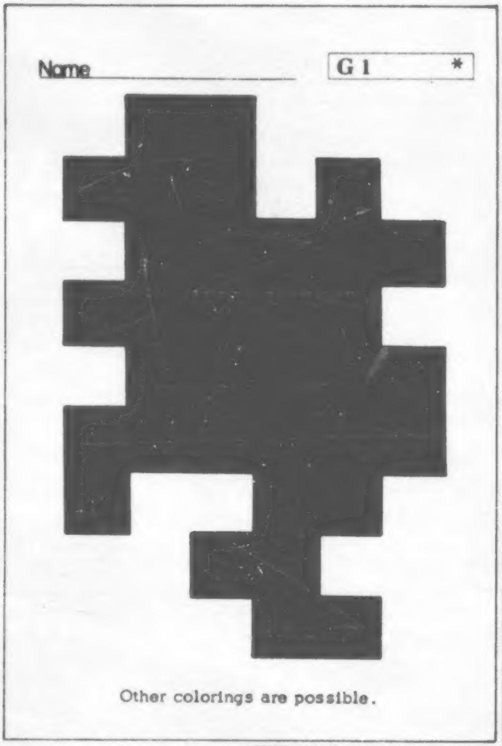
S: Color four red and four blue and leave the other one uncolored.

T: But I want to color the whole shape.

S: Cut the extra one into two halves.

Let a student color the shape in this way. If someone else has another method, tape the other copy of shape D from Geometry Poster #3 onto the board and let that student color it. Two possibilities are given below.

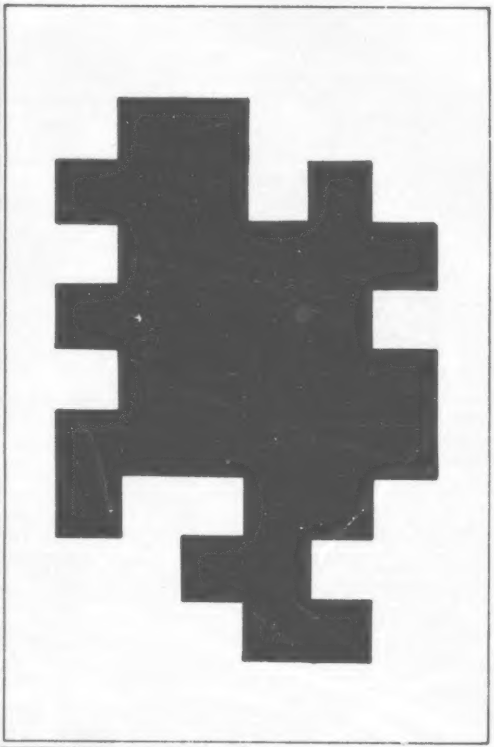
If time remains, let the students complete Worksheet G1**.

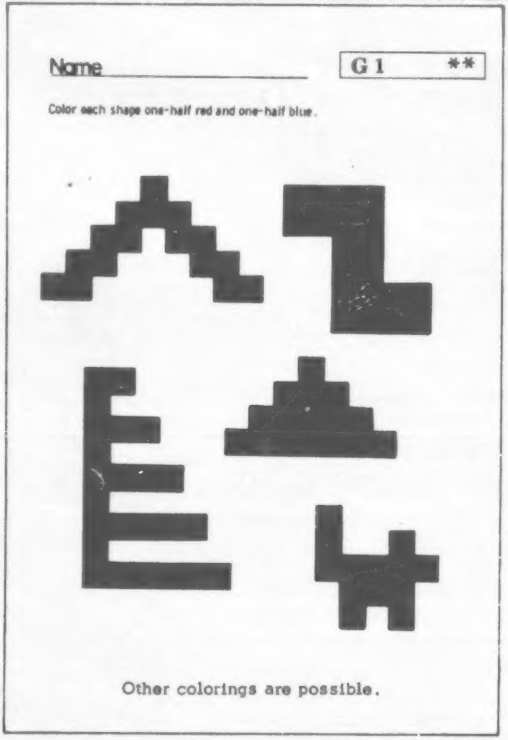


Name

G 1 *

Other colorings are possible.





Name

G 1 **

Color each shape one-half red and one-half blue.

Other colorings are possible.

G2 Taxi-Distance

CAPSULE LESSON SUMMARY

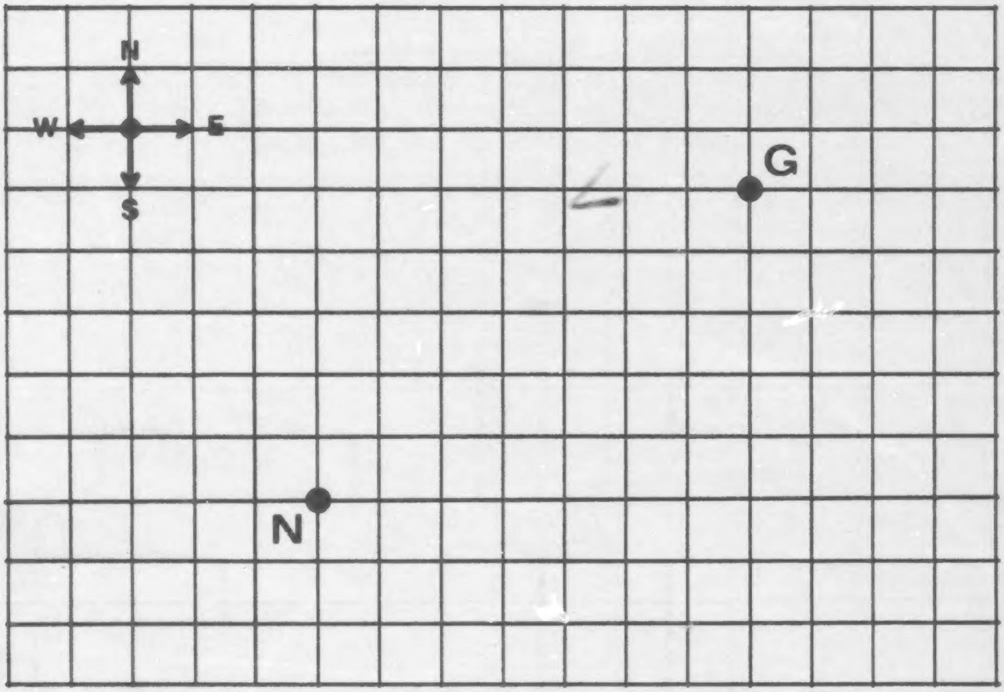
Introduce paths which follow the lines on a grid and describe some paths in terms of direction: North, South, West, and East. Observe the lengths of paths and look for shortest paths between two points. Introduce the notion of taxi-distance.

MATERIALS

Teacher: Grid Board*; colored chalk
Student: Worksheets G2* and **; colored pencils

DESCRIPTION OF LESSON

Display a grid board. Mark and label two points "N" and "G" respectively as in the diagram below. Indicate directions at a grid point to the far side of one of the dots.



T: This is a map of a city. The lines are for streets. Starting at any corner, there are four directions to go: North, East, South, and West. (Point to where directions are indicated.)

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* See the NOTE ON GRIDS in the introduction to this strand.

G2

Let students show how to move each of these four directions starting from N.

T (pointing to the dot labeled "N"): Does anyone remember who lives here?

S: Nora.

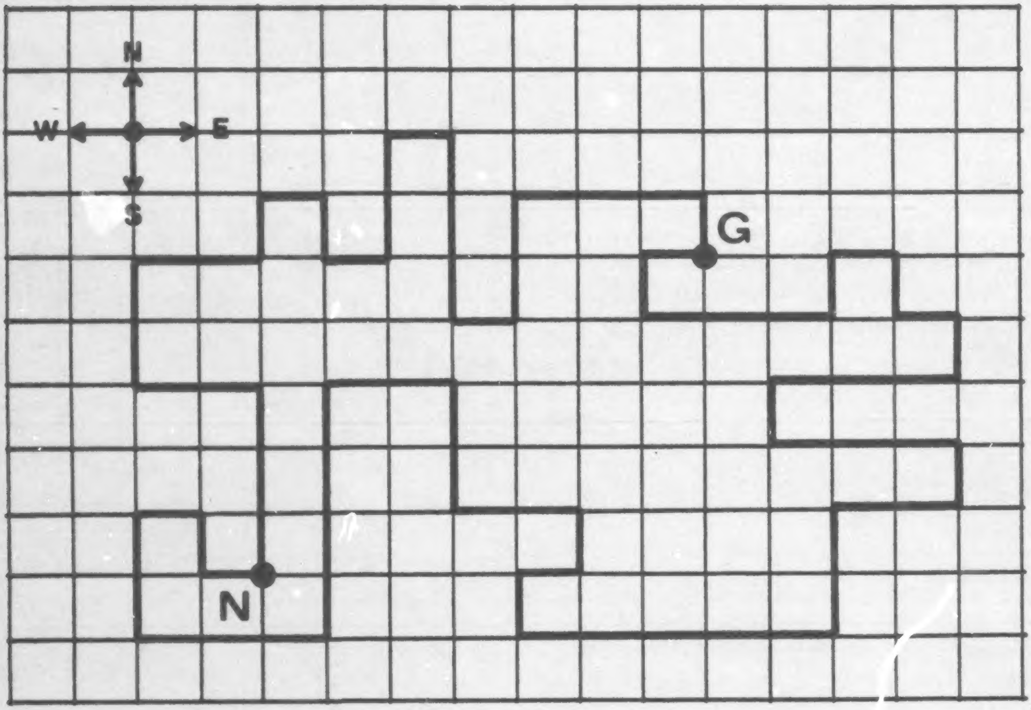
T (pointing to the dot labeled "G"): Who lives here?

S: Nora's grandmother.

NOTE: If students do not recall this situation, simply give them the information.

T: Nora very often goes to visit her grandmother, but she likes to walk all around the city. Who would like to show a very long path from Nora's house to her grandmother's house? Remember . . . Nora only walks on sidewalks along the streets.

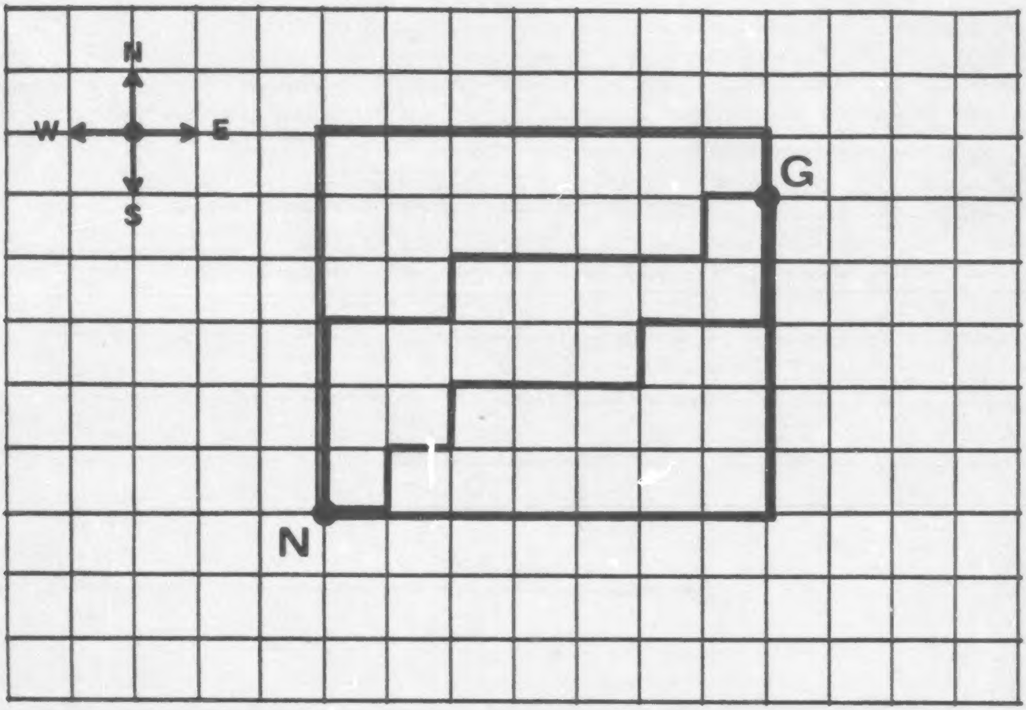
Let some students trace long paths from N to G. Do not draw them. The following illustration gives two examples of long paths.



T: This week has been a rainy week in Nora's city, and Nora's grandmother hasn't been feeling very well. Nora has visited her every day. Since Nora doesn't like to walk a long distance in the rain, she has been trying to find some short paths. Who can show a very short path from Nora's house to her grandmother's house?

Invite four or five students to trace some very short paths. There are many shortest paths, all of which have the same length. Accept any of these or any close to them in length. Draw the paths yourself, using a different color for each path.

Very likely your students will have traced several shortest paths and one or two paths which are slightly longer. We will use the paths in the next illustration for the lesson description. The red, blue, and green paths are shortest paths; the yellow is longer.

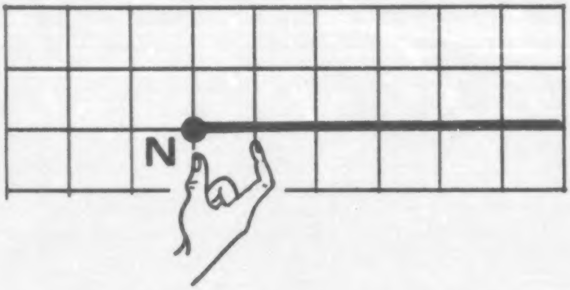


T: Which path is the shortest? How do you know?

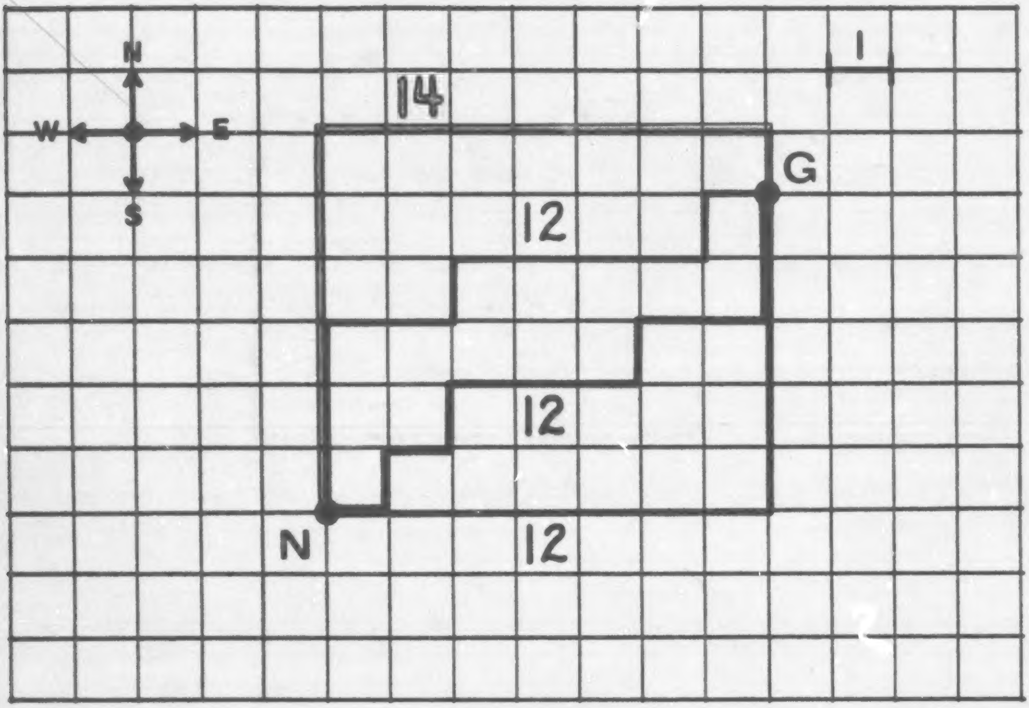
G 2

If some students suggest certain paths have the same length, ask them how they measured length. In any case, if no one suggests counting city blocks, do so yourself and ask some students to do the counting. Record the unit (1 block) on the grid.

Students might not count the blocks correctly; for example, they might count the dots or the squares. It may be helpful to ask your students to show the blocks using their thumbs and index fingers as illustrated below.



Write the length of each path in the same color as the path.



Erase any paths which are not shortest paths; for example, the yellow one in the lesson description.

T: Why do all the paths on the board have the same length?

Most likely some students will have good ideas about this problem, but it might be difficult for them to verbalize these ideas. Encourage them to come to the board; let them make gestures and use their own words to explain.

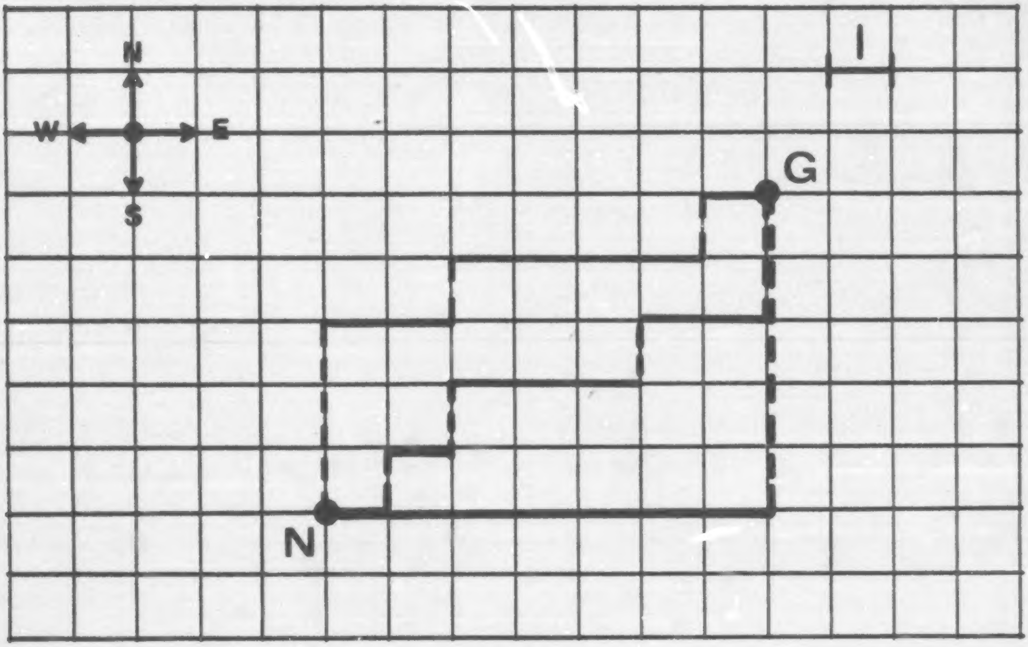
T: In which directions does Nora walk in all of these paths?

S: North and east.

Rephrase directions given as "up" and "that way" (pointing to the right).

T: Let's look at how many blocks Nora walks north in each of these paths.

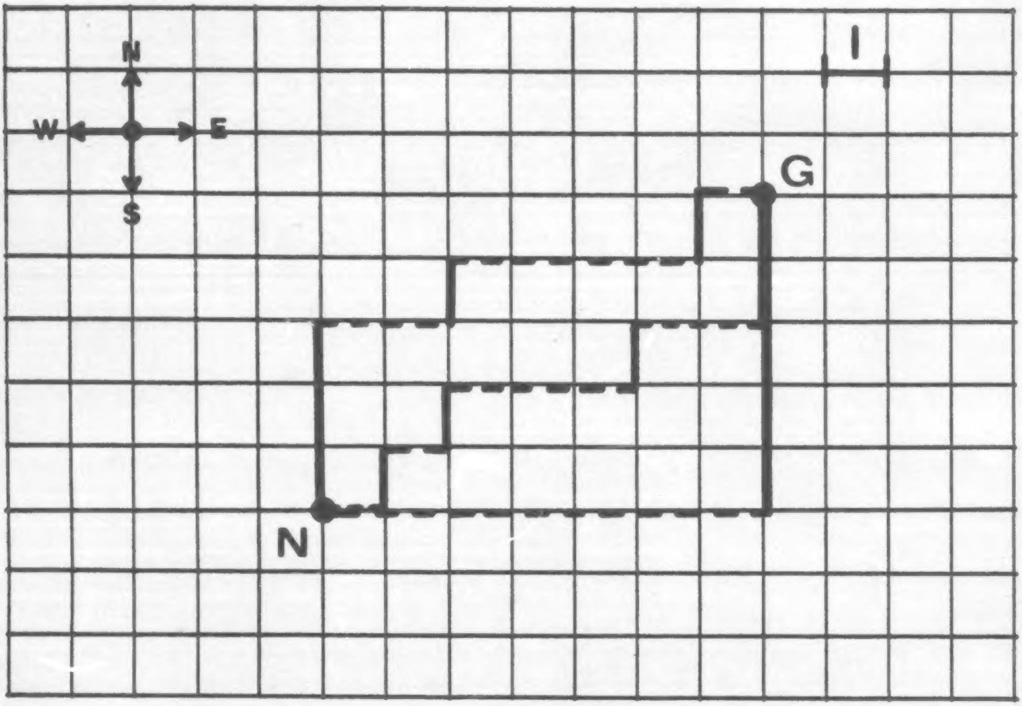
Consider each path on the board. For each one, invite someone to count the blocks at the board. Conclude Nora walks five blocks north in each one. The dotted lines in the next picture show the blocks on which she walks north for the particular paths in the lesson description.



G 2

T: Let's look at how many blocks Nora walks east in each of these paths.

Again let students count the blocks and conclude Nora walks seven blocks east.



T: For each of the shortest paths, Nora walks five blocks north and seven blocks east. 5 + 7 = . . . ?

S: 12.

T: If Nora follows the streets, all the shortest paths from her house to her grandmother's house have exactly the same length: 12 blocks. We say that the taxi-distance from N to G is 12 blocks.

Write this on the board.

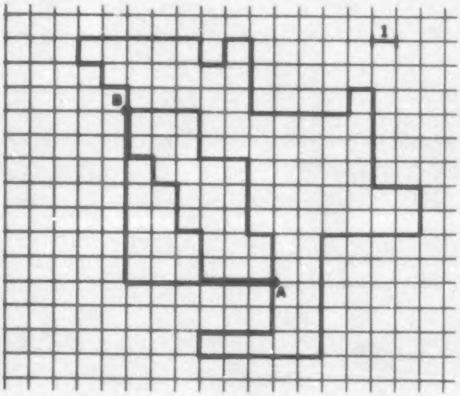
taxi-distance from N to G = 12 blocks

Worksheets G2* and ** are available for individual work.

Name____________________

G 2 *

Draw one long path between Angela's house (A) and Barbara's house (B). Draw several shortest paths between A and B. Use a different color for each path.



The taxi-distance from A to B is 13 blocks.

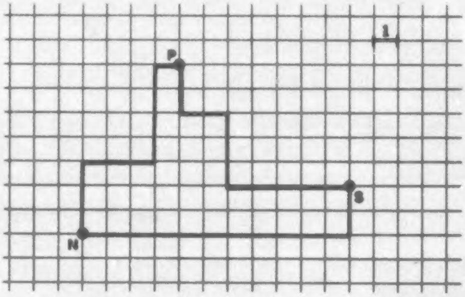
The taxi-distance from B to A is 13 blocks.

Many long paths such as the red one and many shortest paths such as the blue, green, and black ones are possible.

Name____________________

G 2 **

N = Nora's House P = Post Office S = Store



The taxi-distance from N to P is 11 blocks.

The taxi-distance from P to S is 12 blocks.

The taxi-distance from S to N is 13 blocks.

It is raining and Nora must do some errands. She walks from her house (N) to the Post Office (P) and then to the store (S) and then home. What is the length of the shortest trip she can make? 36 blocks

One of the many possible shortest trips is shown in red.

G3 Length #1

CAPSULE LESSON SUMMARY

Find the length in centimeters of zigzags by measuring the component parts and adding their lengths. Find a shorter zigzag with the same end points as a given zigzag both when there is no obstacle to circumvent and when there is.

MATERIALS

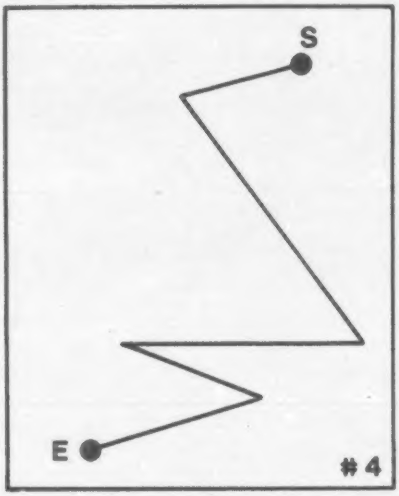
Teacher: Geometry Posters #4 and #5 for UPG-III; centimeter tape measure

Student: Worksheets G3*, **, ***, and ****; centimeter tape measure; straight edge; pencil

DESCRIPTION OF LESSON

Exercise 1

Tape a copy of Geometry Poster #4 to the board and put a centimeter tape measure in an obvious place near the board.



If your class was introduced to the story of a bug walking a zigzag path to find some food last year (Lessons G5 and G9 in UPG-II), recall that situation for the class. Otherwise, begin as here, introducing the bug.

T: Imagine there is a bug here (point to the dot labeled "S") and there is some food here (point to the dot labeled "E"). This bug is very hungry and starts walking, hoping to get to the delicious food he smells. There is an "S" at the dot where he starts and (trace the zigzag path with your finger) an "E" at the dot where he ends his walk. Did he get to the food?

S: Yes.

T: How can we find out how far he walked?

S: Measure his path.

T: Measure it in what unit?

S: Centimeters.

If no one suggests centimeters, suggest it yourself.

Write the unit on the board.

centimeter

T: Do you know the abbreviation for centimeter?

S: The abbreviation is "cm".

Write the abbreviation on the board.

centimeter (cm)

Hold up a tape measure for the students to see.

T: How can we use this tape to measure the bug's path?

S: Measure each piece and then add the numbers.

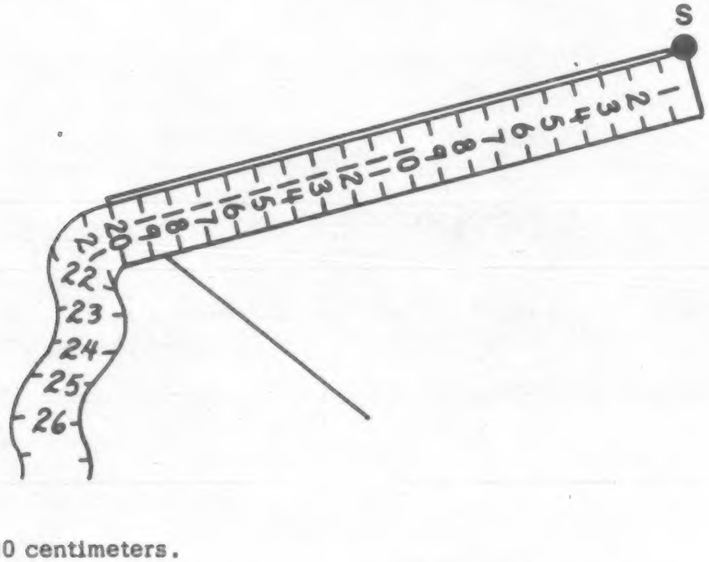
T: That's a good idea. Who can measure the first piece?

Each time a student comes to the board to measure a piece of the path, you will have an opportunity to give individual instruction to that student on how to measure a segment using a tape measure. Call on a different student each time and call on students who most need this extra help.

Suppose a student named "Barbara" measures the first piece.

T: Barbara, make sure to put the end of the tape (the mark for "0") at the end of the segment (or at S).

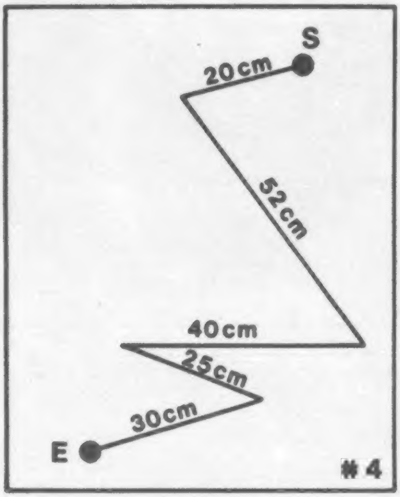
Help the student handle the tape and line it up along the edge of the segment.



S: 20 centimeters.

Ask for the unit if it is not given with the number. Write "20 cm" on the poster near the first piece of the bug's path.

Continue in this manner until all the pieces of the path have been measured.

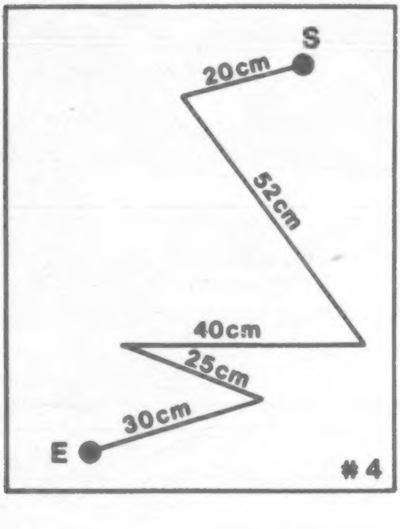

S
20cm
52cm
40cm
25cm
30cm
E
#4

T: We have measured all of the pieces. How can we find how long the entire path is?

S: Add.

T: Tell me what numbers to add and I'll write them on the board.

Write the numbers in column form as a student gives them. Invite a student to do the addition at the board and to explain.

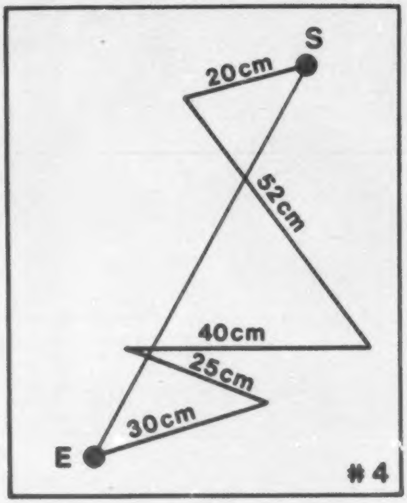

S
20cm
52cm
40cm
25cm
30cm
E
#4

$$\begin{array}{r}20\\52\\40\\25\\+30\\\hline 167\end{array}$$

centimeter (cm)

T: The bug walked 167 centimeters. Do you think there is a very short path that the bug could have walked?

Let students trace paths until someone traces the segment from S to E. Draw that path.



Ask a student to measure the segment from S to E.

S: 66 centimeters.

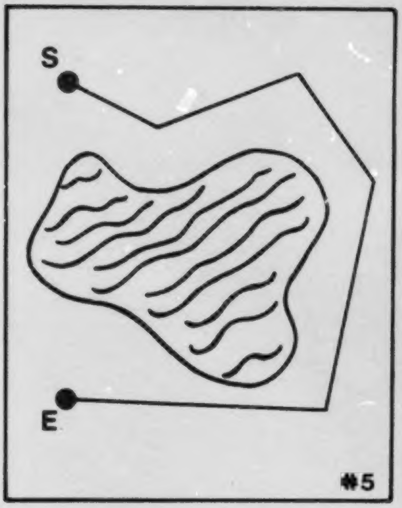
T: How much shorter is this path than the zigzag path that the bug walked?

S: 101 centimeters.

Erase the addition problem on the board and remove the poster.

Exercise 2

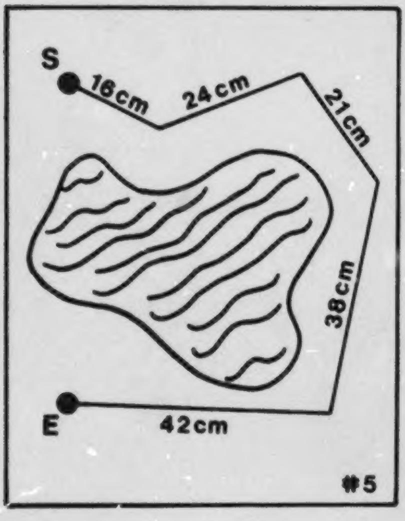
Tape Geometry Poster #5 on the board.



T: This bug is still hungry and manages to find some more food. Only this time there is a big puddle of water in his way. Our bug cannot swim so he walks around the puddle. (Trace the zigzag from S to E with your finger.) How can we find how far the bug walked?

Continue as in the previous exercise letting students measure each piece of the bug's path.

Write the appropriate addition problem on the board and call on one of your more capable students to do the addition aloud at the board.



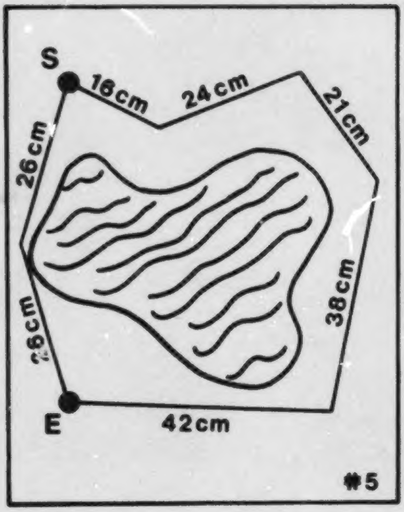
$$\begin{array}{r} 16 \\ 24 \\ 21 \\ 38 \\ +42 \\ \hline 141 \end{array}$$

centimeter (cm)

3

terward, let students trace shorter paths. Draw one of them and let a udent measure it.

r example,


S
16cm
24cm
21cm
26cm
38cm
26cm
E
42cm
#5

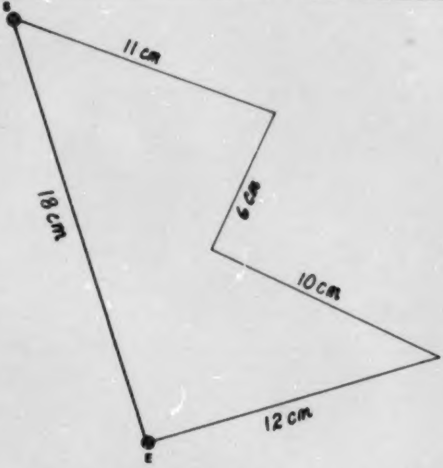
: 26 + 26 = 52. This path is 52 cm and is shorter than the path the bug walked. How much shorter . . . ?

: 89 cm.

orksheets G3*, **, ***, and **** are available for independent work. Students will need tape measures and straightedges.

Name

G 3 *



S
11 cm
6 cm
10 cm
18 cm
12 cm
E

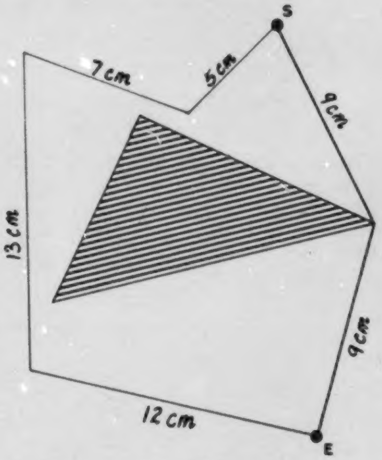
How long is this zigzag from S to E? 39 cm

Draw a shorter path from S to E. Try to make it as short as possible.

How long is your path? 18 cm

Name

G 3 **



S
7 cm
5 cm
9 cm
13 cm
9 cm
12 cm
E

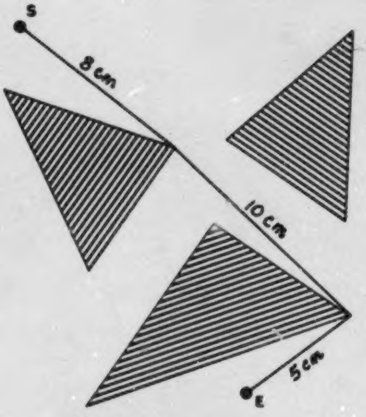
How long is this zigzag from S to E? 37 cm

Draw a shorter path from S to E. Try to make it as short as possible.

How long is your path? 18 cm

Name

G 3 ***



S
8 cm
10 cm
5 cm
E

Draw as short a zigzag path as you can from S to E.

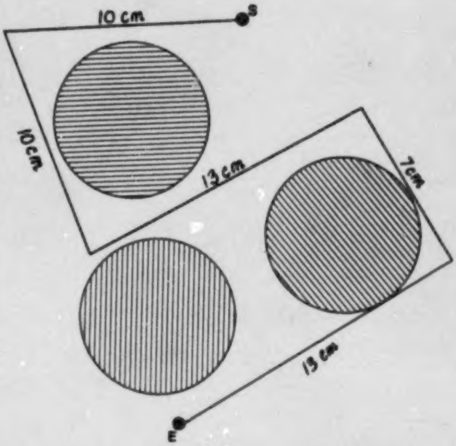
Try to make it shorter than 25 cm.

How long is your zigzag? 23 cm

Other solutions are possible.

Name

G 3 ****



10 cm
S
10 cm
13 cm
7 cm
13 cm
E

Draw a zigzag path from S to E which is longer than 50 cm.

How long is your zigzag? 53 cm

Many solutions are possible.

G 4

G4 Area #2

CAPSULE LESSON SUMMARY

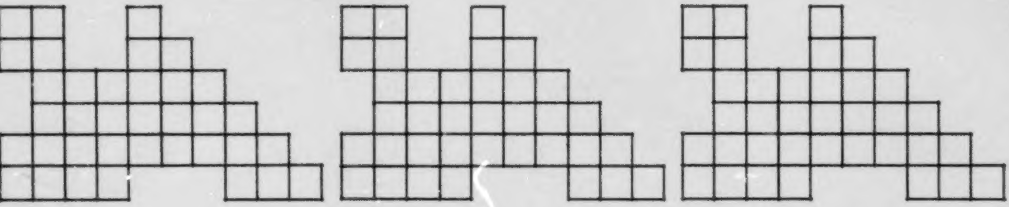
Divide a shape into halves, into thirds, and into fourths. Find the number of little squares in the whole shape and in one-half, in one-third, and in one-fourth of the shape. Divide into thirds another shape which lends itself naturally to such a division.

MATERIALS

Teacher:	Geometry Posters #6 and #7 for UPG-III; tape; colored felt-tip markers
Student:	Worksheets G4* and **; colored pencils

DESCRIPTION OF LESSON

Tape three copies of Shape A from Geometry Posters #6 and #7 to the board.



T: I put three shapes on the board. What do you think of these shapes?

Let the students comment freely.

T (pointing to the left most shape): I would like this shape to be colored one-half red and one-half blue. How do you think we could do this?

S: Count the squares. Then color one-half of that many squares red and the others blue.

T: How many squares are there in each of these shapes?

Invite someone to count at the board the squares in one of the copies of Shape A.

S: Thirty-six.

T: What number is $\frac{1}{2} \times 36$?[*]

S: 18.

T: Let's check.

Near the shape furthest to the left write this addition problem.

$$\begin{array}{r} 18 \\ +18 \\ \hline \end{array}$$

T: What number is 18 + 18?

S: 36.

Erase the addition problem and then write this multiplication number story.

$$\frac{1}{2} \times 36 = 18$$

If students offer the division fact "36 ÷ 2 = 18, write it on the board as well.

---

[*] Read "$\frac{1}{2} \times 36$" as "one-half of thirty-six."

G 4

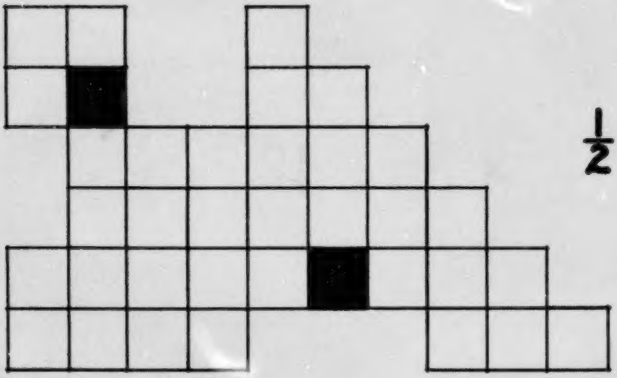
T: We could color eighteen squares red and then color eighteen squares blue. What is another way we could color one-half of this shape red and one-half blue?

S: Color one square blue and one square red; one square blue and one square red; and so on.

Students might suggest other ways of coloring the shape, but the two included in the description are the ones in which we are most interested.

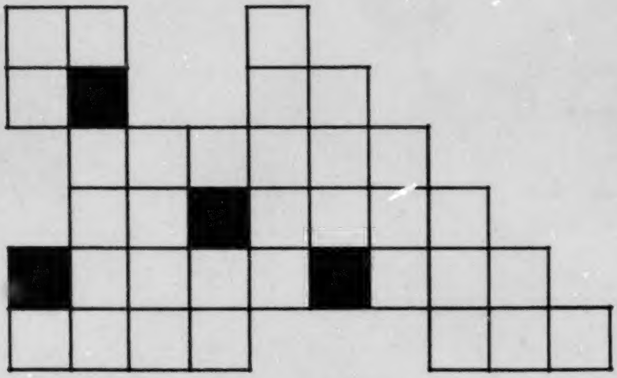
Choose two students to come to the board to color the shape furthest to the left. Give each student a different color marker (or crayon). The rest of the class should be encouraged to count the red and the blue squares as they are being colored by the two students chosen.

T: When I say "Go", I want each of you (look at the two students at the board) to color a square. . . . Go!



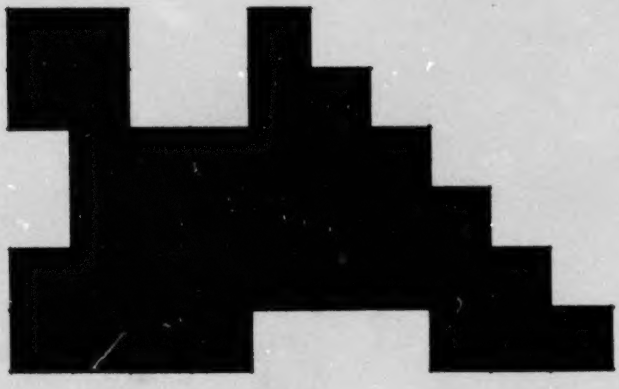
$$\frac{1}{2} \times 36 = 18$$

T: Color another square. . . . Go!



$\frac{1}{2} \times 36 = 18$

Continue signaling the students to color another square until every square in the shape has been colored. This is one possible coloring.



$\frac{1}{2} \times 36 = 18$

NOTE: The students might simply put colored "×'s" in the squares. This would keep the lesson from slowing down from careful coloring.

T: How many red squares are there? (Eighteen)

Point to each red square as the students count them.

T: How many blue squares are there? (Eighteen)

Point to each blue square as the students count them.

Stand by the middle shape.

T: I would like this shape to be colored one-third red, one-third blue, and one-third green. How do you think we could do this?

S: Divide the number of squares by 3 and color that many squares each color.

T: We already know there are thirty-six squares altogether. So how many should be colored red? . . . blue? . . . green?

S: Twelve of each color.

T: How do you know?

S: $36 \div 3 = 12$.

S: $12 + 12 + 12 = 36$.

S: $3 \times 12 = 36$, so $\frac{1}{3} \times 36 = 12$.

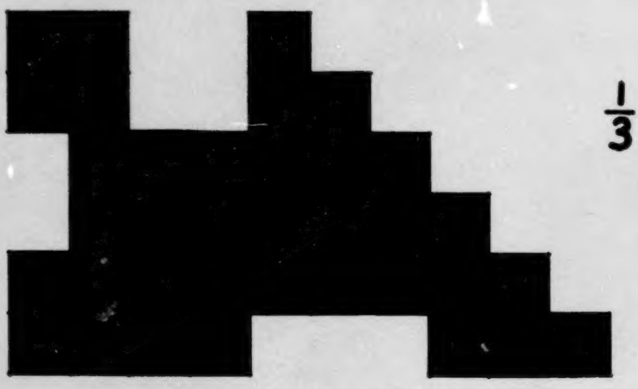
Near the middle shape write these number stories.

$$\begin{array}{r} 12 \\ 12 \\ +\,12 \\ \hline 36 \end{array} \qquad\qquad \frac{1}{3} \times 36 = 12$$

T: We could color twelve squares red, then twelve squares blue, and then twelve squares green. What is another way we could color one-third of this shape red, one-third blue, and one-third green?

S: Do what we did before, only this time let three people color squares.

Choose three students and give them each a different color marker. Signal them to color squares so that three squares are being colored simultaneously each time, until all the squares in the shape have been colored. This is one possible coloring.

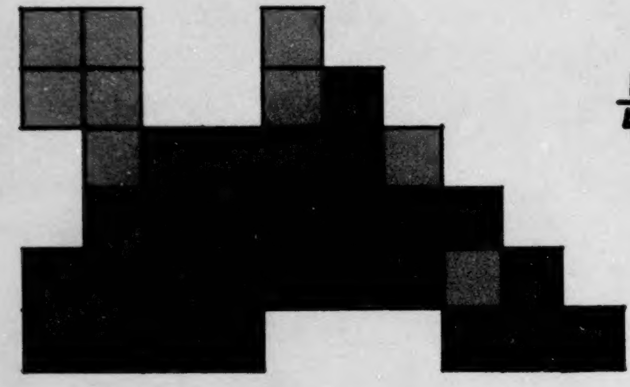


$\frac{1}{3} \times 36 = 12$

Point to all the squares of each color as the students count them to verify that there are twelve squares of each color.

Repeat this activity with the remaining shape except color one-fourth of the shape red, one-fourth blue, one-fourth green, and one-fourth yellow. You will need four students to come to the board.

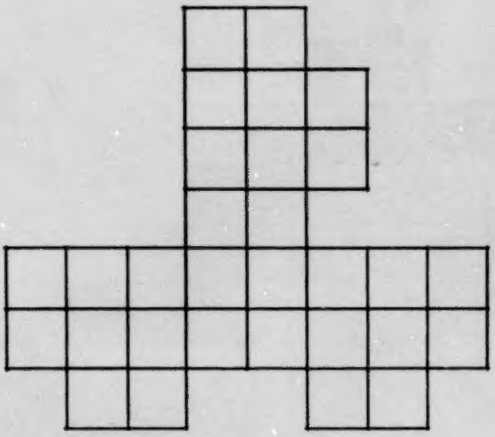
This is one possible coloring.



$\frac{1}{4} \times 36 = 9$

Exercise 2

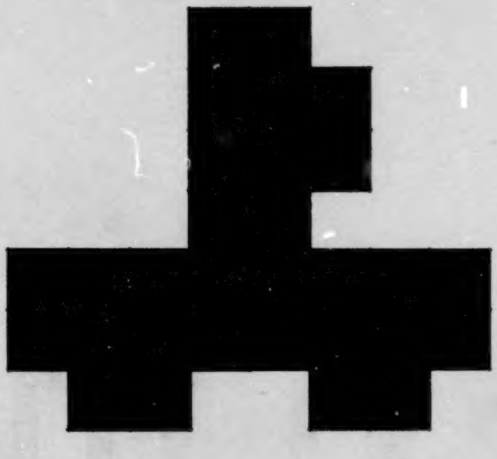
Tape Shape B from Geometry Poster #7 to the board. If necessary for space, remove the copies of Shape A from the board.



T: Look at this shape carefully. I would like to color it so that it would be easy to see that I colored one-third red, one-third blue, and one-third green. How should I do it?

Let students come to the board and point to the pieces they would color red, blue, and green. If someone suggests letting three students color the squares as in the previous exercise, accept this as a good method but suggest that quite likely someone not watching while the coloring was being done would need to count the squares of each color to check to see if there were exactly one-third red, one-third blue, and one-third green. Then ask for another way to color the shape so that it would be easy to see one-third of each color.

When a student suggests this coloring[*] . . .



. . . follow the suggestion and color the poster yourself. (Again, colored X's will suffice.)

Worksheets G4* and ** are available for individual work. Students will need colored pencils.

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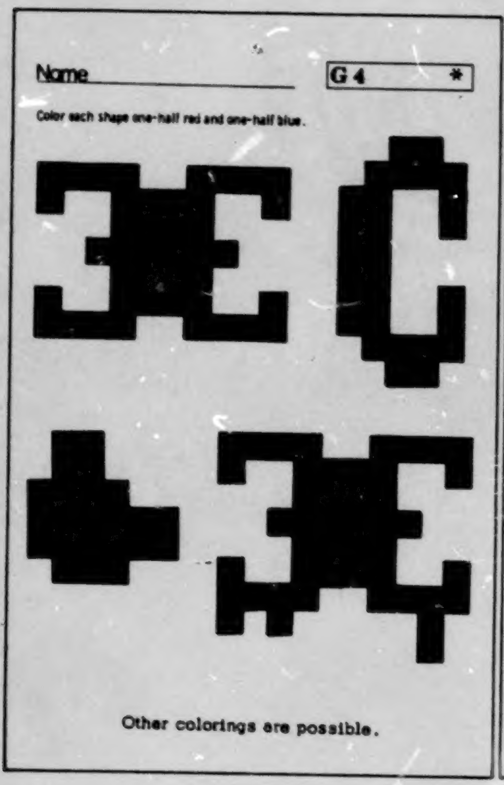
[*] The fact that there are symmetric pieces is important, but which piece is colored a particular color is not.

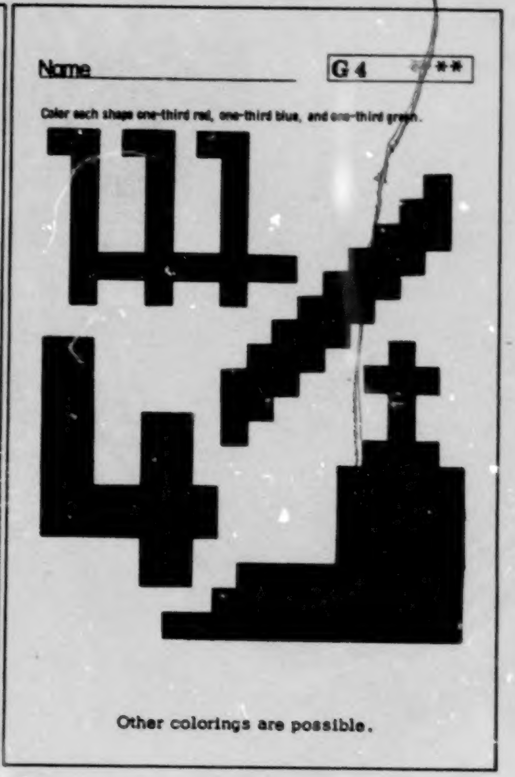


Name

G 4 *

Color each shape one-half red and one-half blue.

Other colorings are possible.



Name

G 4 ***

Color each shape one-third red, one-third blue, and one-third green.

Other colorings are possible.

G5 Nine-Square and Sixteen-Square

CAPSULE LESSON SUMMARY

Find the areas of the possible shapes which can be made using two squares which are each divided into 1 cm by 1 cm squares. You can overlap the two squares; but when you cover a little square of one of them, you must cover all of it.

MATERIALS

Teacher: Geometry Poster #8 for UPG-III; tape; grid board or poster grid sheets; colored chalk

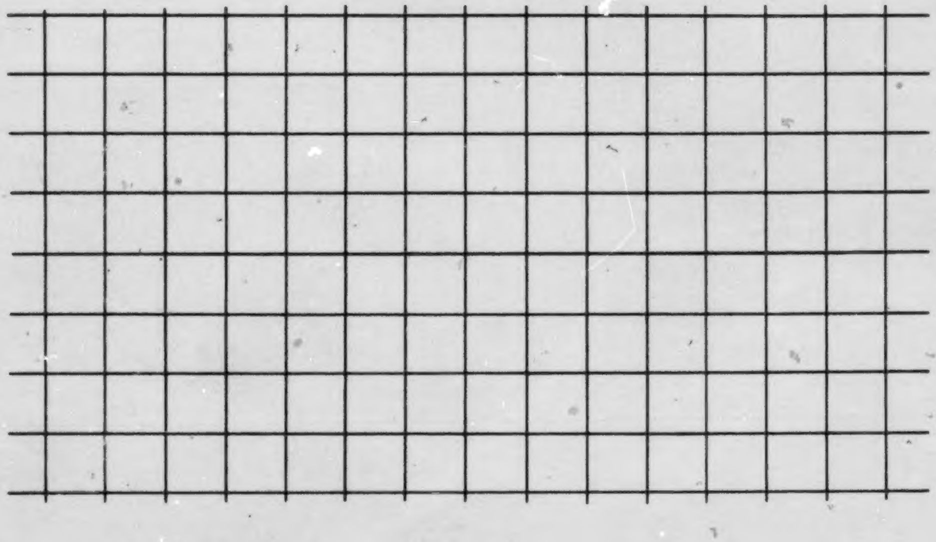
Student: Worksheet G5 (no stars), *, and **; colored pencils; scissors (one pair for every three students will suffice)

NOTE: Cut out shapes A and B from Geometry Poster #8. If possible, paste them individually to heavy paper or cardboard cut exactly to the size of the shapes.

DESCRIPTION OF LESSON

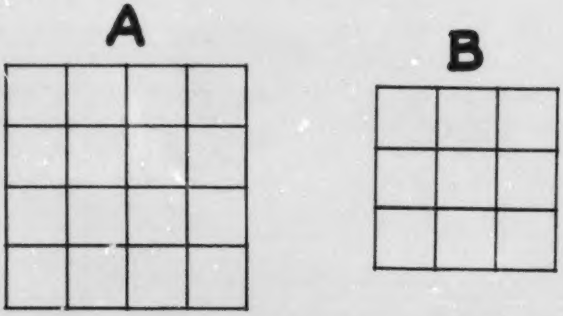
Exercise 1

Display a grid board with 8 cm sided squares or put at least two sheets of the grid poster together to form a grid you can tape to the board. The squares on the grid should be exactly the same size as the ones in the demonstration shapes; that is, the edge of each small square should be eight centimeters long.



G 5

Hold up the two demonstration shapes.



T: Do you know what shape these are?

S: Squares.

T (indicating B): What is the area of this square?

S: 9.

T: 9 what?

S: 9 little squares.

T: How did you get 9?

S: I counted the little squares.

S: 3 + 3 + 3 = 9.

S: 3 x 3 = 9.

Record the area on the board.

Area: 9 little squares

T (indicating A): What is the area of this square?

S: 16 little squares.

T: How did you get 16?

S: 4 + 4 = 8; 4 + 4 = 8; and 8 + 8 = 16.

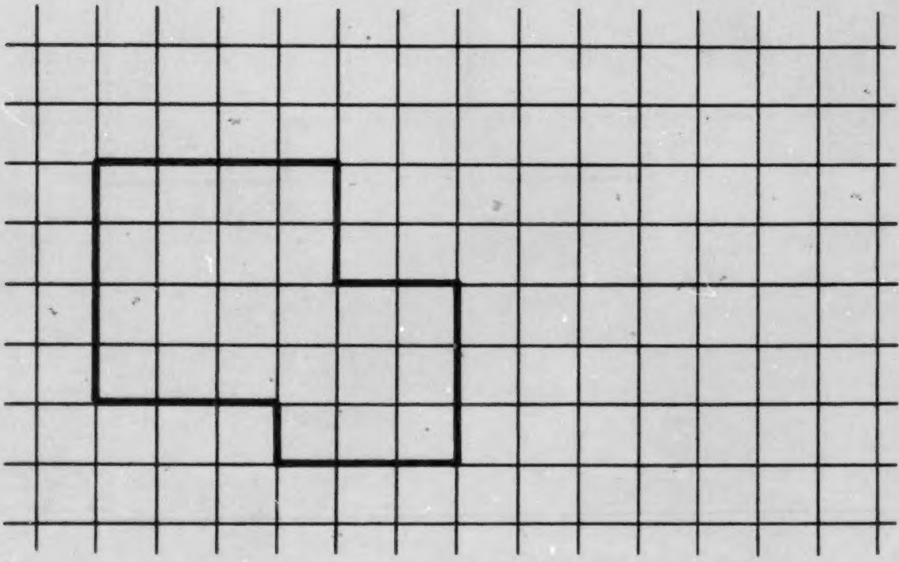
S: I counted the little squares.

S: 4 x 4 = 16.

Write "16 little squares" on the board.

Area : 9 little squares
16 little squares

Tape the demonstration shapes on the grid board, carefully overlapping the squares to get the shape in the next illustration. Outline in red the border of the resulting shape.



G5

Remove the demonstration shapes leaving only the outline on the grid. (Touch up the outline due to removing the tape.)

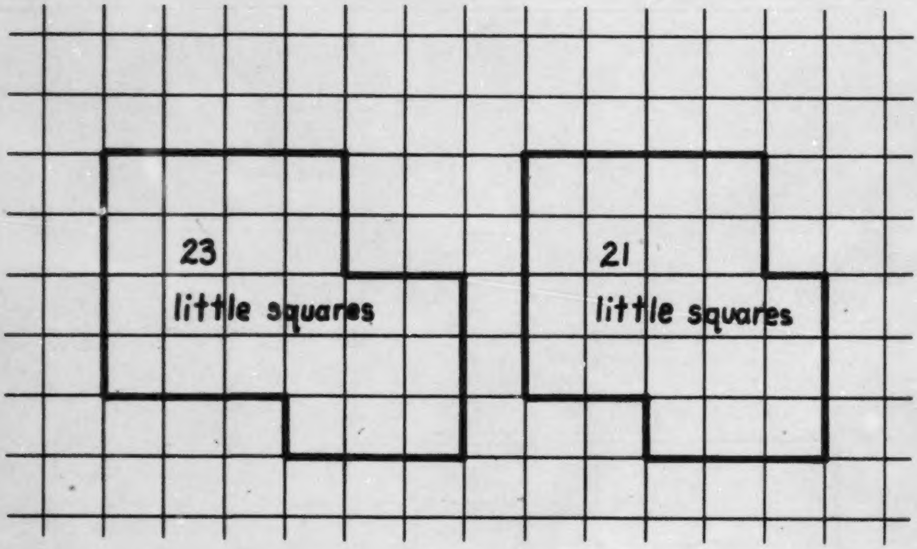
T: What is the area of this new shape?

S: 23 little squares.

If there is disagreement, collectively add the number of squares in each row: 4 + 4 = 8; 8 + 6 = 14; 14 + 6 = 20; and 20 + 3 = 23.

Write "23 little squares" inside the shape.

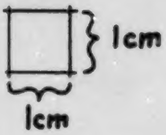
Repeat this activity for the shape on the right in the next illustration.



Distribute Worksheets G5 and G5*, red pencils, and scissors. Ask students to cut out the 16-square and the 9-square from the upper half of Worksheet G5.

After the 16-square and the 9-square have been cut out, run your finger along each of two adjacent sides of one of the grid squares on a copy of Worksheet G5* as you give its length.

T: Each of the little squares on your paper is exactly 1 centimeter long.



T: We say that the area of one of these (point to a square) is one square centimeter.

This shape (hold up demonstration shape A) has area 16 square centimeters. This one (hold up demonstration shape B) has area 9 square centimeters.

Replace "little squares" on the board with "square centimeters".

Area: 9 square centimeters

16 square centimeters

Introduce "cm^2" as an abbreviation for square centimeters and write "cm^2" under "square centimeters" where it occurs on the board.

Area: 9 square centimeters
cm^2
16 square centimeters
cm^2

T: On the worksheet there is a picture of the first shape I drew on the board. (Point to the shape on a copy of the worksheet.) The area of this shape is 23 cm^2 and is written to the side of it.

Point to the second shape you put on the board.

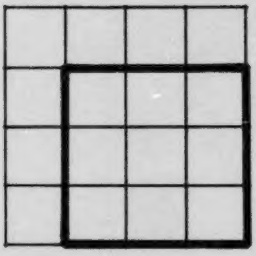
G5

T: I want you to make this shape with your 9-square and 16-square. Then trace the shape on your worksheet. Record its area next to it.

Check to make sure the students are outlining the shape on grid lines.

T: What do you think is the smallest area you can make using these two squares (hold up the demonstration shapes)? (Answer: 16 cm^2)

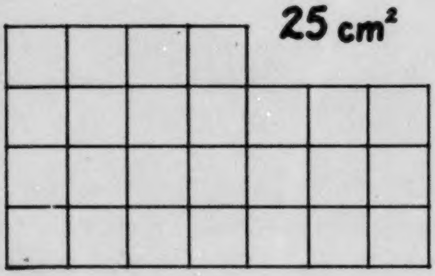
Let the students experiment and then ask a student to give a solution using the demonstration shapes.

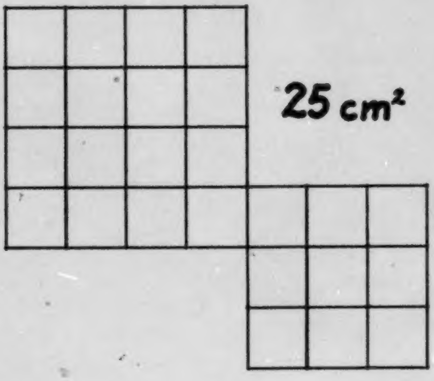


Ask the students to make that shape, to trace it on their worksheets, and to record its area next to it.

T: What is the largest area we could ever get? (Answer: 25 cm^2)

Let students experiment and then let several students come to the board one at a time to show some of the various solutions. Two out of several possible solutions are illustrated below.





Ask the students to draw one of the shapes whose area is 25 cm^2 on their worksheets and to record the area next to it.

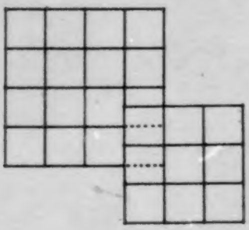
T: The smallest area we can get with the 9-square and the 16-square is 16 cm^2 and the largest area is 25 cm^2. We also found a shape with area 21 cm^2 and a shape with area 23 cm^2.

Record the whole number areas from 16 cm^2 to 25 cm^2 on the board. Circle those which have already been found.

[✓ 16 cm^2]
17 cm^2
18 cm^2
19 cm^2
20 cm^2
[✓ 21 cm^2]
22 cm^2
[✓ 23 cm^2]
24 cm^2
[✓ 25 cm^2]

T: Do you think we could find shapes for any of these areas? (Indicate the areas which are not circled in the list on the board.) If you find such a shape, trace it on the grid on the back of the worksheet and record its area. When you cover part of a little square, you must cover all of it.

NOTE: You may need to illustrate that your last comment prohibits overlapping the squares as shown below.



G5

Encourage the students to work independently. Make sure that the sides of the shapes they are tracing fall on grid lines. When several students have found a shape with a new area, circle the appropriate area on the board.

Shapes with areas 19 cm^2, 22 cm^2, and 24 cm^2 can be found. Invite some students to show shapes with these areas using the demonstration shapes. Sample solutions are shown on page 46.

Under the given restrictions, it is impossible to make shapes of areas 17 cm^2, 18 cm^2, and 20 cm^2 with the 9-square and the 16-square. Ask the students for explanations.

S: We can't get 17 cm^2 because with 16 cm^2 in the larger square we would need just one more little square to get 17 cm^2. Using the 9-square, the fewest (other than none) which we can get is three more little squares.

S: We can't get 18 cm^2 because with 16 cm^2 in the larger square we would need two more little squares to get 18 cm^2. Using the 9-square, the fewest (other than none) which we can get is three more little squares.

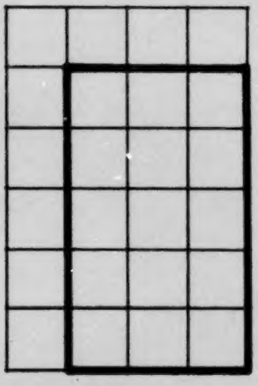
S: We can't get 20 cm^2 because with 16 cm^2 in the larger square we would need four more little squares to get 20 cm^2. Using the 9-square, we can get three more or six more, but not four more little squares.

Another approach would be to look at how many little squares of the 16-square would need to be covered by the 9-square to get such areas.

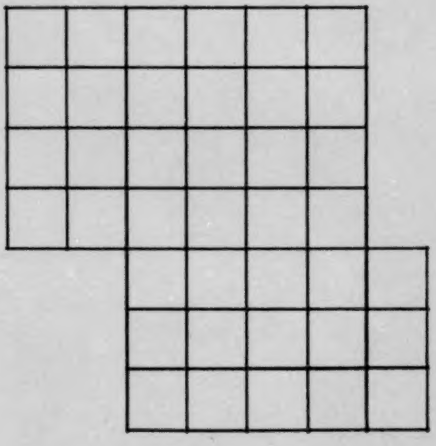
Exercise 2

As in Exercise 1, ask the class to find all possible areas using the rectangles on the lower half of Worksheet G5. Students should work individually tracing shapes and recording their areas on Worksheet G5**.

The smallest shape that can be made with these rectangles has an area of 24 cm^2. The largest shape has an area of 39 cm^2.



24 cm^2

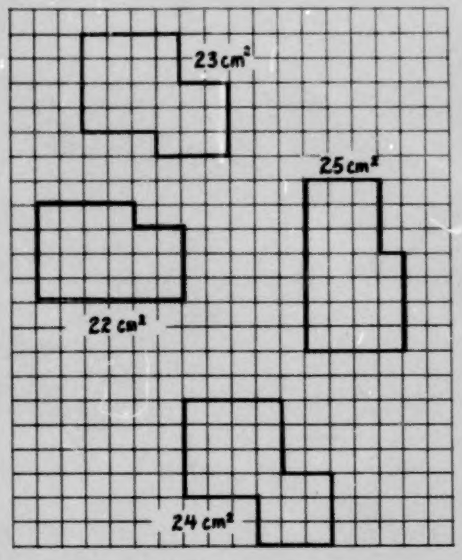


39 cm^2

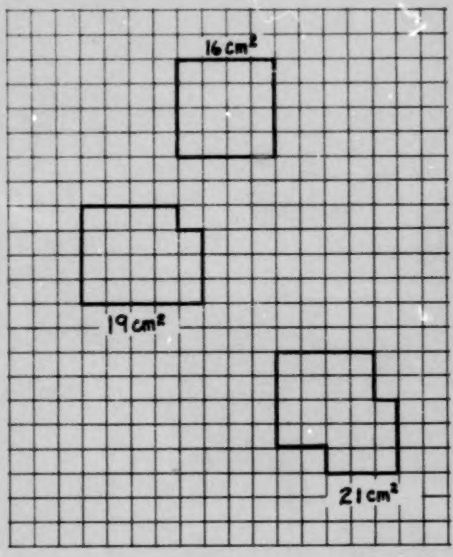
Shapes with (whole number) areas from 24 cm^2 to 39 cm^2 can be made with the exception of 32 cm^2, 28 cm^2, 26 cm^2, and 25 cm^2.

Name ____________________

G 5 *

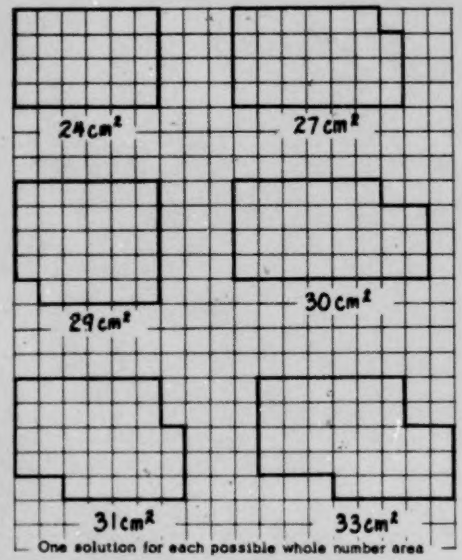

23 cm²
25 cm²
22 cm²
24 cm²

One solution for each possible whole number area from 16 cm^2 to 25 cm^2 is shown here. There are other solutions for some of these areas.

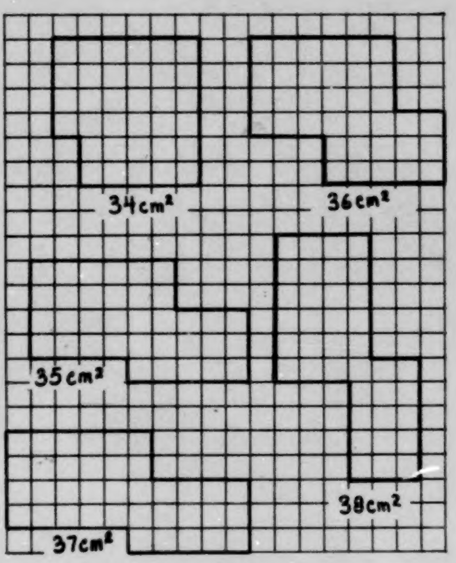

16 cm²
19 cm²
21 cm²

Name ____________________

G 5 **


24 cm²
27 cm²
29 cm²
30 cm²
31 cm²
33 cm²

One solution for each possible whole number area from 24 cm^2 to 38 cm^2 is shown here. It is also possible to make a shape with area 39 cm^2 as shown on page 45. There are other solutions for some of these areas.


34 cm²
36 cm²
35 cm²
38 cm²
37 cm²

G6 Length #2 (Halloween Message)

CAPSULE LESSON SUMMARY

Do a dot-to-dot drawing by making measurements and discover a secret message.

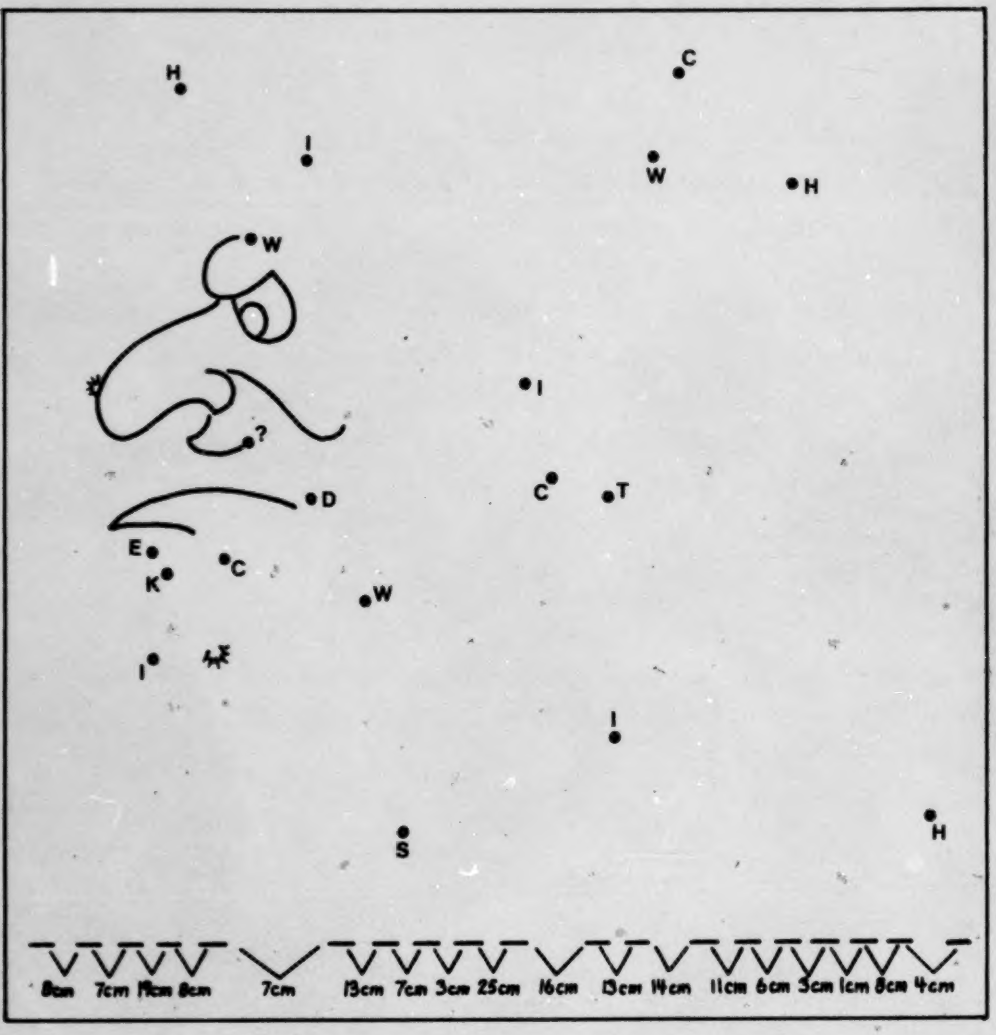
MATERIALS

Teacher: Centimeter ruler (30 cm); Geometry Poster #9 for UPG-III; felt-tip pen

Student: Worksheet G6; pencil; centimeter ruler

DESCRIPTION OF LESSON

Tape Geometry Poster #9 to the board.


H
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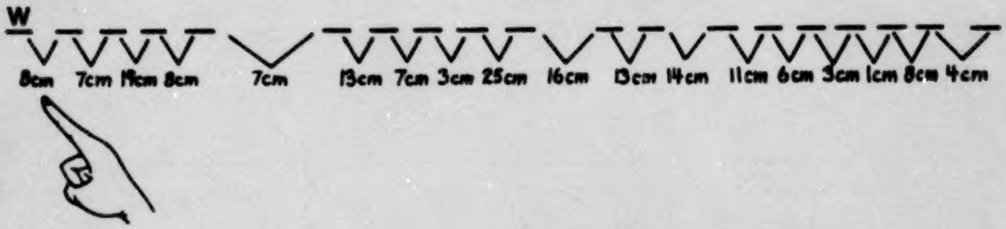
7cm
13cm
7cm
3cm
25cm
16cm
13cm
14cm
11cm
6cm
3cm
1cm
8cm
4cm

G 6

Guide your students to complete the dot-to-dot picture using this method:

- Start the picture at the dot labeled "W" above the eye. Refer to the first entry (8 cm) in the given sequence of measurements to find the distance from W to the second dot.



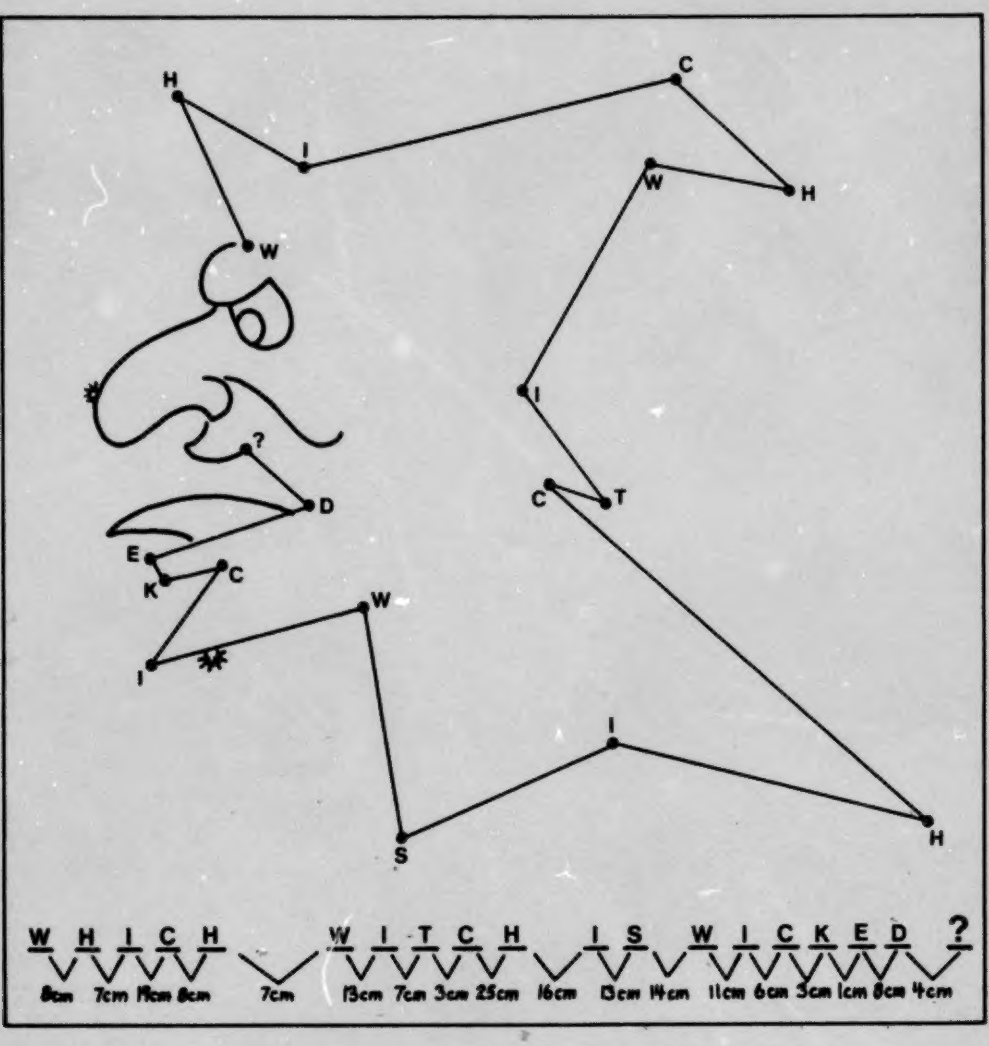
- Measure the distance from the starting dot W to other dots until the dot which is 8 cm from W is found. Connect the two dots with a segment and record the appropriate letter in the second blank. (H)

- Refer to the given sequence of measurements to find the distance from the second dot (labeled "H") to the third dot. (7 cm) Then proceed as before by making measurements. Once the third dot is found, connect it to the second dot and record its corresponding letter in the third blank. (I)

- Continue in this manner until the message is complete.

NOTE: A dot can be the endpoint of at most two segments in this picture. Do not "double back" to a dot already used even though the distance is appropriate.

The completed picture is shown below.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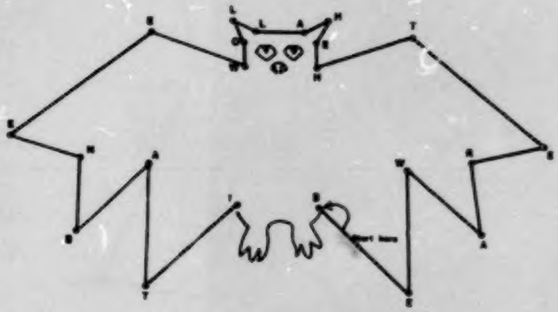
W H I C H WITCH IS WICKED ?
8cm 7cm 19cm 8cm 7cm 13cm 7cm 3cm 25cm 16cm 13cm 14cm 11cm 6cm 3cm 1cm 8cm 4cm

Worksheet G6 is available for individual work.

Name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E

B E W A R E T H E H A L L O W E E N B A T !

5cm 5cm 4cm 5cm 5cm 2cm 4cm 1cm 1cm 1cm 2cm 1cm 1cm 1cm 4cm 2cm 5cm 5cm 4cm 5cm 5cm

G6

G7 Where Shall We Meet? #1

CAPSULE LESSON SUMMARY

Draw and compare several shortest paths from Nora's house to her grandmother's house. Find places which are the same taxi-distance from a rollerskating rink and a movie theater. Do a related problem involving Nora's house and a friend's house.

MATERIALS

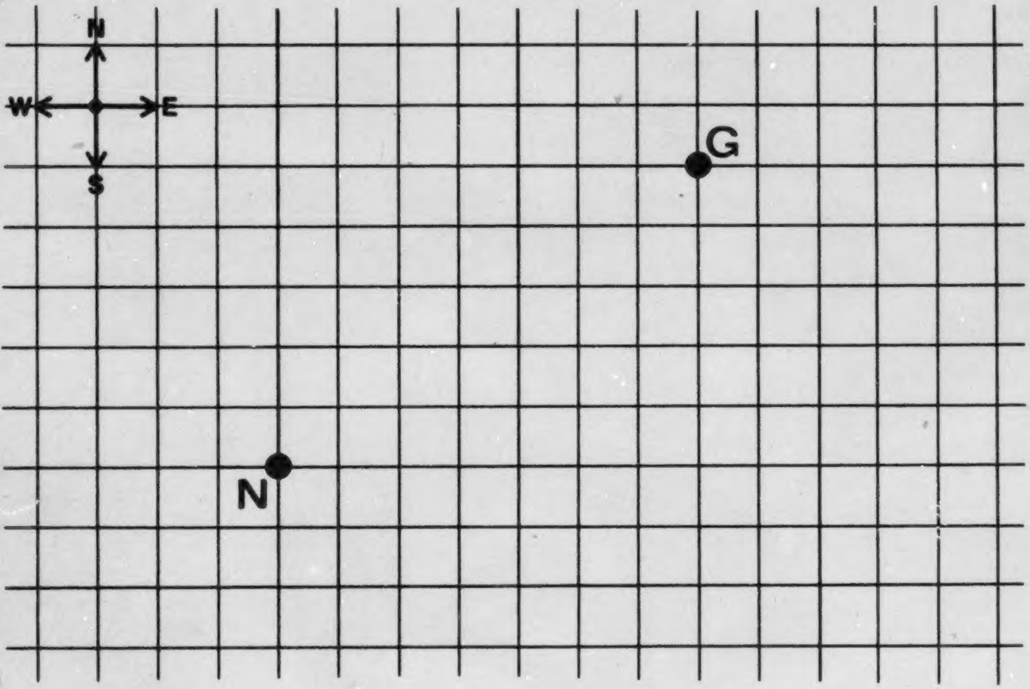
Teacher: Grid board; colored chalk
Student: Worksheets G7* and **; colored pencils

DESCRIPTION OF LESSON

Exercise 1

NOTE: This exercise is a review of Nora's neighborhood and of taxi-geometry and should move quickly.

Display a grid board. Draw and label two dots as shown below. In the upper left corner of the grid, indicate the four directions.



603

G 7

T: Remember this map of Nora's neighborhood. Who can tell us something about Nora?

S: Nora likes to visit her grandmother.

S: Nora sometimes takes long walks and sometimes short walks.

S: Nora doesn't cut through people's yards.

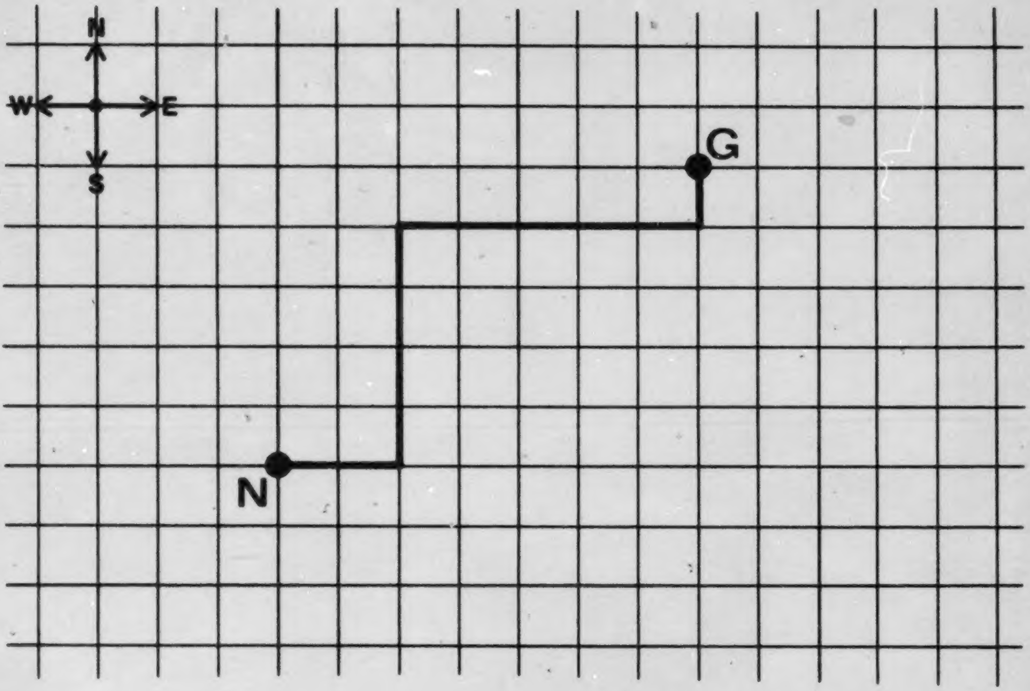
Ask someone to trace a path from N to G. Accept any path that follows the lines on the grid board.

T: Can someone show us a longer path (than the one just previously traced)?

Invite a student to trace another path from N to G.

T: When it is raining, Nora finds a shortest path to her grandmother's. Who can show us a shortest path?

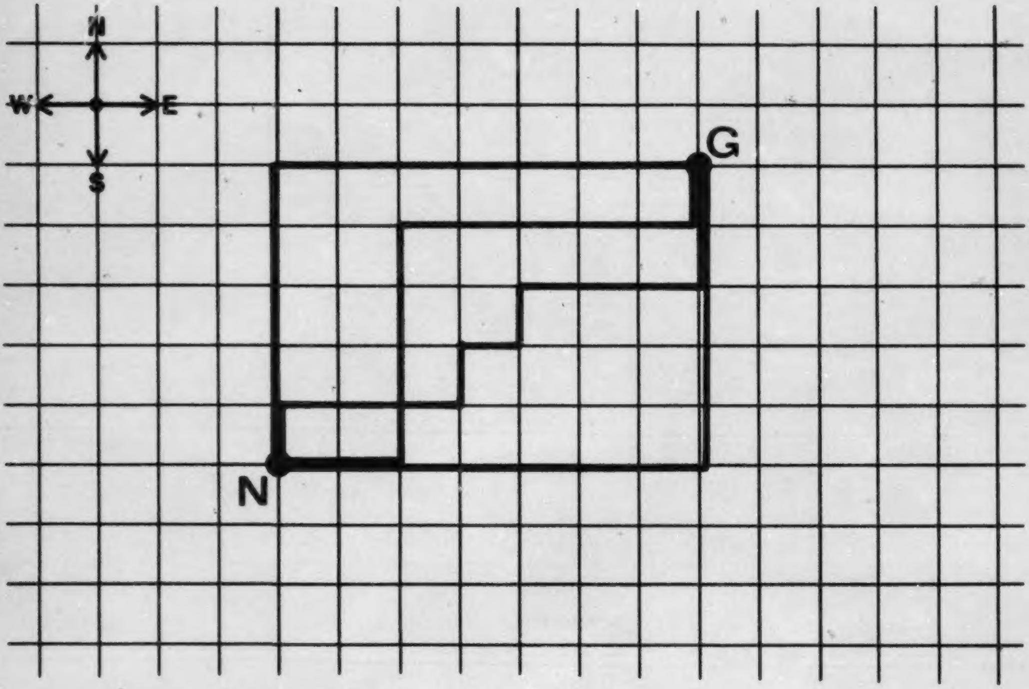
Invite students to trace paths until a shortest path (in this case, one in which Nora walks only in the north and the east directions) is suggested and then draw it on the board. Help the student whose path is drawn to count how many blocks are in the path.



S: My path is 12 blocks long.

T: Does anyone think that Nora could find a shorter path?

Some students might see that there are several shortest paths and that these paths all have a length of 12 blocks. But for the students who are not yet convinced, continue asking students to trace paths until several shortest paths have been suggested and have been drawn on the grid board.

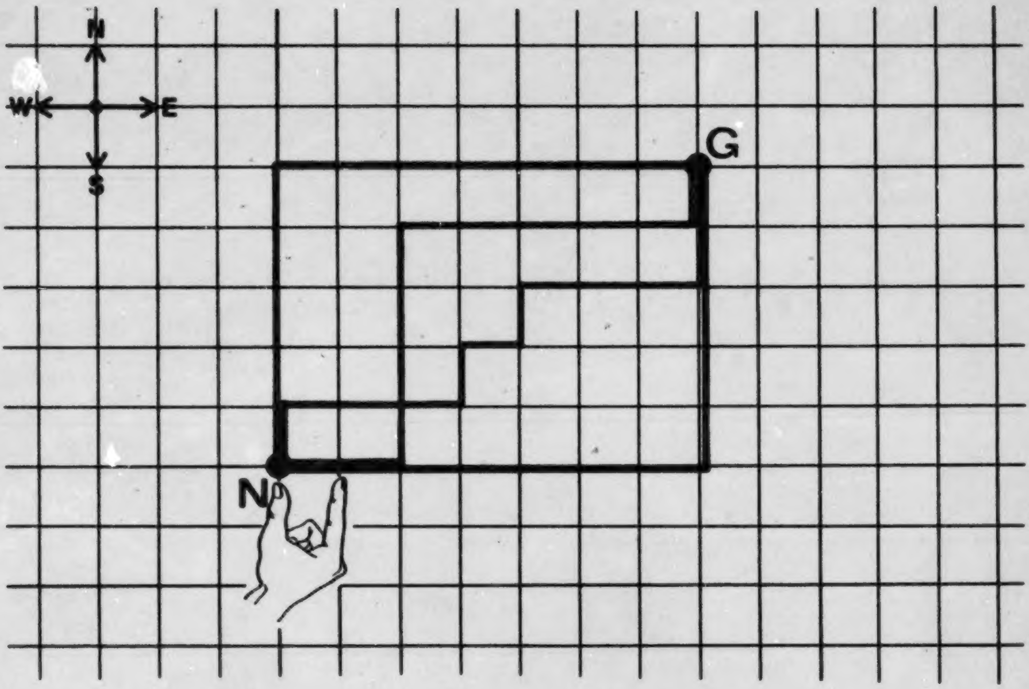


T: Who can explain why there are many shortest paths each having a length of 12 blocks?

A student might suggest looking at how many blocks Nora walks east and how many blocks north, but most likely you will need to encourage this discussion yourself.

T: Let's look at the red path. How many blocks would Nora walk east (indicate the east direction of the map)?

Ask the class to count the blocks of the red path in the east direction as you indicate the blocks one at a time using the "index finger and thumb" method (see the next illustration).



In the same manner, count the blocks on which Nora would walk east in each path on the board. Conclude that on any shortest path Nora would walk seven blocks in the east direction.

T: Let's count the number of blocks Nora would walk north in each of these paths (indicate the north direction of the map).

Ask a student to count the blocks Nora would walk north on one of the paths. Conclude there are five blocks in the north direction.

Write this information on the board.

7 blocks east
5 blocks north

T: How many blocks are there altogether in each of these paths?

S: 12 blocks.

G 7

T: Does anyone remember what we call the length of the shortest walks?

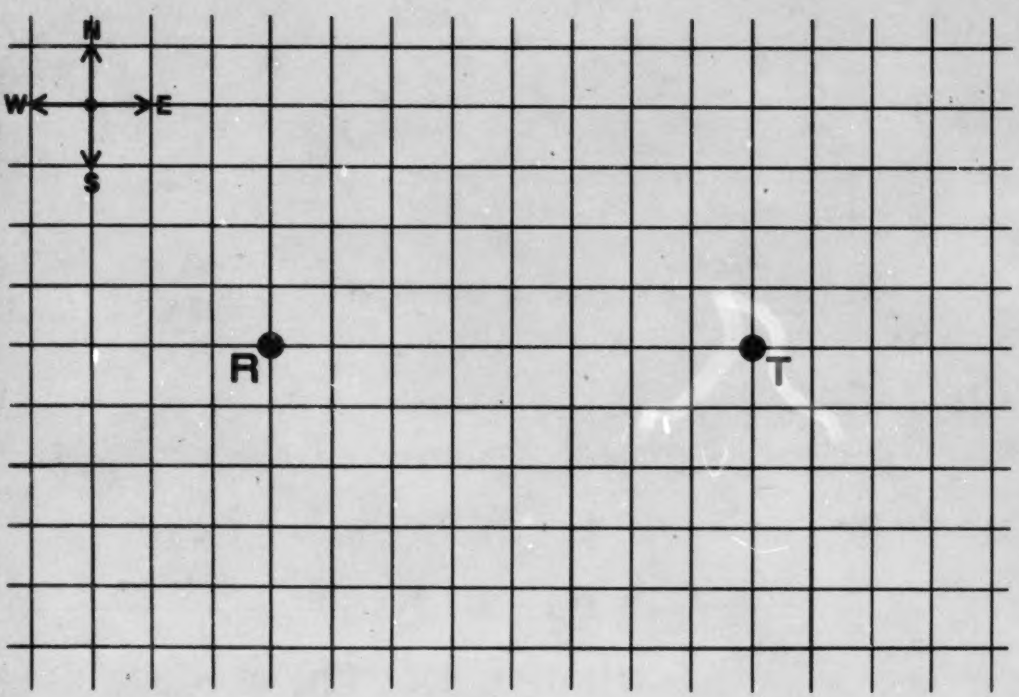
S: Taxi-distance.

Write this on the board to the side of the grid board.

taxi-distance from N to G = 12 blocks

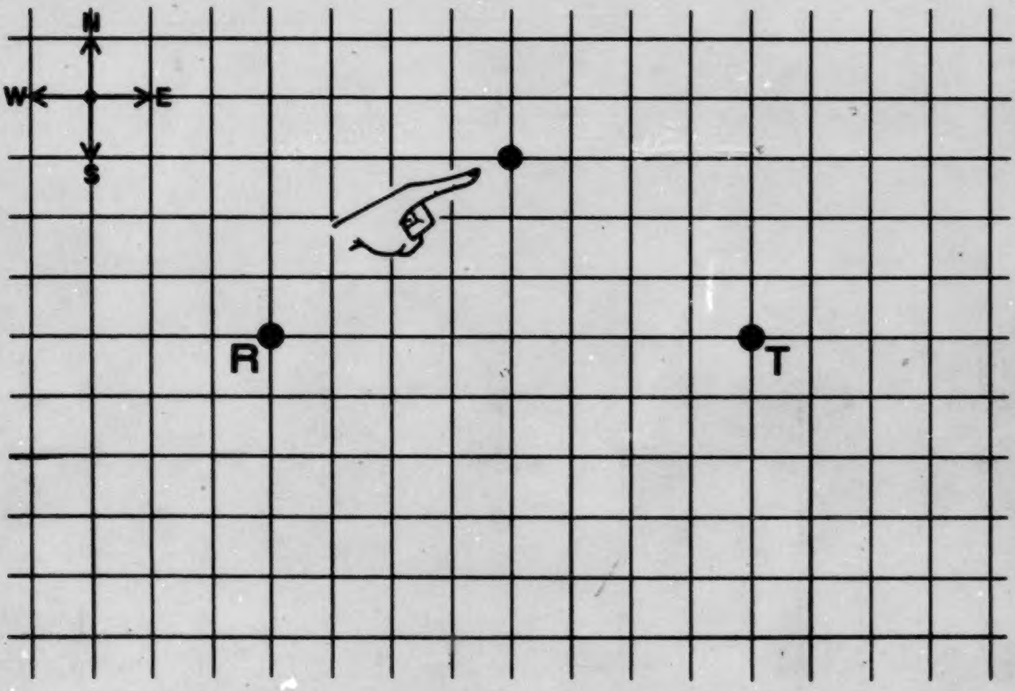
Exercise 2

Clear the grid board and then draw two dots one labeled "R" and one labeled "T" as in the next illustration.



T: Early one afternoon Nora's friend Janet calls her and suggests that she meet her at a place the same taxi-distance from the rollerskating rink (point to "R") as from the movie theater (point to "T"). From there they can decide whether to go skating or to a movie. Shortly after Nora hangs up the telephone she becomes confused. She realizes that Janet could be referring to different places. Does anyone see one of those places?

Suppose a student suggests this point (see illustration). Draw the dot in red.



T: What is the taxi-distance from the rollerskating rink to this dot (tap the red dot)?

Let a student count the blocks. Make sure the student is considering a shortest path between the red dot and R.

G7

S: The taxi-distance is 7 blocks.

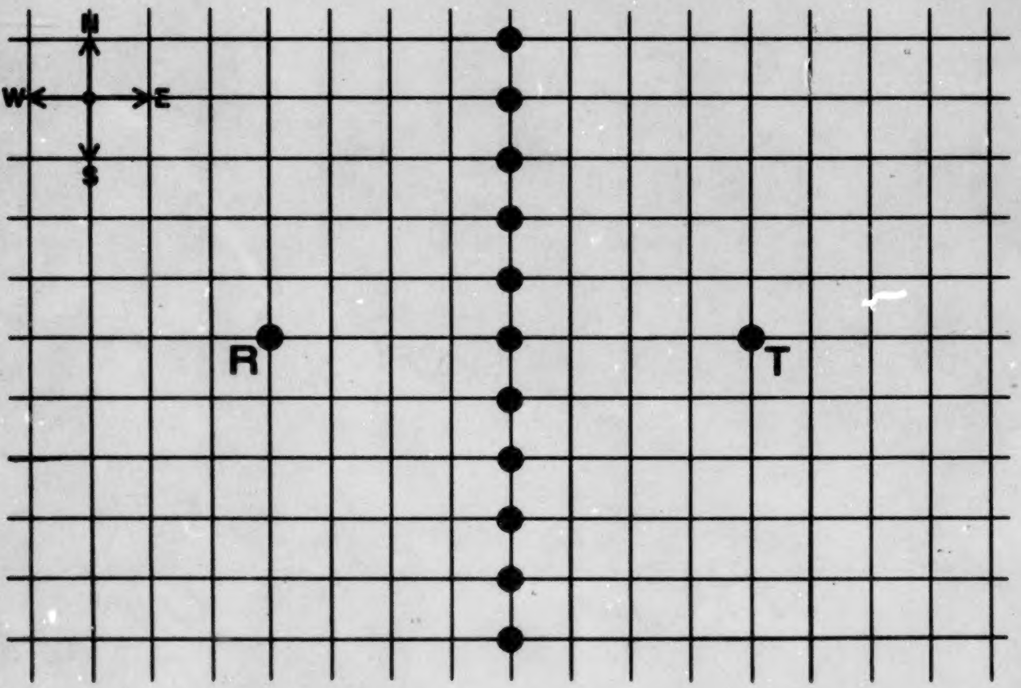
T: What is the taxi-distance from the movie theater to this dot (tap the red dot)?

Let a student count the blocks.

S: 7 blocks.

T: So if Nora were standing here (red dot), she would have the same distance to walk to R or to T.

Continue asking for other places which are the same taxi-distance to R and to T until you have this picture. (Check the taxi-distance for several but not all of the suggested dots.)



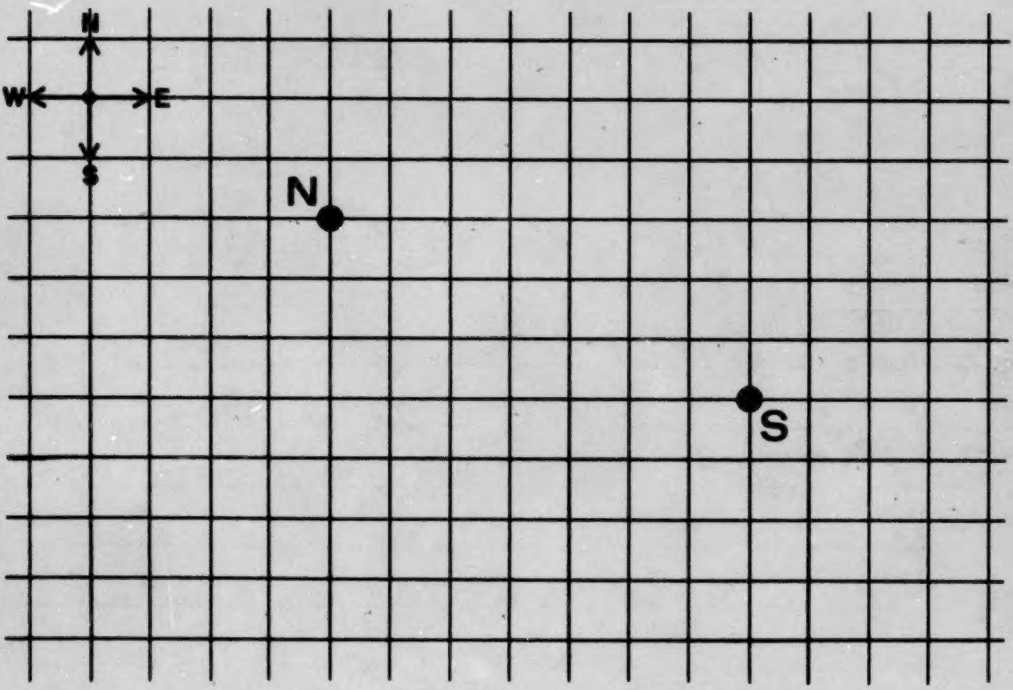
T: If this map were larger, do you think we could draw some more places the same taxi-distance from R and from T?

S: Yes, in a long line.

Let a student come to the board and indicate where they would be.

Exercise 3

Clear the grid board and then draw two dots as in the next illustration.



T: Nora has a friend named "Sally". Sally lives here (point to "S") and Nora lives here (point to "N").

Distribute copies of Worksheet G7* and red pencils.

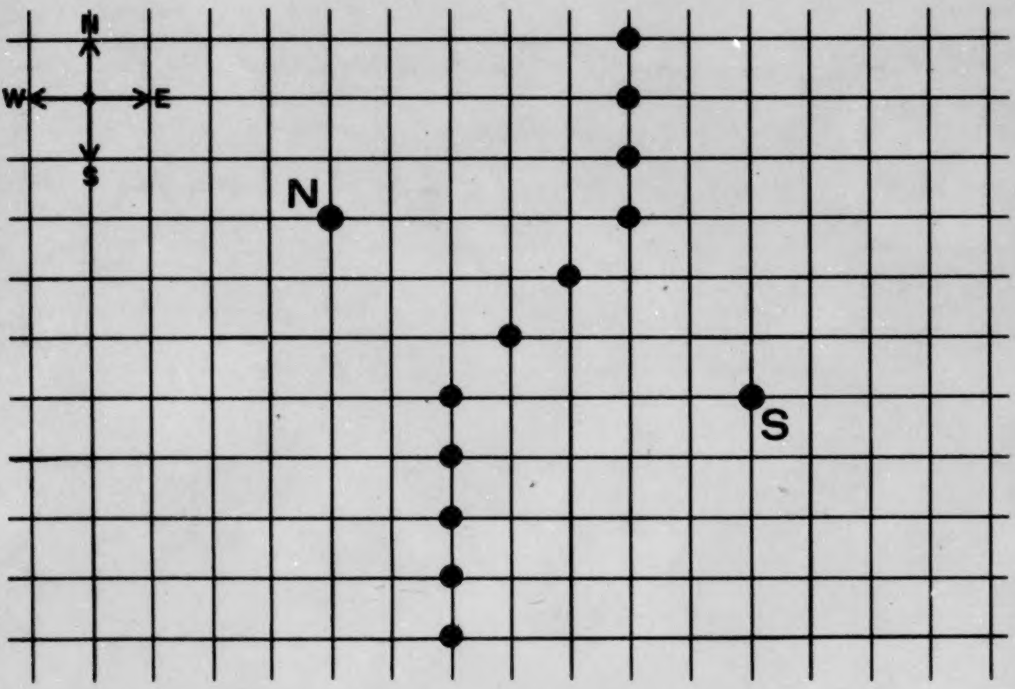
G7

Hold up a copy of the worksheet.

T: On Your copy of this worksheet, try to find places that are the same taxi-distance from Nora's house and from Sally's house. As you find correct places, I'll let you put them on the board. Use red pencil to mark the places you find.

Walk around the room observing students' work. Help students who cannot find any correct places by suggesting one and then checking with them that the taxi-distance from that dot to N is the same as the taxi-distance from that dot to S. Ask students with correct solutions to put them on the grid board. You may need to help students transfer dots from their worksheets to the demonstration grid board. Suggest that they count blocks over and up (down) from either N or S.

A completed picture should look like this one.



Afterward check several of the red dots on the board by tapping each and asking, "How far is it from Nora's house to this dot? . . . from Sally's house?"

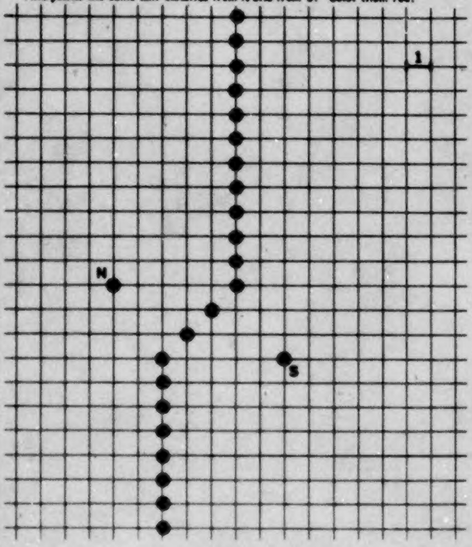
T: What would the picture look like if the map were larger?

S: We would get more dots going straight up and more going straight down.

On Worksheet G7**, find the places the same taxi-distance from Nora's house (N) and Tom's house (T). Students should work individually on this worksheet.

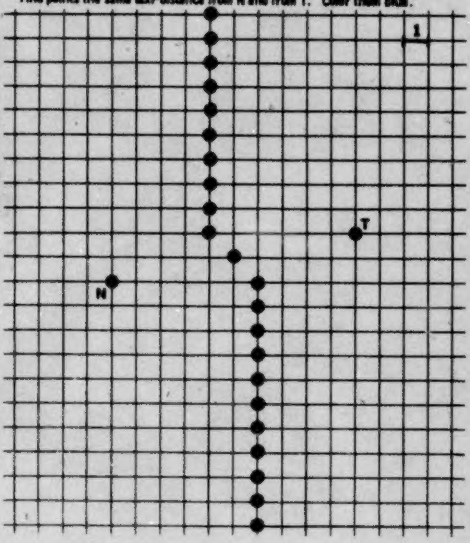
Name ______________________ | **G 7 ***

Find points the same taxi-distance from N and from S. Color them red.



Name ______________________ | **G 7 ****

Find points the same taxi-distance from N and from T. Color them blue.



G8 Where Shall We Meet? #2

CAPSULE LESSON SUMMARY

Find places which are the same taxi-distance from Nora's house and from her friend Alice's house. Do a related problem involving Nora's house and another friend's house, but observe a different pattern in the solution.

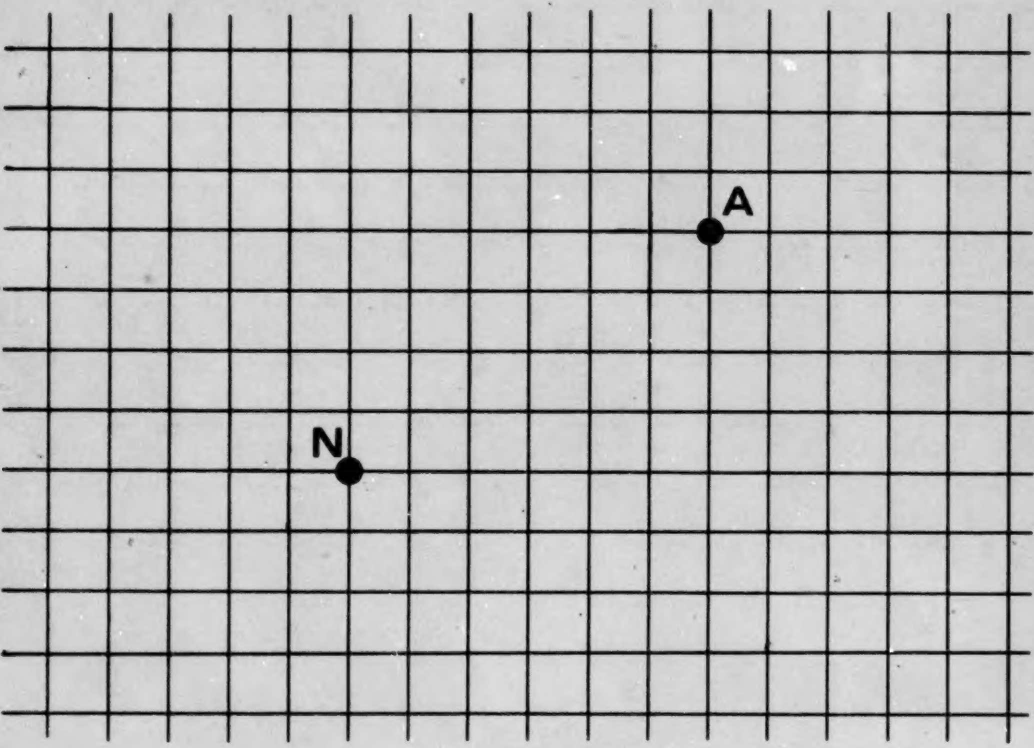
MATERIALS

Teacher: Grid board; colored chalk
Student: Worksheet G8; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Display a grid board. Draw and label two dots as shown below.

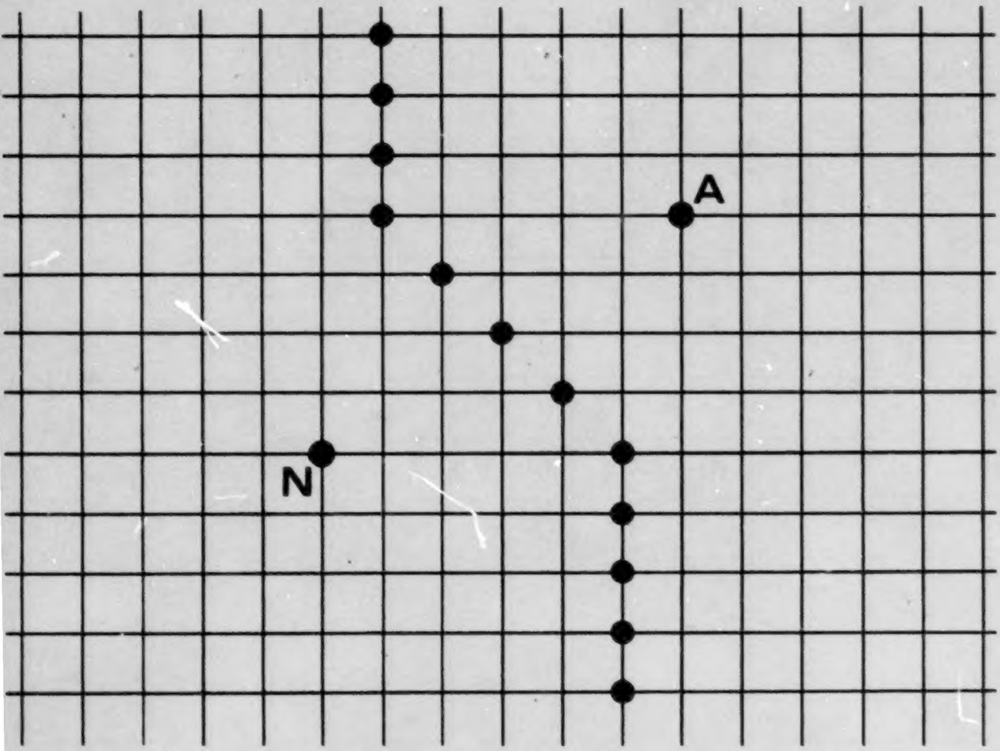


G8

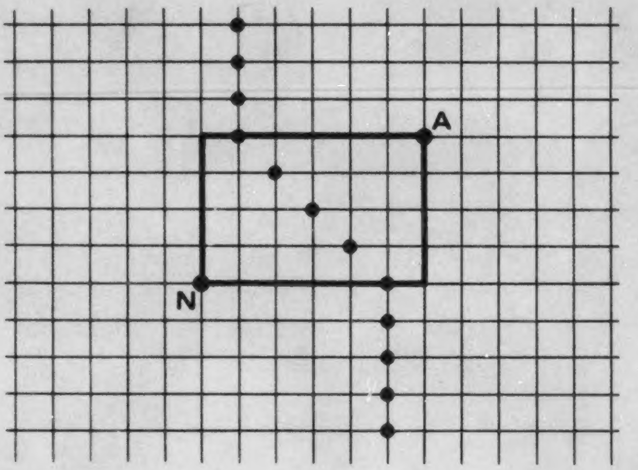
T: Last week we were looking at places where Nora and her friends could meet. Nora has a friend whose name is "Alice". Alice lives here (point to the dot labeled "A") and Nora lives here (point to the dot labeled "N"). Who can show us a place where Nora and Alice could meet? Remember we are looking for places the same taxi-distance from Nora's house and from Alice's house.

Let students draw dots in red for possible meeting places. Each time one student draws a dot in the picture, choose another student to check the taxi distance from the suggested dot to Nora's house and then to Alice's house. Erase any incorrect dots. After many dots have been drawn and the pattern is clear, let students plot correct points without checking or commenting.

A completed picture (within the limits of the grid) should look like this one.

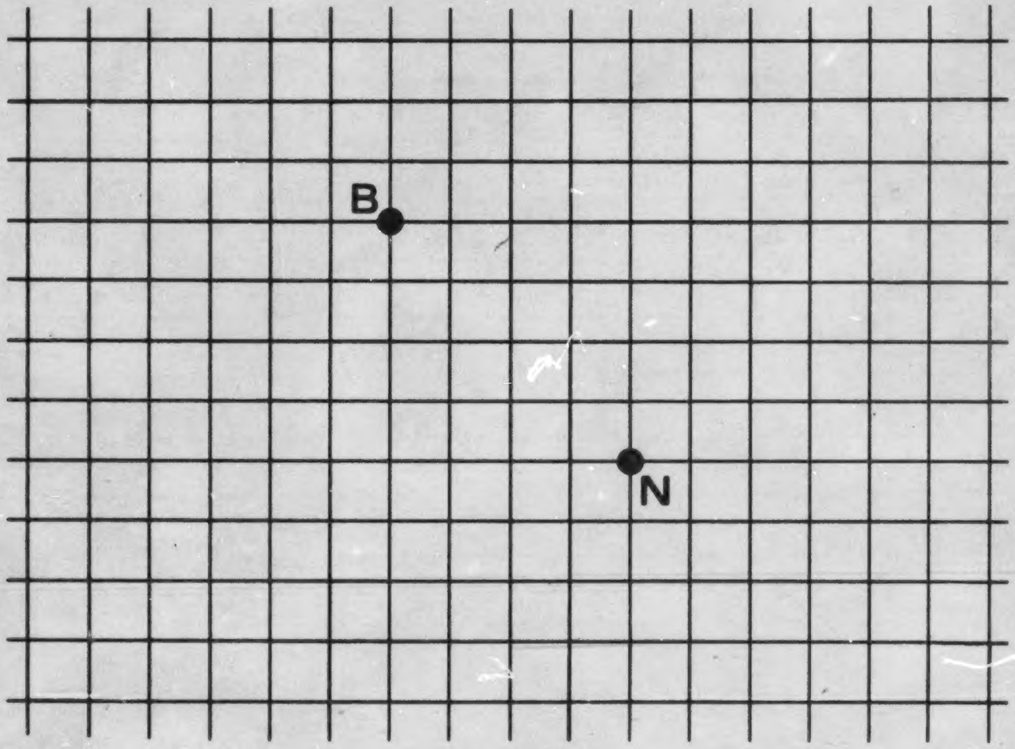


NOTE: A rectangle with N and A as opposite corners can be drawn in the picture to highlight the pattern, but should be done so only after several dots have been drawn.



Exercise 2

Clear the grid board and then draw and label two dots as shown in the next illustration.



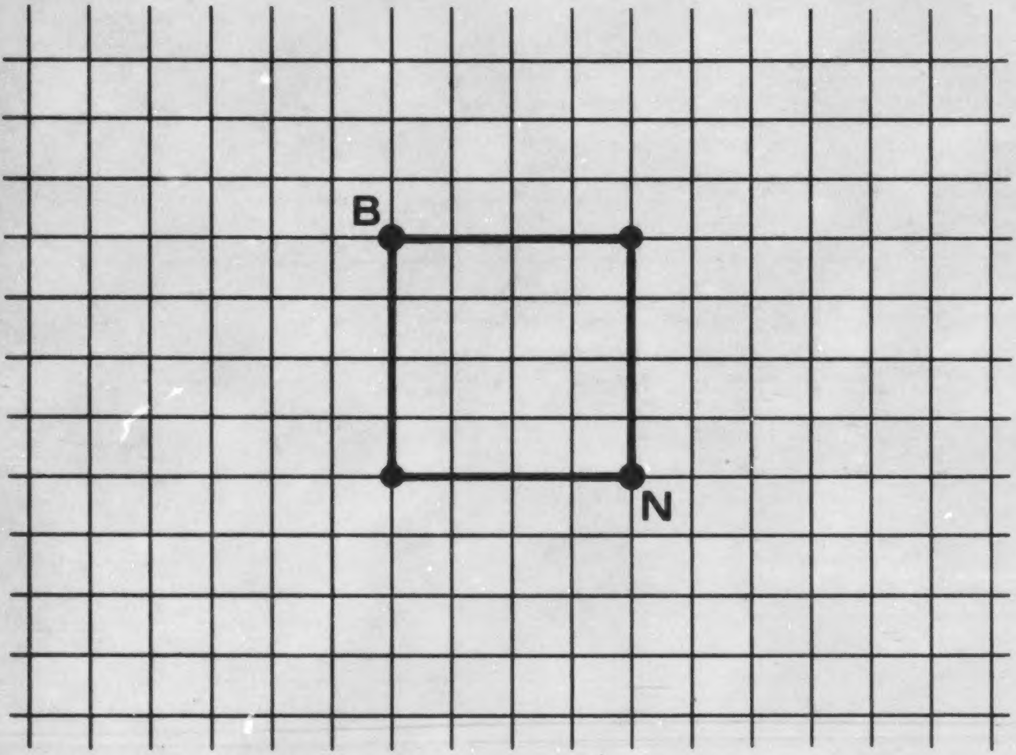
G8

T: Nora's friend Bill lives here (point to the dot labeled "B") and Nora lives here (point to the dot labeled "N").

Distribute copies of Worksheet G8 and red pencils. Hold up a copy of the worksheet.

T: Each of you has a copy of this worksheet. Try to find places that are the same taxi-distance from Nora's house and from Bill's house. Draw red dots at those places.

Give individual help to students having difficulty and, as necessary, encourage others to find more dots. After several minutes, direct the class' attention to the grid board. Draw a square and two dots as in the next illustration.

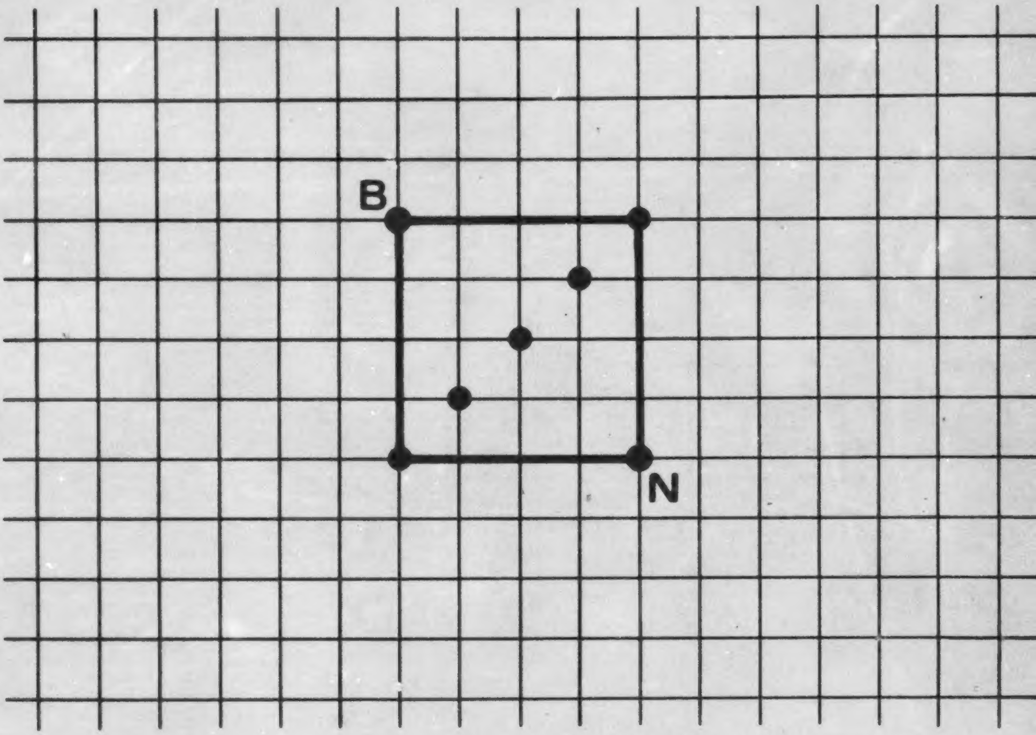


T (pointing to the red dots): Almost everyone has found these two dots.

Check the taxi-distance from one of the red dots to Nora's house and then to Bill's house (each will have a taxi-distance of 4 blocks). Students should check their papers and draw these dots if they do not have them.

T (tracing the square): Are there any places inside this square which are the same taxi-distance from both of their houses?

Let students point to and, if correct, draw the dots along the diagonal of the square as in the next illustration.



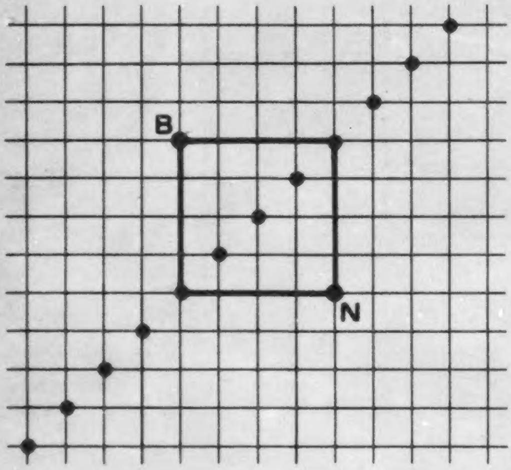
T: There are still more places which are the same taxi-distance from both houses. Try to find some of them.

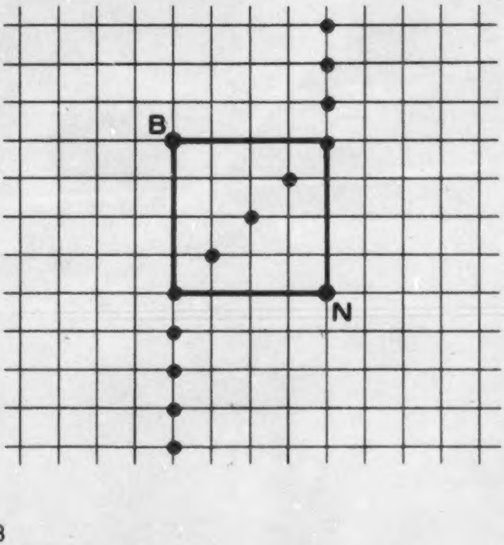
Students should continue working independently for about ten minutes. Then call the entire class' attention back to the board.

T: You have been finding lots of places which are the same taxi-distance from Nora's house and from Bill's house. Let's see if we can combine all the dots which class members have found and draw them on the board.

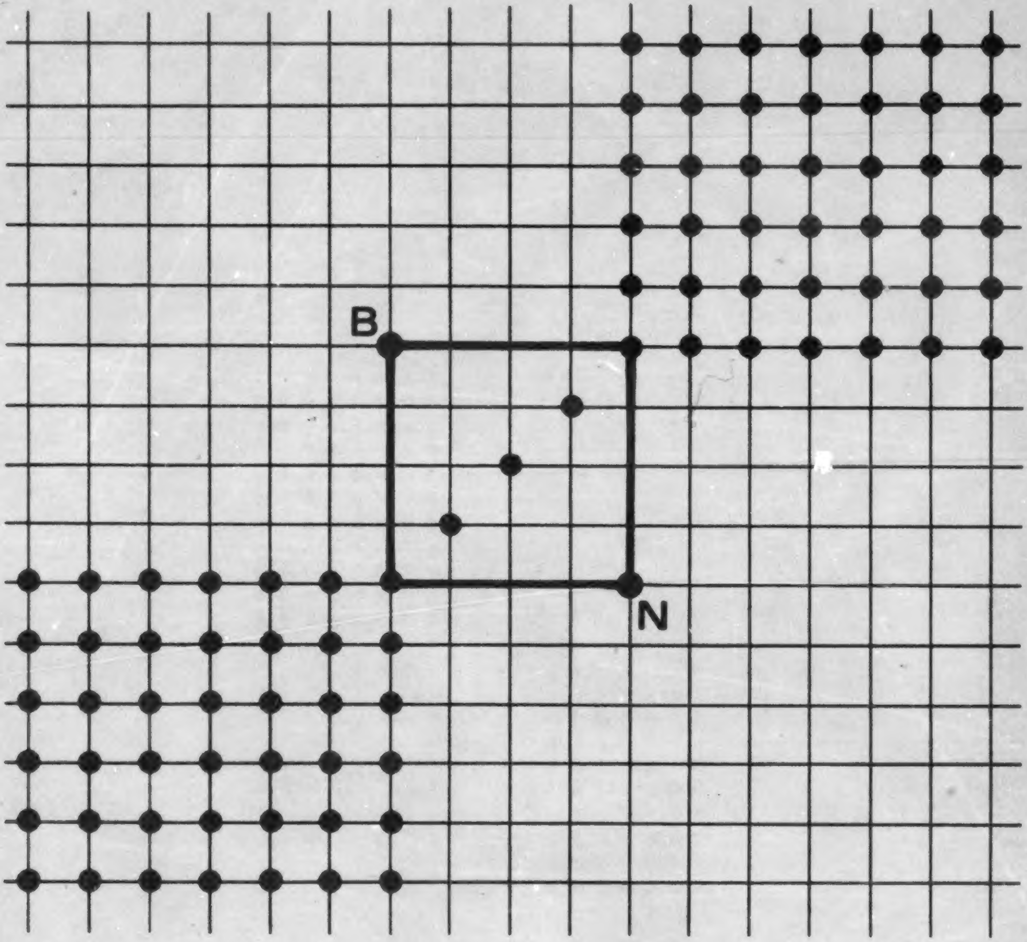
Let students draw dots in the picture three at a time. When three dots are suggested by someone, ask another student to check the taxi-distance for one of the dots. Since more and more dots are being added to the picture, put your finger on the dot being checked.

There are two patterns of dots which most students will discover, and very likely these dots will be the first ones suggested. They are shown below.





After many dots have been drawn, let students draw more than three dots at a time. The completed picture (within the limits of the grid) is given below.

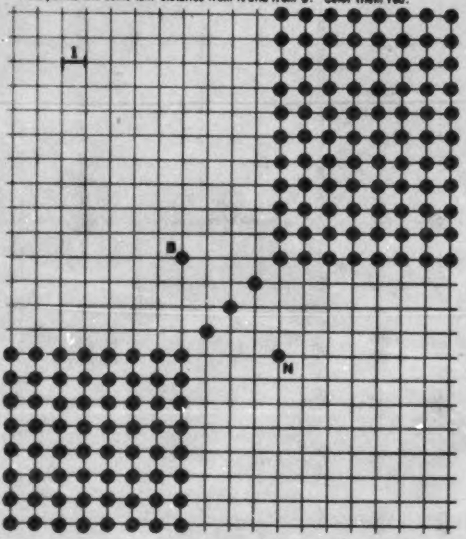


Encourage students to comment on any patterns they see. For example, some may observe that you get all the corners in two regions of the grid and that those regions have boundaries obtained by extending the sides of the black square with B and N as diagonal corners.

Name ______________________

G 8

Find points the same taxi-distance from N and from B. Color them red.



G9 Area #3

CAPSULE LESSON SUMMARY

Divide discs into halves, into fourths, into eighths, and into sixteenths. Use the resulting pictures to suggest equivalent fractions. Divide a rectangular shape into thirds and then into sixths. Use the resulting picture to suggest equivalent fractions.

MATERIALS

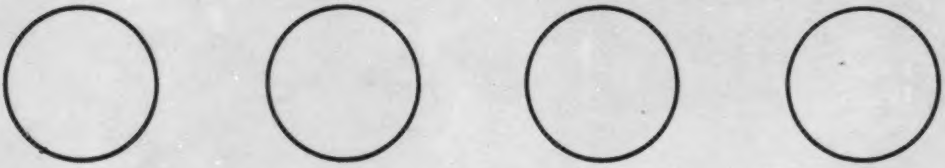
Teacher:	Chalkboard compass (or any tool with which to draw a circle); meterstick or straightedge; colored chalk
Student:	Worksheets G9*, **, ***, and ****; colored pencils

NOTE: See Lesson W8 A Spinner Game #1 for a description of how to draw a circle using a piece of string attached to a piece of chalk.

DESCRIPTION OF LESSON

Exercise 1

Draw four circles of the same size (approximately 50 cm in diameter) on the board.



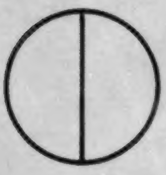
T: What shapes are these on the board?

S: Circles.

T (pointing to the left most circle): Who can show how to divide this shape into two pieces of the same size?

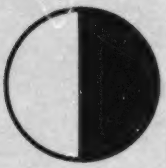
G9

Give a student a meterstick to indicate where the disc could be divided. Draw the line segment yourself while the student holds the meterstick in position.



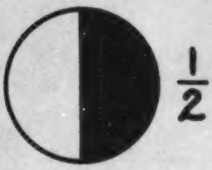
T: Who would like to color one-half of this shape red?

Show the student who volunteers how to use the side of the chalk to color a large region quickly.



T: Does anyone know how to write "$\frac{1}{2}$"?

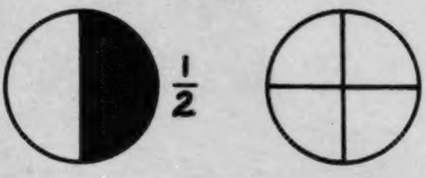
Point to the side of the circle to show a volunteer where to write "$\frac{1}{2}$".



NOTE: To prevent the rest of the lesson from dragging, one student could be coloring a portion of a disc while another is writing its fractional name on the board. Or you could be talking about how you want to divide the next disc while someone writes an equivalent fraction to one already on the board.

T (pointing to the second circle from the left): Who can show how to divide this shape into four pieces of the same size?

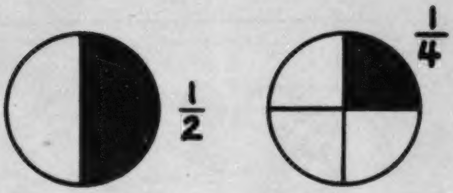
Let a student hold a meterstick in positions which indicate where the disc could be divided while you draw the appropriate line segments.



T: What part of the shape is each of these pieces?

S: One-fourth.

T: Who would like to color one-fourth of this shape green?
Who can write "$\frac{1}{4}$"?



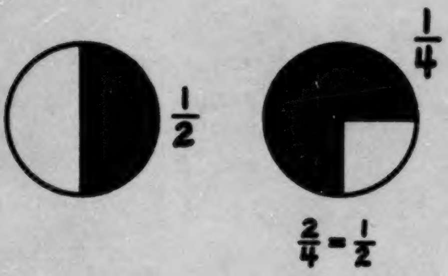
T: We colored one-half of this shape (point to the first circle) red; who would like to color one-half of this shape (point to the second circle) red?

While a student is coloring one-half of the disc, ask:

T: How many fourths make $\frac{1}{2}$? (Two)
Who can write "$\frac{2}{4}$" on the board?

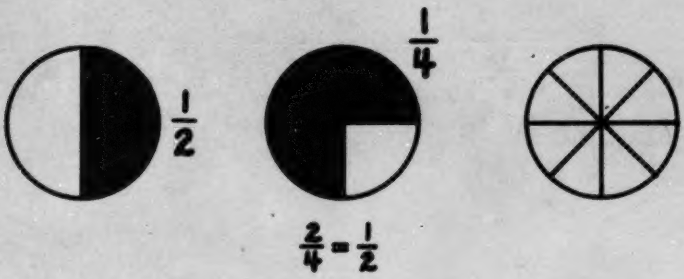
G 9

Let a student write "$\frac{2}{4}$" on the board and then write "$=\frac{1}{2}$" next to it yourself.



T (pointing to the third circle from the left): Who can show how to divide this shape into eight pieces all the same size?

Let a student hold a meterstick in positions which indicate where the disc could be divided while you draw the appropriate line segments. If a student suggests a way to divide the disc into eight pieces but the pieces are not the same size, trace how the disc was divided for four pieces and suggest dividing each of those pieces in half.

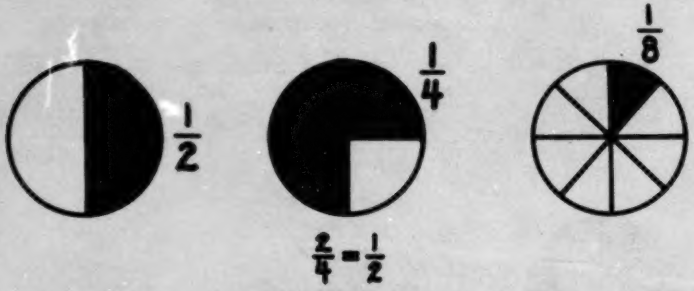


T: What part of the shape is each of these pieces?

S: One-eighth.

T: Who would like to color one-eighth of this shape?
Who can write "$\frac{1}{8}$"?

Let a student color one-eighth of the disc blue and let another write "$\frac{1}{8}$".



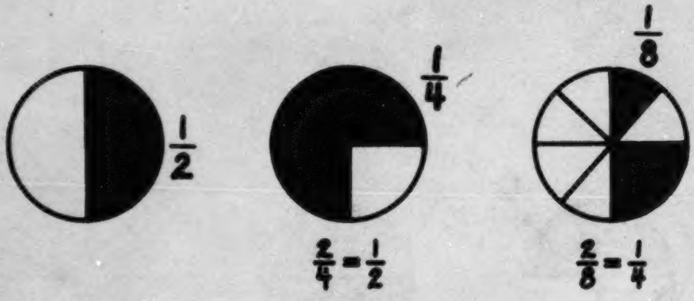
$\frac{1}{2}$
$\frac{1}{4}$
$\frac{2}{4} = \frac{1}{2}$
$\frac{1}{8}$

T: We colored one-fourth of this shape green (trace the green section of the second disc); who would like to color one-fourth of this shape (point to the third disc) green?

While a student is coloring the picture, ask:

T: How many eighths make $\frac{1}{4}$? (Two)
Who can write "$\frac{2}{8}$"?

Let a student write "$\frac{2}{8}$" on the board and then write "$= \frac{1}{4}$" next to it yourself.

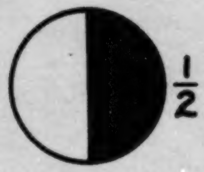


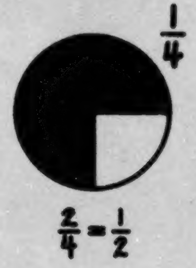
$\frac{1}{2}$
$\frac{1}{4}$
$\frac{2}{4} = \frac{1}{2}$
$\frac{1}{8}$
$\frac{2}{8} = \frac{1}{4}$

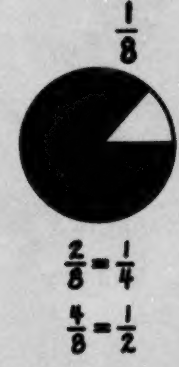
T: How many eighths make $\frac{1}{2}$? (Four)
Who can write "$\frac{4}{8}$"?

Let a student write "$\frac{4}{8}$" on the board and then write "$= \frac{1}{2}$" next to it yourself.

NOTE: If you think it would aid the students to continue coloring one-half of each shape red, do so. However, if it is unnecessary you would omit this extra coloring and save some time. The pictures in the lesson will show all the possible colorings.



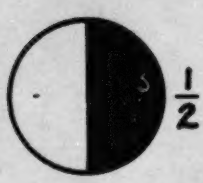

$\frac{1}{4}$
$\frac{2}{4}=\frac{1}{2}$

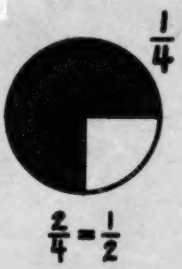

$\frac{1}{8}$
$\frac{2}{8}=\frac{1}{4}$
$\frac{4}{8}=\frac{1}{2}$

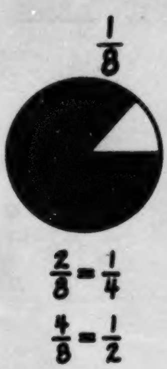
T: We divided this shape (point to the third disc) into eight pieces. How can we divide this shape (point to the fourth disc) into sixteen pieces all the same size?

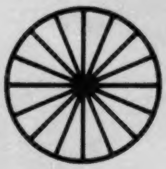
S: First divide it into eight pieces and then divide each piece in half.

If no one suggests this, suggest it yourself and let a student hold the meter-stick appropriately while you draw the line segments.

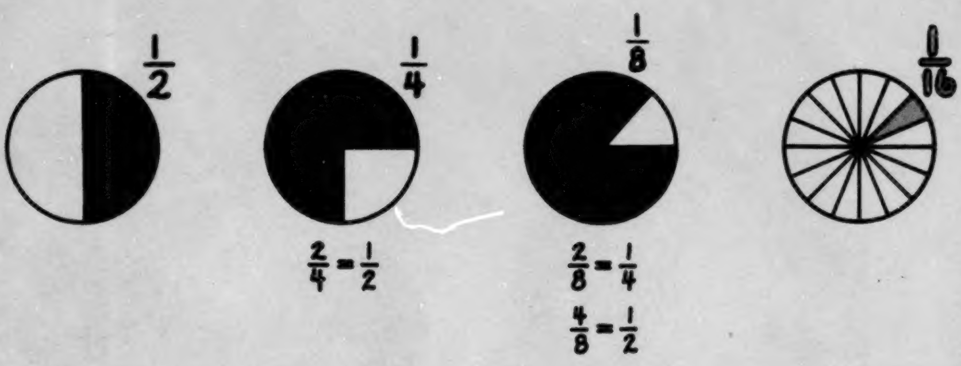



$\frac{1}{4}$
$\frac{2}{4}=\frac{1}{2}$


$\frac{1}{8}$
$\frac{2}{8}=\frac{1}{4}$
$\frac{4}{8}=\frac{1}{2}$



Ask someone to color one-sixteenth of the fourth disc yellow and ask another to write "$\frac{1}{16}$" on the board.


$\frac{1}{2}$
$\frac{1}{4}$
$\frac{1}{8}$
$\frac{1}{16}$
$\frac{2}{4} = \frac{1}{2}$
$\frac{2}{8} = \frac{1}{4}$
$\frac{4}{8} = \frac{1}{2}$

NOTE: In the following discussion, you could if you feel it is appropriate, color one-eighth of the fourth disc blue, one-fourth of it green, and one-half of it red. Again, let understanding and time concerns direct you.)

T (tracing the blue section of the third disc): How many sixteenths make $\frac{1}{8}$?

S: Two.

Write "$\frac{2}{16} = \frac{1}{8}$" under the fourth circle.

T (tracing the green section of the second disc): How many sixteenths make $\frac{1}{4}$?

S: Four.

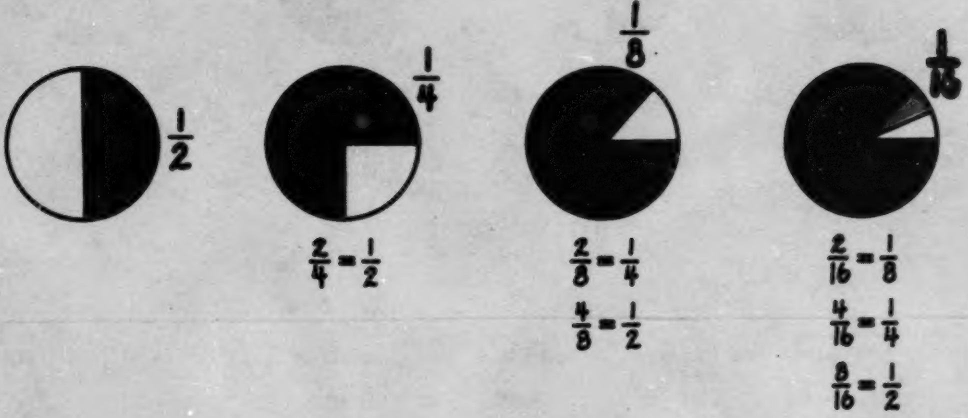
Write "$\frac{4}{16} = \frac{1}{4}$" under the fourth circle.

G 9

T (tracing the red section of the first disc): How many sixteenths make $\frac{1}{2}$?

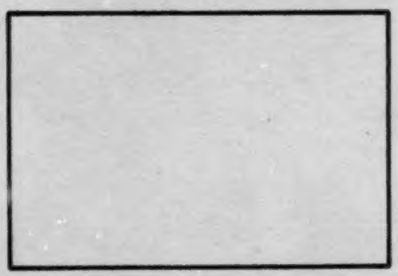
S: Eight.

Write "$\frac{8}{16} = \frac{1}{2}$" under the fourth circle.


1/2
1/4
1/8
1/16
2/4 = 1/2
2/8 = 1/4
4/8 = 1/2
2/16 = 1/8
4/16 = 1/4
8/16 = 1/2

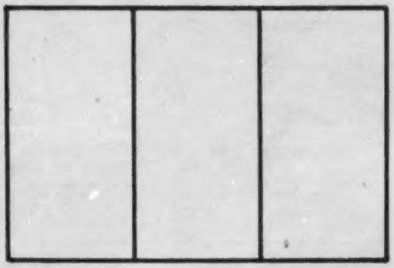
Exercise 2

Draw a rectangle on the board.



T: How can I divide this shape into three pieces of the same size?

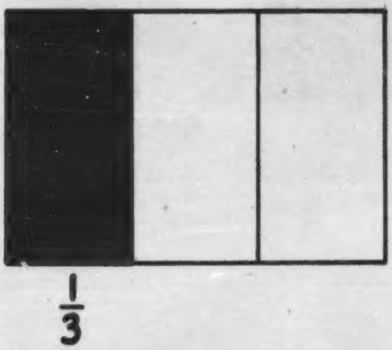
Let a student hold a meterstick in positions which indicate a way to divide the shape while you draw the appropriate line segments. One possibility is illustrated below.



T: What part of the shape is each of these pieces?

S: One-third.

T: Who would like to color one-third of the shape green?
Who can write "$\frac{1}{3}$"?

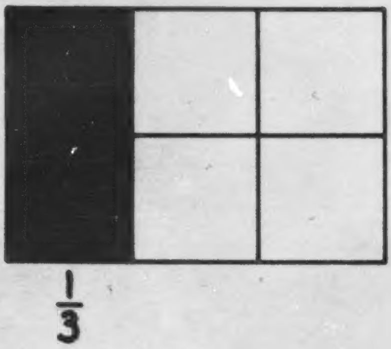


T: How can we divide the same shape into six pieces of the same size?

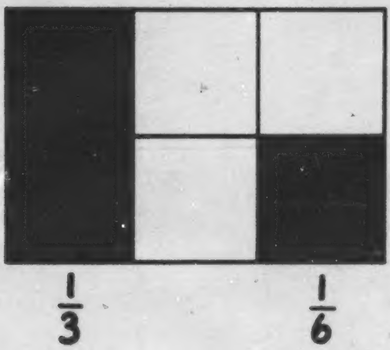
S: Divide each of the three pieces in half.

G 9

Again let a student indicate with a meterstick a way to divide the shape while you draw the appropriate line segment(s). One possibility is shown in the next illustration.



Ask someone to color one-sixth of the shape red and someone else to write "$\frac{1}{6}$".



T:	How many sixths make $\frac{1}{3}$?
S:	Two.
T:	How many sixths make $\frac{1}{2}$?
S:	Three.

Write the appropriate equalities on the board.

$$\frac{2}{6} = \frac{1}{3}$$

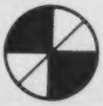
$$\frac{3}{6} = \frac{1}{2}$$

Worksheets G9*, **, ***, and **** are available for individual work.

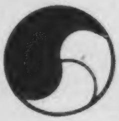
Name ______________________ G 9 *

Color exactly one-half of each shape. Use the picture to write another name for $\frac{1}{2}$.

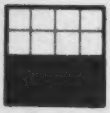
EXAMPLE



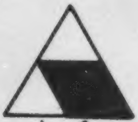
$\frac{1}{2} = \frac{4}{8}$



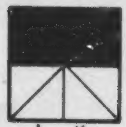
$\frac{1}{2} = \frac{2}{4}$



$\frac{1}{2} = \frac{8}{16}$



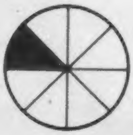
$\frac{1}{2} = \frac{2}{4}$



$\frac{1}{2} = \frac{4}{8}$

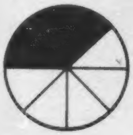
Name ______________________ G 9 **

Color one-eighth of this shape.



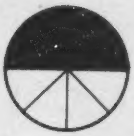
$\frac{1}{8}$

Color three-eighths of this shape.



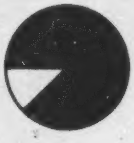
$\frac{3}{8}$

Color four-eighths of this shape.



$\frac{4}{8}$

Color seven-eighths of this shape.



$\frac{7}{8}$

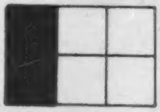
Circle the name for $\frac{1}{2}$.

$\frac{1}{8}$ $\frac{3}{8}$ [✓ $\frac{4}{8}$] $\frac{7}{8}$

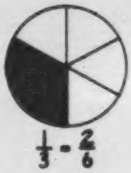
Name ______________________ G 9 ***

Color exactly one-third of each shape. Use the picture to write another name for $\frac{1}{3}$.

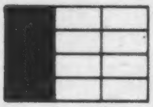
EXAMPLE



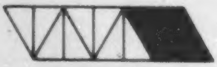
$\frac{1}{3} = \frac{2}{6}$



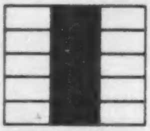
$\frac{1}{3} = \frac{2}{6}$



$\frac{1}{3} = \frac{4}{12}$



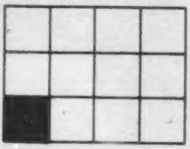
$\frac{1}{3} = \frac{4}{12}$



$\frac{1}{3} = \frac{5}{15}$

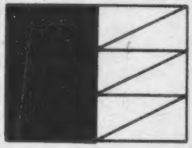
Name ______________________ G 9 ****

Color one-twelfth of this shape.



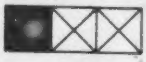
$\frac{1}{12}$

Color six-twelfths of this shape.



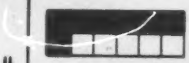
$\frac{6}{12}$

Color four-twelfths of this shape.



$\frac{4}{12}$

Color seven-twelfths of this shape.



$\frac{7}{12}$

Circle the name for $\frac{1}{2}$: $\frac{1}{12}$ [✓ $\frac{6}{12}$] $\frac{4}{12}$ $\frac{7}{12}$

Circle the name for $\frac{1}{3}$: $\frac{1}{12}$ $\frac{6}{12}$ [✓ $\frac{4}{12}$] $\frac{7}{12}$

G10 Length #3

CAPSULE LESSON SUMMARY

Review centimeter and meter units of length and estimate the heights of some people in the room. Find things in the room which measure about 1 m, about 10 cm, and about 1 cm. Check estimates by measuring with a centimeter tape measure. Draw a zigzag with three pieces of specified lengths and then calculate the length of the entire zigzag path.

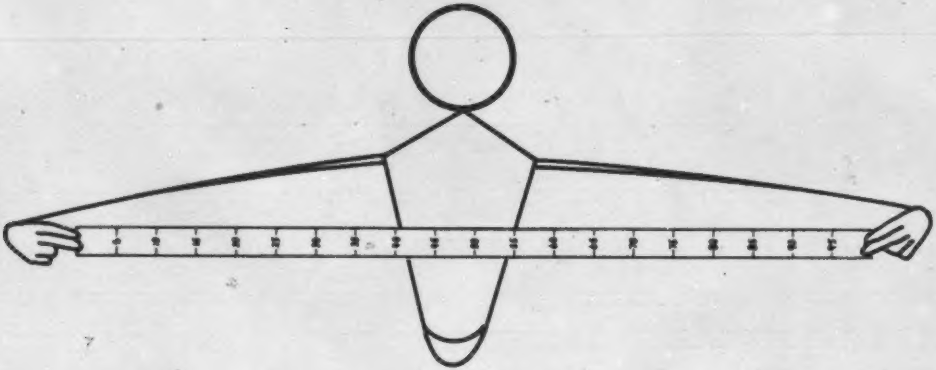
MATERIALS

Teacher:	Centimeter tape measure; meter stick or centimeter ruler
Student:	Worksheets G10* and **; centimeter tape measure; straightedge; pencil

DESCRIPTION OF LESSON

Exercise 1

Write "1 m" on the board and display 1 meter with a tape measure.



T: I wrote the length of this tape measure on the board.
Do you know what the "m" is for?

S: Meter.

T: Do you know how many centimeters that is?

S: 100 centimeters.

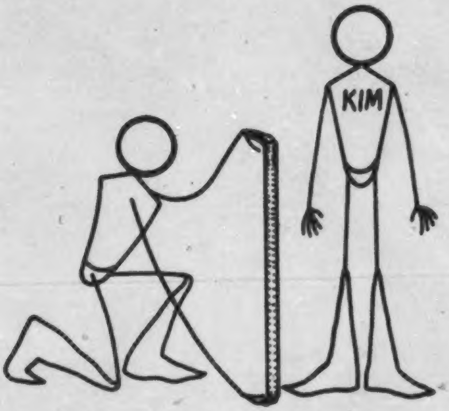
Record this equivalence on the board.

1 m = 100 cm

Ask one of your students to stand up. Suppose her name is "Kim".

T: Do you think Kim is taller or shorter than 1 meter?

Let students give their opinions and then compare the length of the tape measure (1 m) to the height of the particular student (Kim).



T: Kim is taller than 1 meter. Do you think she is taller or shorter than 2 meters?

S: Shorter.

With a students' help display a length of 2 meters using two tape measures next to the particular student (Kim).

T: Do you think that there is anyone in the room shorter than 1 meter?

Compare the length of the tape measure (1 m) to the height of any students suggested.

T: Do you think that there is anyone in the room taller than 2 meters?

Perhaps some of your students will suggest you or any adult in the room. You may want to let the students compare your height to the length of 2 meters.

T: Look around the room and see if there is something which measures 1 meter.

Look at the difference between 1 meter and the actual measurement of each item suggested. A sample dialogue follows.

S: The cabinet.

T: How do you want to measure the cabinet?

S: The width across (as indicated by a hand motion).

T (measuring the width of the cabinet): 106 centimeters. Is that longer than or shorter than 1 meter?

S: Longer.

T: 106 centimeters is how much longer than 1 meter?

S: 6 centimeters.

Continue in this manner until five or six things have been measured. Commend students on good estimates, but do not criticize bad guesses.

Repeat this activity looking for things in the room which measure 10 centimeters in some direction and which measure 1 centimeter in some direction.

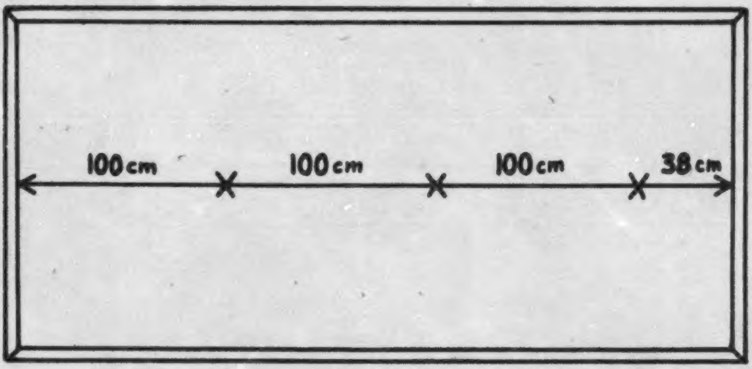
T: Everyone try this: hold your hands 1 meter apart.
. . . hold two fingers 10 centimeters apart.
. . . hold two fingers 1 centimeter apart.

Check at least one of the three estimates with each student.

T: How long do you think the chalkboard is?

G 10

Write the estimates on the board, and then measure the length of the chalkboard in the usual way. Ask a student to help you. For example, your chalkboard might be like the one shown below.



Acknowledge the students who made the closest guess, determining with the class how close that guess is.

Exercise 2

Distribute blank paper and centimeter tape measures. Students should have straightedges of some kind.

T: I want you to draw a zigzag with three pieces on your paper. The first piece should be 25 centimeters; the second, 15 centimeters; and the third, 10 centimeters. There are many ways to draw such a zigzag.

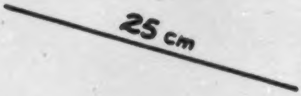
Write the measurements on the board.

25 cm
15 cm
10 cm

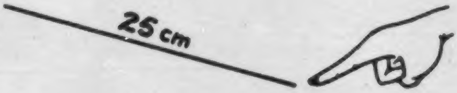
Call the class' attention to the board after most of the students have drawn a zigzag.

T: Let's draw on the board a zigzag with these measurements.
Who will help me draw the 25 centimeter piece?

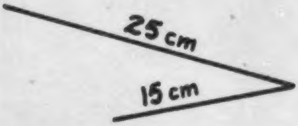
A student who volunteers should show you the "25 cm" mark on the meter stick and then should hold the stick in place while you draw the segment. Label it "25 cm". One possibility is illustrated below.



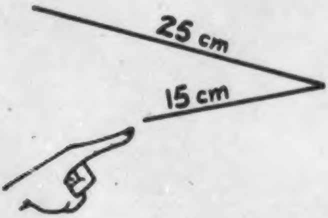
T: The second piece is to be 15 centimeters. I'll start it at the end of the first piece (point to it).



Ask another student to help you. This student should show you the "15 cm" mark on the meter stick and then should hold the stick in place while you draw the segment. Label it "15 cm". One possibility is illustrated below.

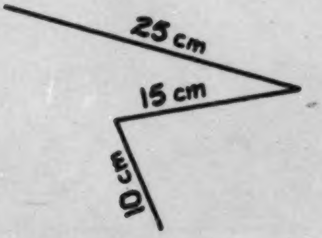


T: The third piece is to be 10 centimeters. I'll start it at the end of the second piece (point to it).



G 10

As before, draw the third segment with student's help.



T: How can we find length of the zigzag (trace the whole thing)?

S: Carefully hold a tape measure to it.

S: Add: 25 + 15 + 10.

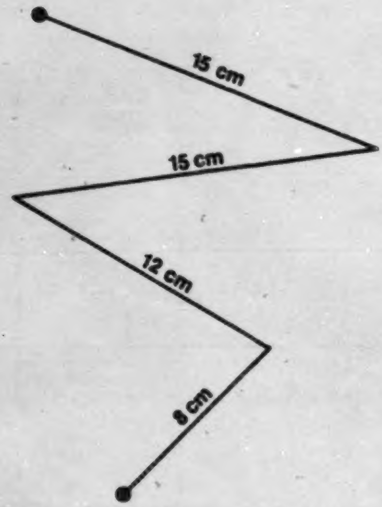
As soon as a student suggests adding the lengths of the three pieces, write the addition problem on the board and invite a student to do the calculation at the board.

$$\begin{array}{r} \overset{1}{2}5\text{ cm} \\ 15\text{ cm} \\ +\ 10\text{ cm} \\ \hline 50\text{ cm} \end{array}$$

Worksheets G10* and ** are available for individual work. Explain to the students that they are to draw a zigzag which has four pieces and which is 50 centimeters long. Remind them that the zigzag on the board is 50 centimeters long but has only three pieces.

Name ____________________ G 10 *

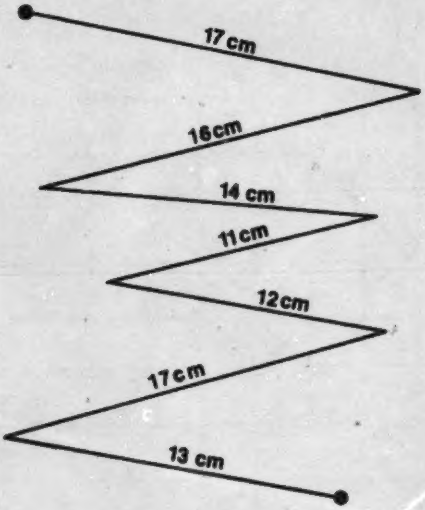
Draw a zigzag with four parts. The total length of the zigzag should be 50 cm.


15 cm
15 cm
12 cm
8 cm

Many solutions are possible.

Name ____________________ G 10 **

Draw a zigzag which is one meter (100 cm) long.


17 cm
16cm
14 cm
11cm
12cm
17cm
13 cm

Many solutions are possible.

G 11

G11 Area #4

CAPSULE LESSON SUMMARY

Solve an area puzzle in which all shapes having a specified area are colored one color.

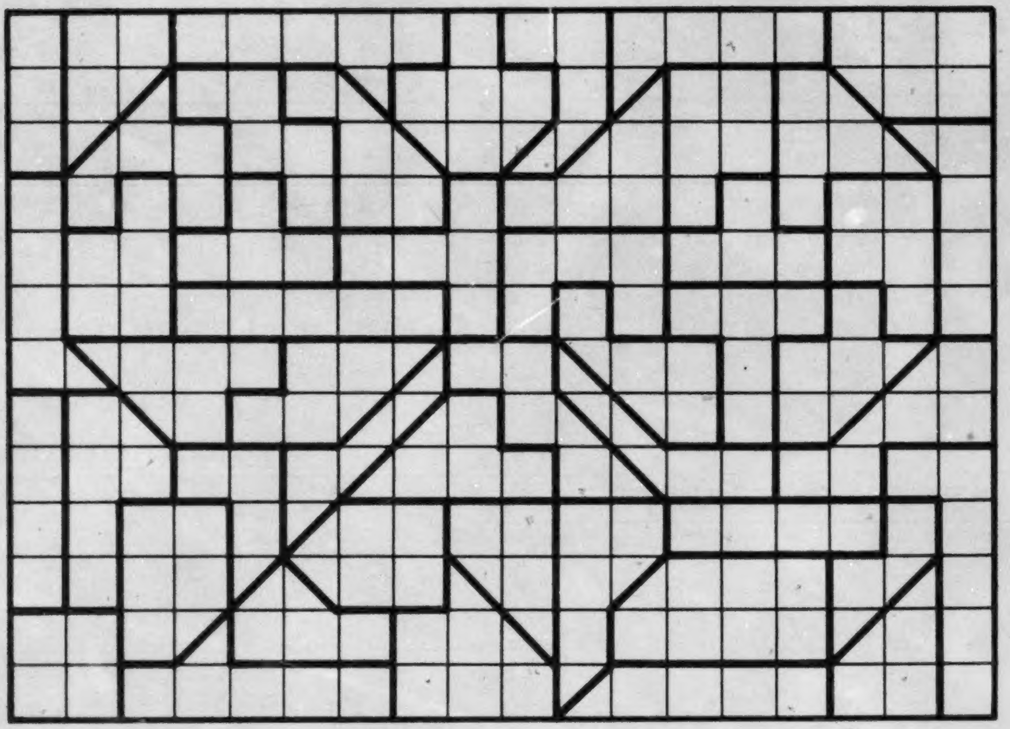
MATERIALS

Teacher: Geometry Poster #10 for UPG-III; red crayon or felt-tip pen

Student: Worksheets G11* and **; colored pencils

DESCRIPTION OF LESSON

Tape Geometry Poster #10 to the board.



Let students discuss what they see in the poster. Do not comment on their ideas.

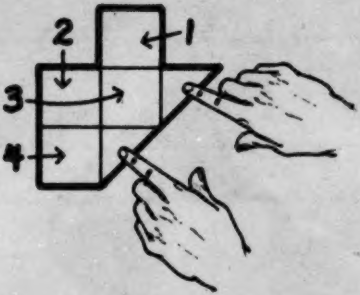
T: To see what's in the picture, we need to find all the shapes which have an area of 5 squares. We'll color these red.

Write the following information on the board. Draw the square the same size as those in the poster.

Area: 5 □

Color: red

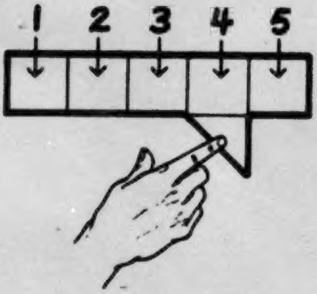
Invite students one at a time to trace the shapes with an area of 5 squares. After a student traces a shape, count the squares in the shape aloud. Count the whole squares first and then pair halves when possible. For example in this shape (see the next illustration), count: 1, 2, 3, 4,



Do not write these numerals on the poster. They are here to indicate a manner of counting.

. . . and two halves make 5.

A student may suggest a shape whose area is not 5 squares; count the squares to "discover" this fact. For example in this shape (see the next illustration), you would count: 1, 2, 3, 4, 5,



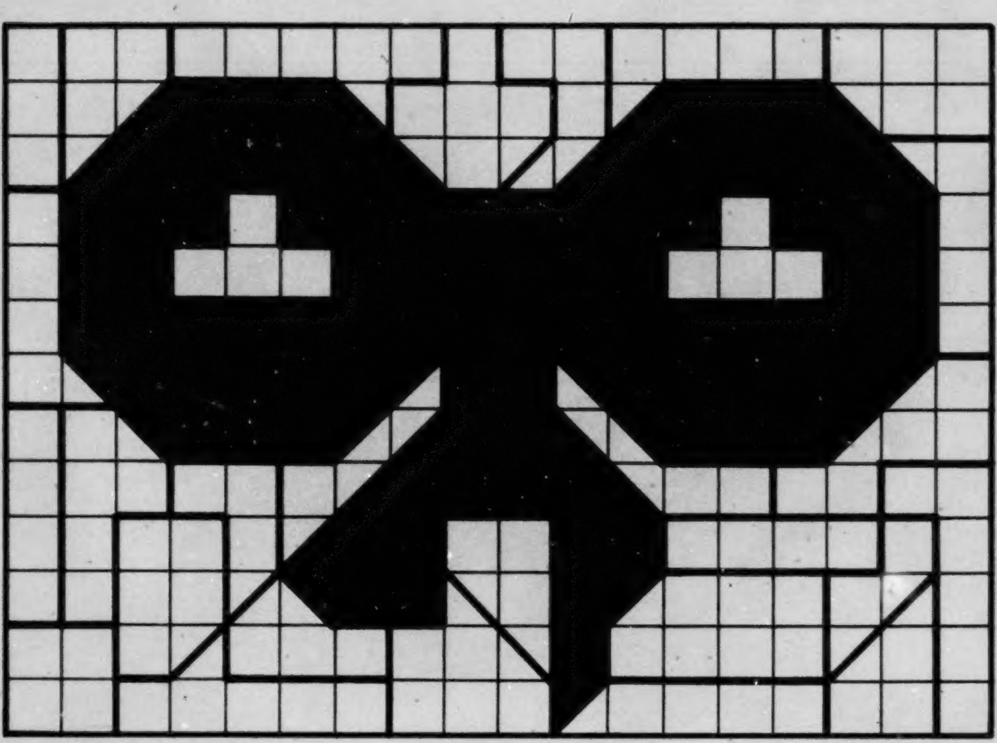
Do not write these numerals on the poster. They are here to indicate a manner of counting.

. . . $5\frac{1}{2}$.

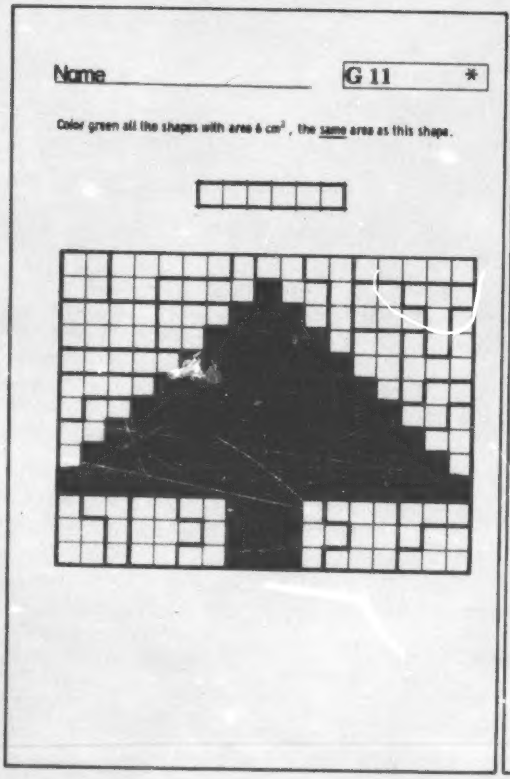
G 11

Let a student who finds a shape of area of 5 squares color that shape with a red marker. Several red markers would be useful here. Draw an "X" lightly with pencil through suggested shapes whose areas are not 5 squares.

When all the shapes of area 5 squares have been found, the students will discover a Christmas bow.



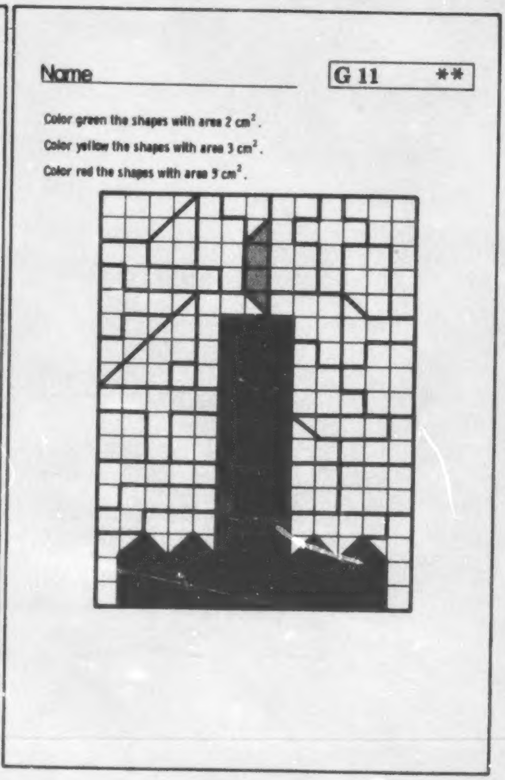
Worksheets G11* and ** are available for individual work. Students will need colored pencils (or crayons).



Name

G 11 *

Color green all the shapes with area 6 cm², the same area as this shape.



Name

G 11 **

Color green the shapes with area 2 cm².

Color yellow the shapes with area 3 cm².

Color red the shapes with area 5 cm².

G 12

G12 Hiking Trails #1

CAPSULE LESSON SUMMARY

Find trails between campsites on a map of a campground. Discuss what is meant by a round-trip trail and find round-trip trails which do not take the same path more than once. Examine a given round trip trail and find ways to shorten it into trails which are still round-trips. Introduce the idea of a cycle and find ways of partitioning a round-trip trail into cycles.

MATERIALS

Teacher:	Geometry Poster #11 for UPG-III; colored felt-tip pens or crayons
Student:	Worksheets G12* and **; colored chalk

NOTE: There are two copies of the same map on the poster. Cut the poster in half before the lesson begins so that they may be displayed separately.

DESCRIPTION OF LESSON

T: Have any of you ever gone camping?

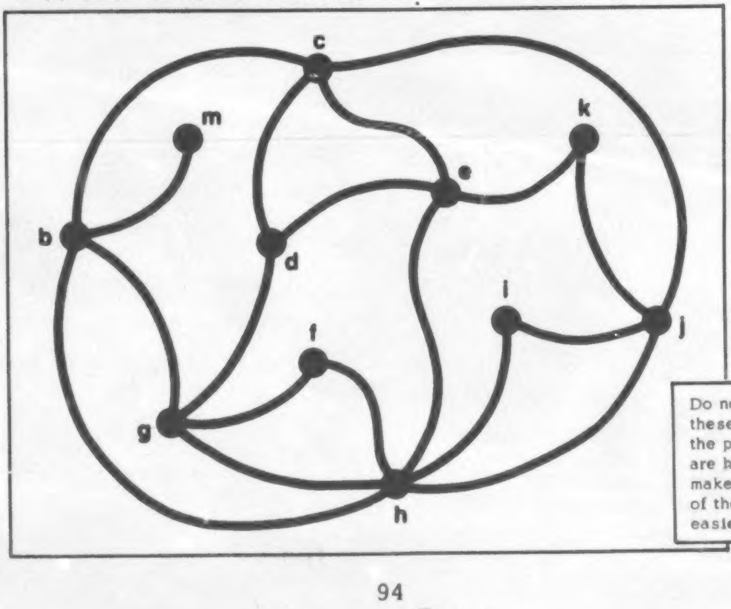
Were there some hiking trails?

Let students freely relay their experiences for a few minutes.

T: Today I want to tell you about a campground with some hiking trails.

This is a map of that campground.

Tape one copy of the map on Geometry Poster #11 to the board.



Do not write these letters on the poster. They are here just to make the description of the lesson easier to follow.

T: What do you think the dots are for?

S: Campsites.

Invite a student to count aloud the campsites while pointing to each.

S: 1, 2, 3, . . . 11. There are eleven campsites.

Trace several cords in the map as you say:

T: These are paths which were made through the woods so people could walk from one campsite to another. How many paths are there?

Invite a student to count the paths while pointing to each. Carefully watch the student and help if necessary. There are eighteen paths.

NOTE: Although this method probably will not be suggested, a way of counting paths is to count all the paths from each campsite and then to divide by 2 since each path will have been counted twice. Commend any student who suggests this approach.

T: Suppose we are at this campsite (b) and we want to take a hike. Because we want to see as much of the campgrounds as we can, we will not take a path more than once. Who can trace the paths in a trail we could follow on our hike?

Elicit many examples of trails both long and short. Do not allow a student to use a path more than once in a trail. Emphasize that for us a trail does not make use of a path more than once. Very likely some students will suggest trails which return to b.

T: Many of you have shown trails which come back to this campsite (b). If this is our campsite, it is reasonable that we might want to end the hike back at this campsite. These trails have a special name; we call them "round-trip trails."

Let students comment on the reason for this name as you write it on the board.

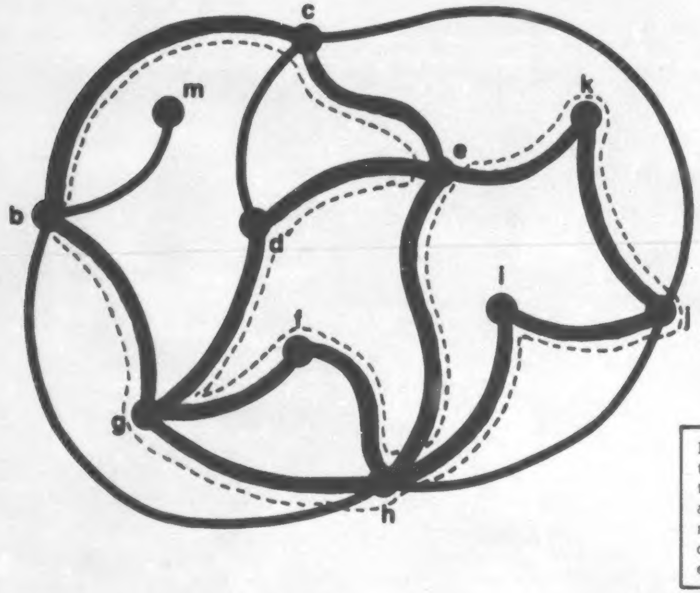
round – trip trails

Invite several students to trace round-trip trails on the poster. Then point to the dot labeled "m" in the previous illustration and ask:

T: Can we find a round-trip trail which begins here (m)?

S: No, because we would need to leave and return by the same path.

Draw this red trail as you would walk if you were following it. There are many ways to do this; one is indicated by the dotted curve in the next illustration.



Do not write these letters on the poster. They are here just to make the description of the lesson easier to follow.

Count aloud the paths in the red trail.

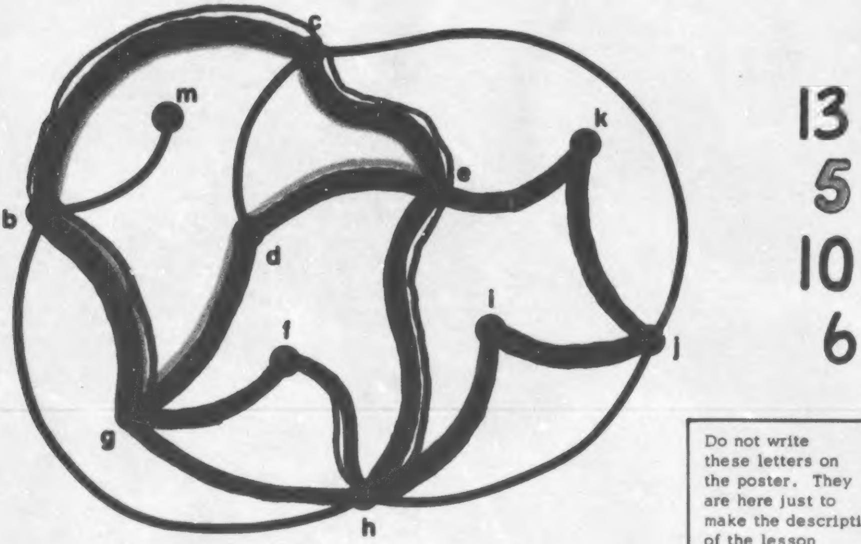
T: We'll say that the length of this trail is 13 (paths).

Write "13" in red to the side of the map.

T: Who can trace a shorter round trip using only paths followed in this red trail?

When a student traces a correct sub-trail, distinguish it by drawing it in color not yet used on the poster. Record the length of a sub-trail in the same color in which the sub-trail is drawn. Continue until several sub-trails have been suggested.

These are some of the many possibilities.



Do not write these letters on the poster. They are here just to make the description of the lesson easier to follow.

As in the dialogue which follows, draw the class' attention to any sub-trail starting at b which visits other campsites only once each and then returns to b; for example, the yellow and the blue trails in the preceding illustration. Draw such a trail if one has not been suggested.

T: Can you find a shorter round trip using only paths followed in the yellow trail? (No)

Can you find a shorter round trip using only paths followed in the blue trail? (No)

Observe with your class that if you followed the red trail starting and ending at b, you would visit campsites e, g, and h more than once.

T: Are there campsites we would visit more than once if we follow the yellow trail?

S: We would start and end at the same campsite, but all the others we would visit only once.

Ask the same question concerning the blue trail.

T: So except for where we start, we never come to a campsite more than once following the yellow or the blue trail. These trails have a special name; they are called "cycles."

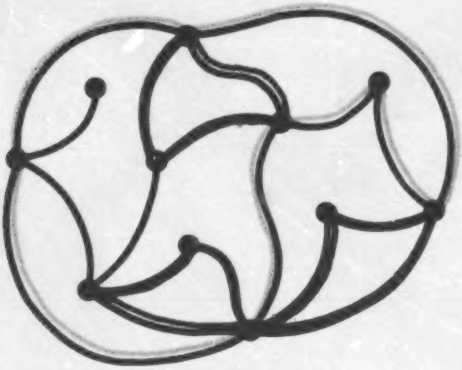
Write this name on the board.

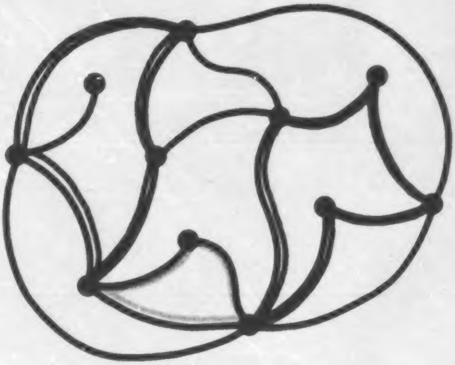
round-trip trails
cycles

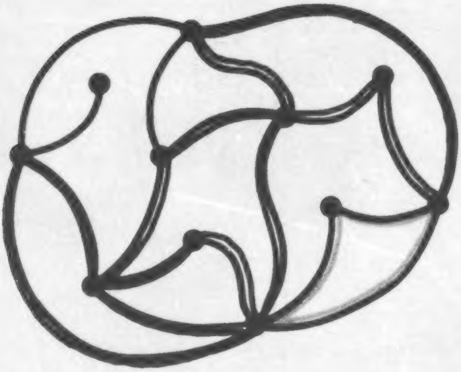
Remove the poster and tape the other copy to the board.

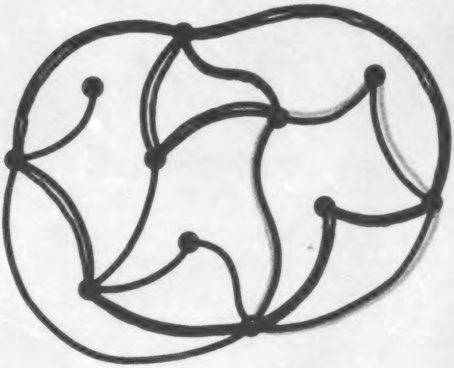
T: Imagine we are working at this campground and we have the job of planning some trails which are cycles. We must show people which way to walk by putting color-coded signs along each trail. We don't want signs of different colors along any one of the paths because that might confuse the hikers. How can we set this up?

Let the students collectively divide the map as much as possible into non-overlapping cycles. There are many ways to do this ; some are given below.





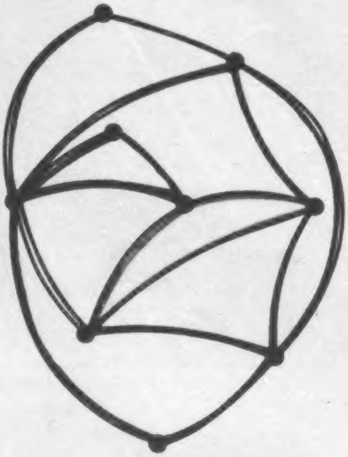




Worksheets G12* and ** are available for individual work.

Name __________________ G 12 *

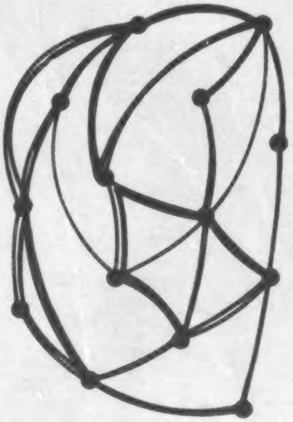
Find cycles which do not overlap. Use different colors to show different cycles. Try to use each path in some cycle.



Other solutions are possible.

Name __________________ G 12 **

Find cycles which do not overlap. Use different colors to show different cycles. Try to use as many paths as possible in some cycle.



Other solutions are possible, but none that uses more paths.

G13 Hiking Trails #2

CAPSULE LESSON SUMMARY

Review the notions of trails in a campground, round-trip trails and cycles. Divide the paths in a campground into cycles which do not overlap and use as many paths as possible in some cycle. Using the cycle decomposition of the campground find a round-trip trail starting at a given campsite and using as many paths as possible.

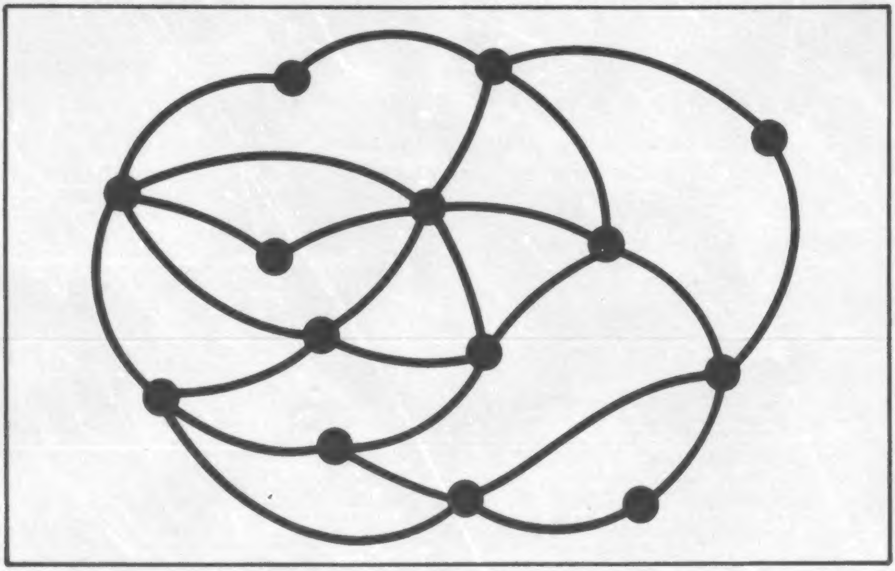
MATERIALS

Teacher:	Geometry Poster #12 for UPG-III; colored felt-tip pens or crayons
Student:	Worksheets G13* and **; colored pencils

NOTE: There are two copies of the same map on the poster. Cut the poster in half before the lesson so that they may be displayed separately.

DESCRIPTION OF LESSON

Tape one copy of the map on Geometry Poster #12 to the board.



T: This is a map of another campground. How many campsites are there?

Invite a student to count the campsites pointing to each.

G 13

S: 1, 2, 3, . . . , 14. There are fourteen campsites.

T: How many paths are there?

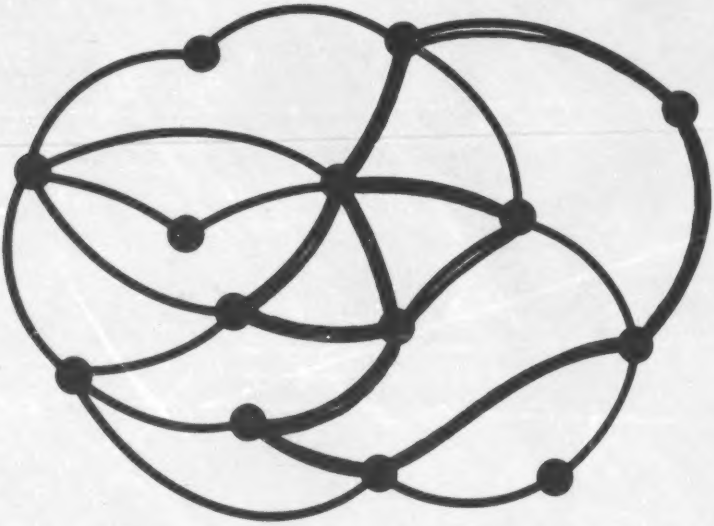
Put checkmarks lightly in pencil by the paths as a student counts them to insure that no path is counted twice and that no path is overlooked. Carefully erase the checkmarks when the number of paths (twenty-five) has been determined.

T: We were looking at a map of a campground last week. What were we trying to find?

S: Trails.

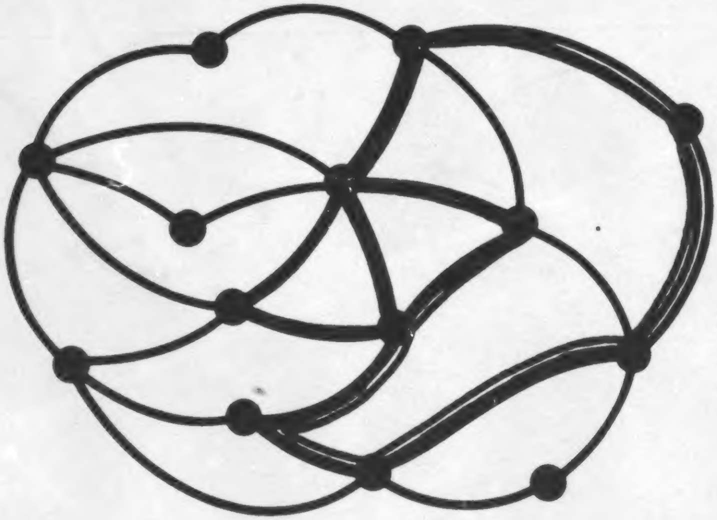
T: Remember, a trail does not follow any path more than once. Who can trace a trail at this campground?

Elicit many examples of trails both long and short. Very likely some student will suggest trails which start and end at the same campsite; choose one of these round-trip trails and draw it on the poster with a felt-tip marker. There are many possibilities; one is shown in blue below.



T: Can you find a shorter round-trip using only paths already in this blue trail?

Let students trace sub-trails until a cycle is suggested. (Remember, a cycle is a round-trip trail which starts at a campsite, visits some (not necessarily all) other campsites once each, and then returns to the original campsite.) In the next illustration, the red trail is a cycle and a sub-trail of the blue round-trip trail.



T: Can we find a shorter round trip using only paths followed in the red trail?

S: No.

T: Do you remember what we call trails like the red one?

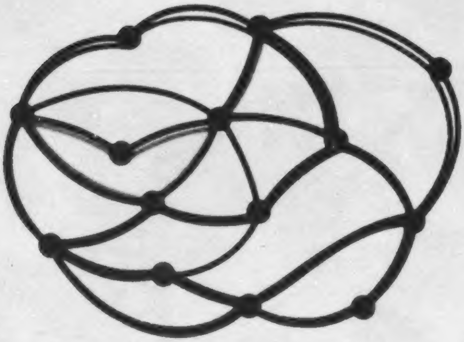
S: Cycles.

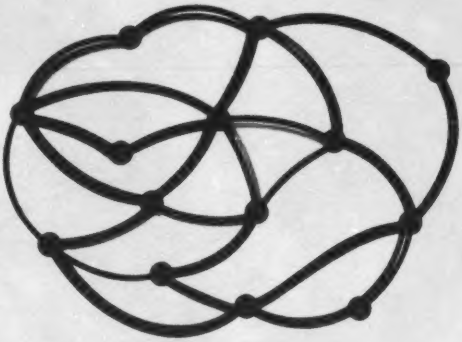
Remove the poster and tape the other copy to the board.

G 13

T: How can we divide this campground so that no path is in two different cycles? Let's try to use many paths in some cycles.

Let the students progressively divide the map into cycles. There are many possibilities; two different ones are shown below.



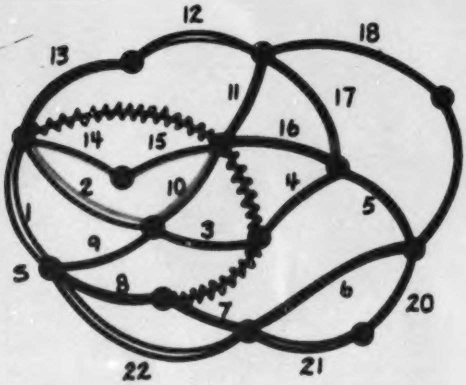


NOTE: The picture on the right shows one way to decompose this map into cycles using all but two of the paths. There is no way to include more than twenty-three (25 - 2) paths in cycles for this picture.

Scribble through (see the next illustration) paths not included in any of the cycles in your picture. Tell your students to pretend that these paths have been closed or blocked off. If you chose to draw the picture on the board instead of using the poster, simply erase the paths not included in any of the cycles.

T: Suppose you are visiting this campground and staying at this campsite (point to any one of the dots and label it "s"). Is there a way to walk along all the paths which are not closed and then to return to your campsite? Remember you want to walk along each path only once.

Let two students, one at a time, trace a trail which begins and ends at s and which includes all the paths (which are not closed). As the first student traces a trail, number the paths lightly in pencil in the order suggested. Check to see that each path has one numeral by it and that the trail returns to s. Then carefully erase the numerals and let the other student trace a trail. There are many possibilities. One way of transversing the paths for each of the pictures in the previous illustration is given below.


12
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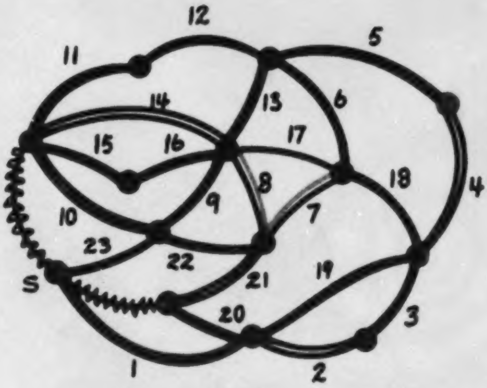
12
11
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4
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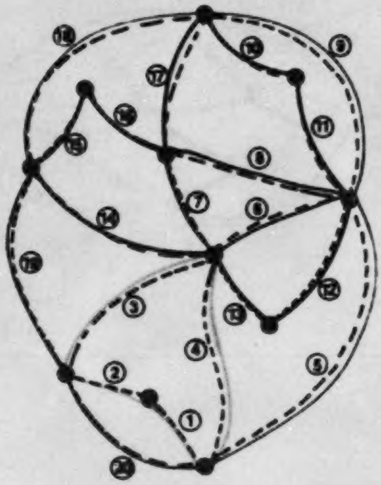
9
7
23
22
19
3
s
21
20
1
2

Worksheets G13* and ** are available for individual work. Check to make sure your students can identify each cycle by color on Worksheet G13*.

Name ______________________

G 13 *

Find a trail which uses every path just once.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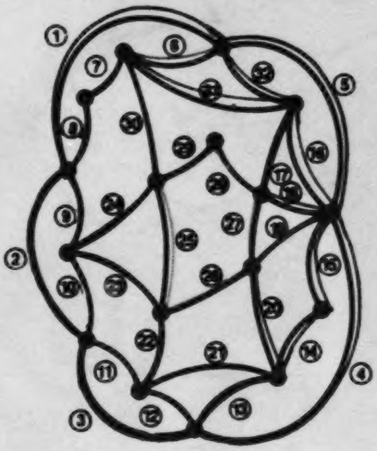
Follow the numbered paths in order to find a trail using every path exactly once.

Name ______________________

G 13 **

Find cycles which do not overlap. Use different colors to show different cycles.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Find a trail which uses every path just once.

Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades
Part III

Workbooks

CEMREL, Inc.

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WORKBOOKS

INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the other content strands. In the Workbook strand, however, it is this individualized work which becomes the principal mode of the majority of lessons. It has been our goal in this strand to provide the students with opportunities:

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking;
- to learn how to read and use mathematics workbooks.

The following workbooks are provided:

WHICH ROAD?
FESTIVAL OF PROBLEMS #1
FISHING FOR NUMBERS
ROAD MAP
FESTIVAL OF PROBLEMS #2
20? 100?

...and one story-workbook is provided:

ROLLERSKATING 37.

The six workbooks contain problems of varying levels of difficulty all in one booklet. Approximately the first ten pages of each booklet have easy problems, the next ten to twelve pages have problems of average level difficulty, and the last ten pages have more challenging problems. For each workbook, we suggest that all students start work at the easiest level of difficulty (i.e. on page 2) and then work through as many pages as they can handle during the one or two lessons scheduled for that workbook. In an average class we estimate that about two-thirds of the students will finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the booklet. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and suggested collective lessons which present the workbook to the whole class or provide a warm-up activity on a problem similar to one found in the workbook.

Eleven of the eighteen lessons in the strand have the individual work in the workbooks as their main activity. The story-workbook ROLLERSKATING 37, used in two lessons, introduces the student to congruence classes of whole numbers and provides early notions of division. Two lessons (W3 and W8) support the addition, subtraction, and ordering of integers. The other three lessons present detective stories where there is one or two secret numbers and several clues which lead to their discovery. The clues review many ideas from the other content strands as well as involve students in new situations involving patterns and counting techniques.

Use of the Workbook Strand for Evaluation Purposes*

The workbooks provide an excellent instrument to assess the progress of your students on a weekly basis. You may not feel that it is necessary to check every page and problem for each student. You should develop the sort of procedures for checking students' work with which you are most comfortable. These may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at the first few pages to be sure the students have understood the general idea of the problems in that particular book.

A progress chart is provided on pages viii and ix to help you keep track of

1) where each of your students is on any particular workbook day,

---

* See also Role and Use of Workbooks in Section 1 of the General Introduction.

2) how much each student has completed in a workbook,

3) your personal degree of satisfaction with the student's work in each workbook,

4) which students are showing improvement from one workbook to the next,

and

5) which students are having difficulty with even the easiest problems and would benefit from more individual attention during the lessons.

Here is a list of some of the important points to keep in mind for workbooks.

- All students start at the beginning of each workbook and progress as far as they can.
- All students begin a workbook on the same day, even if some students have not finished work on the preceding workbook.
- Not all students are expected to complete a given workbook. Only the faster students will reach the most challenging problems. The slower students may succeed only to do the easiest problems, although you should not assume that this will automatically be the case--surprises are not at all uncommon in this area.
- You should read the introductory material for each workbook and give the short collective lesson(s) described.

Workbook Progress Chart

STUDENT'S NAME	WHICH ROAD?	FESTIVAL OF PROBLEMS #1	FISHING FOR NUMBERS

Workbook Progress Chart

ROAD MAP	ROLLERSKATING 37	FESTIVAL OF PROBLEMS #2	20? - 100?

W1 WHICH ROAD? (Lesson 1)

CAPSULE LESSON SUMMARY

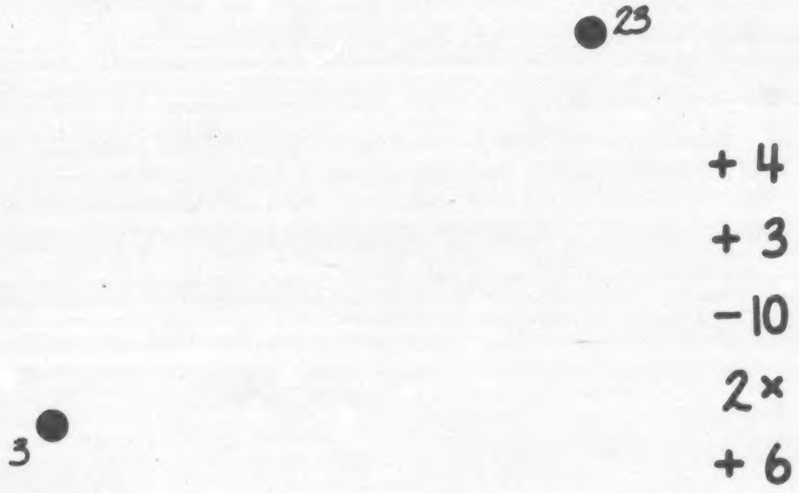
Investigate which of five functions could be used to build a road between two given numbers if only one kind of arrow is used throughout the road. Begin the Workbook WHICH ROAD?.

MATERIALS

Teacher: Colored chalk
Student: Workbook WHICH ROAD?; pencil; colored pencils

DESCRIPTION OF LESSON

Write this information on the board.



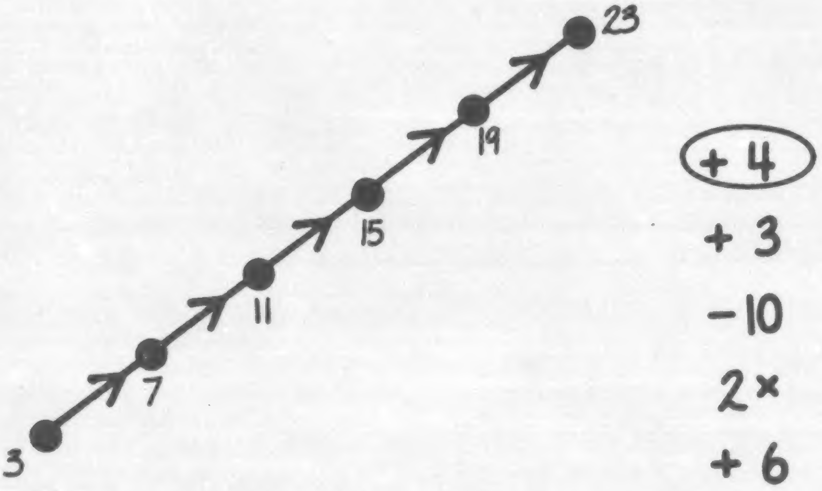
T: Suppose that we want to build a road between 3 and 23 using the same kind of arrow throughout and that these are our choices (point to the list of functions). Could we build such a road?

S: Yes.

T: Who would like to build a road between these numbers? You can start at 3 and go to 23 or you can start at 23 and go to 3.

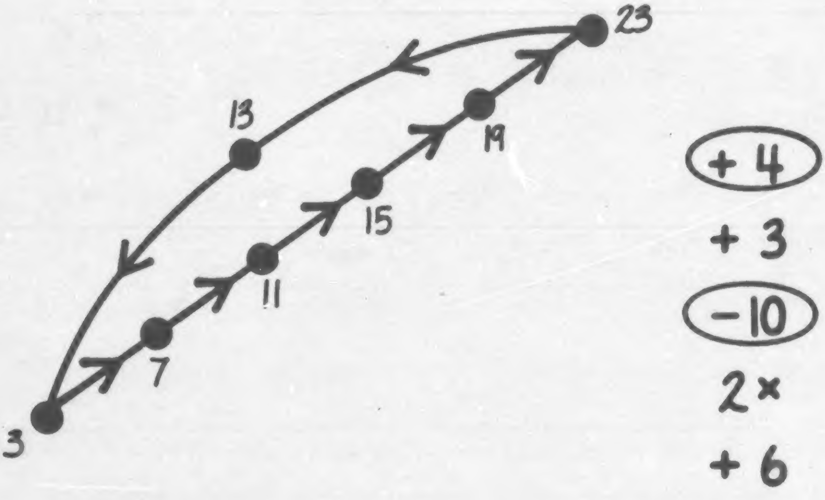
W1

Ask a student who volunteers to circle in red his or her choice of function and then to build the road in the same color. Suppose the student selects +4.



T: This is one way to build a road between 3 and 23. How many arrows were used? (Five) Could we build a road with less than five arrows?

Solicit a volunteer to build a different road between 3 and 23 using another kind of arrow. Perhaps the student will choose −10.



T: How many arrows are in the -10 arrow road? (Two) The -10 arrow road is shorter than the +4 arrow road. Could we build a road between 3 and 23 using one of the other choices for arrows?

Your class should discover that the road could not be built using any of the other three choices.

Distribute copies of the Workbook WHICH ROAD?. Emphasize that on each page of the workbook the students are to build a road between the given numbers. They may only use the functions which are listed on that page, and each road must have only one type of arrow.

Explain that as soon as they have built a road between the two labeled dots, they have solved the problem, but that you would like for them to try to build the shortest possible road.

NOTE: On each page of the workbook there are several functions that could be used successfully to build the road and at least one function that will not "work." Of those functions that do "work," some will give shorter roads than others.

At least one choice on each page is a subtraction function, although the students are not expected to know a subtraction algorithm at this time in the school year. To students seeking help in building a road with subtraction arrows, suggest using a number line, counting backwards, or any other appropriate method.

Allow approximately 30 minutes for the students to work individually on the Workbook WHICH ROAD?; then collect them for use in W2.

W 2

W2 WHICH ROAD? (Lesson 2)

CAPSULE LESSON SUMMARY

Investigate which of five functions could be used to build a road between two given numbers if only one kind of arrow is used throughout the road. Continue working in the Workbook WHICH ROAD?. (This is the second of two lessons using this workbook.)

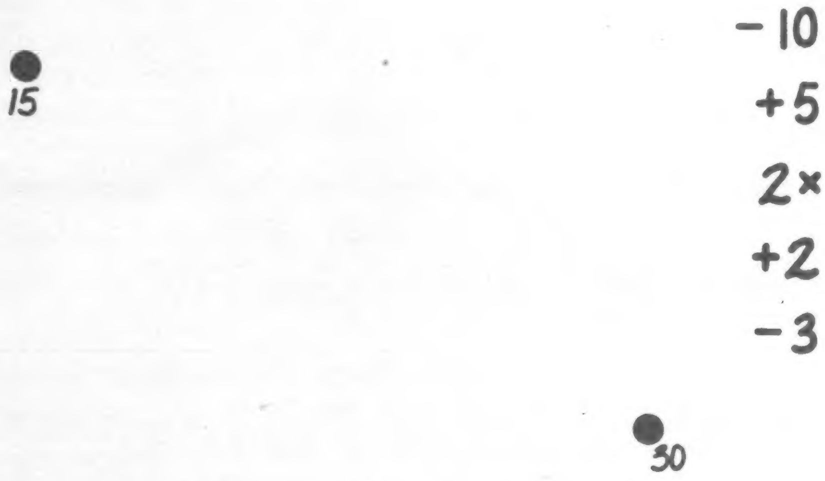
MATERIALS

Teacher: Colored chalk

Student: Workbook WHICH ROAD?; pencil; colored pencils

DESCRIPTION OF LESSON

Write this information on the board.

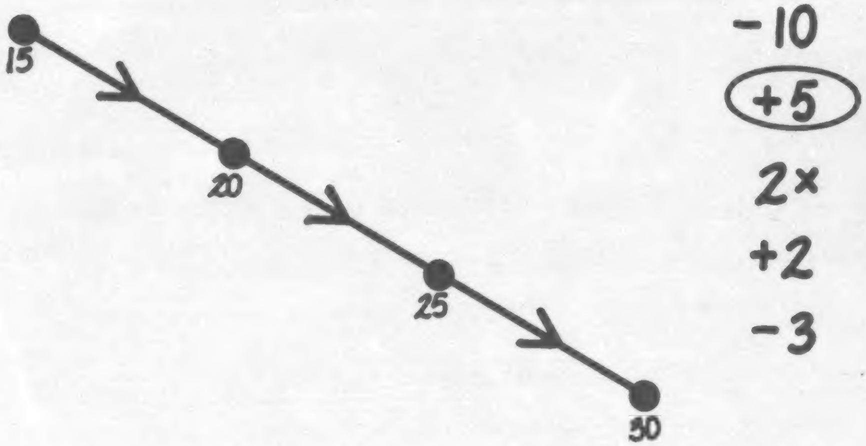


T: Suppose that we want to build a road between 15 and 30 using the same kind of arrow throughout and that these are our choices (point to the list of functions). Could we build such a road?

S: Yes.

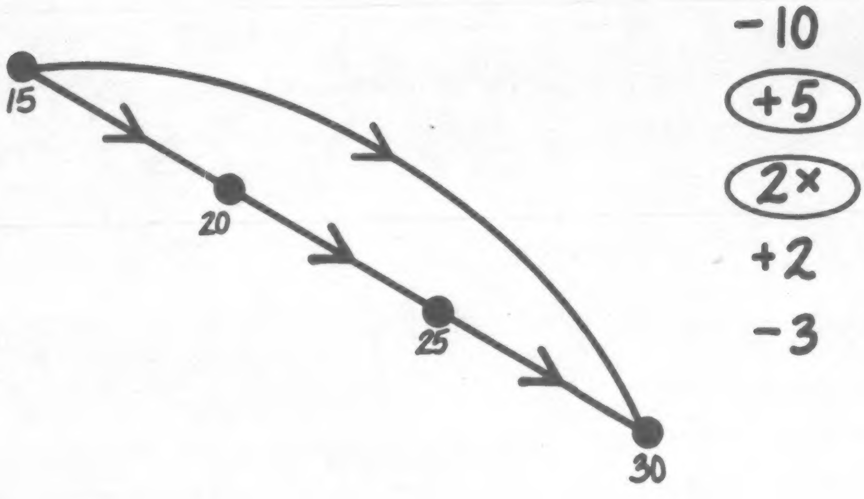
T: Who would like to build a road between these numbers? You can start at 15 and go to 30 or you can start at 30 and go to 15.

Ask a student who volunteers to circle in red his or her choice of functions and then to build the road in the same color. Suppose the student selects +5.



T: This is one way to build a road between 15 and 30. How many arrows were used? (Three) Could we build a road with less than three arrows?

Solicit a volunteer to build a different road between 15 and 30 using another kind of arrow. Perhaps the student will choose 2 x.



W 2

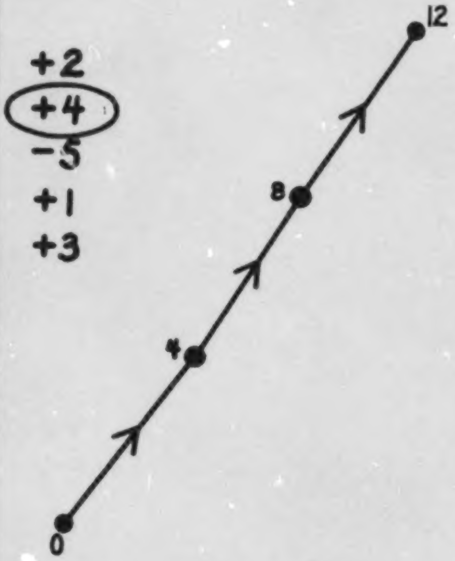
T: How many arrows are in the 2 x arrow road? (One) The 2 x arrow road is shorter than the +5 arrow road. Could we build a road between 15 and 30 using one of the other choices for arrows?

Your class should decide that -3 could be used to build a road between 15 and 30 and proceed to build it. Also they should discover that neither -10 nor +2 could be used successfully.

Distribute students' copies of the Workbook WHICH ROAD?. Review the directions and then allow approximately 30 minutes for individual work. Collect the workbooks for your review.

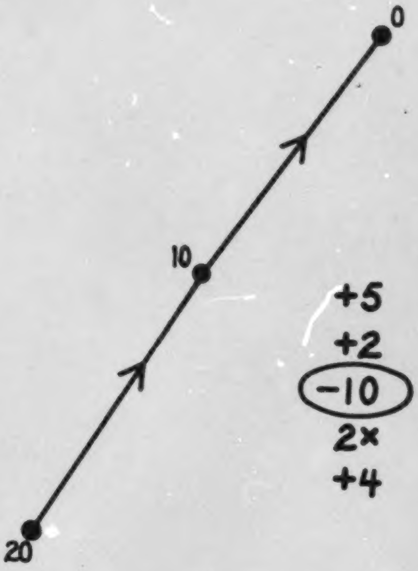
WHICH ROAD?

NOTE: Throughout this answer key for WHICH ROAD? all shortest roads are given. Other solutions are possible as noted.


+2
[✓ +4]
-5
+1
+3
12
8
4
0

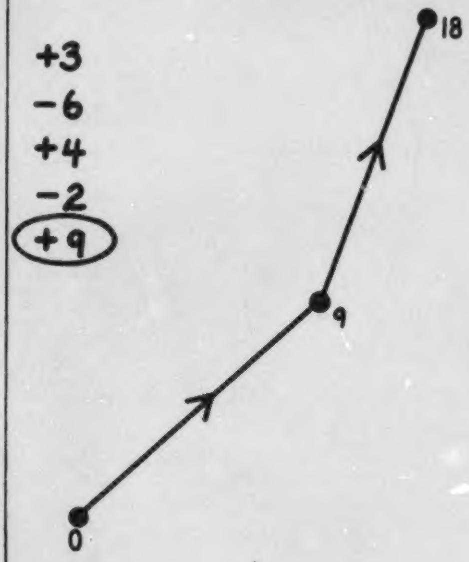
2

Other solutions are possible using + 2, + 1, or + 3.


0
10
+5
+2
[✓ -10]
2×
+4
20

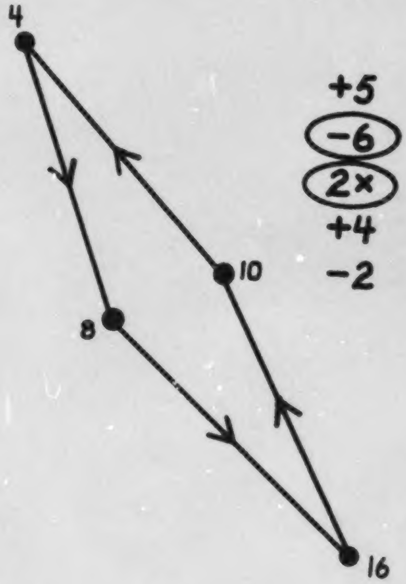
3

Other solutions are possible using + 5, + 2, or + 4.



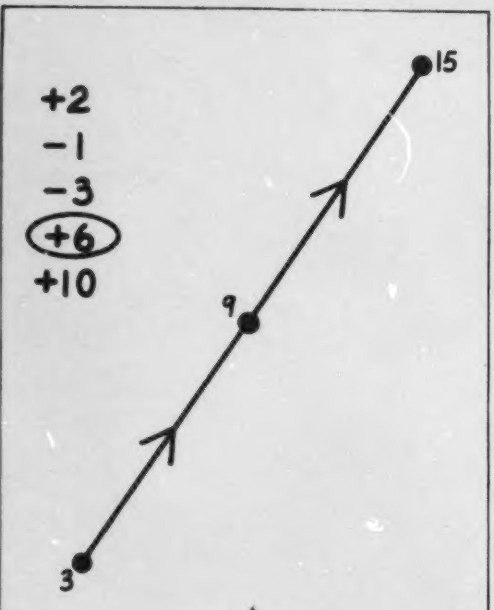
4

Other solutions are possible using + 3, - 6, or - 2.


4
+5
[✓ -6]
[✓ 2×]
+4
-2
10
8
16

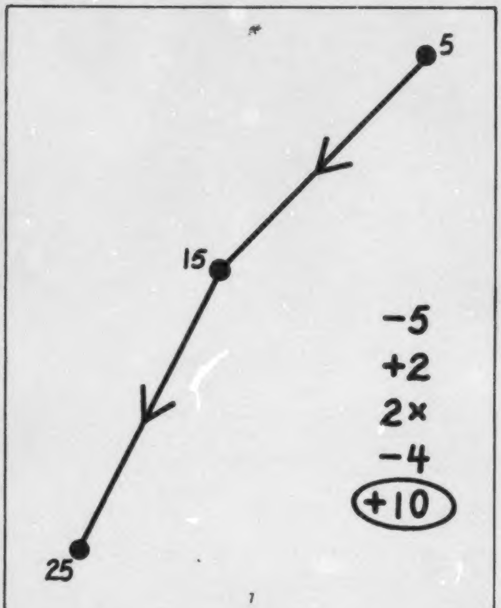
5

Other solutions are possible using + 4 or - 2.



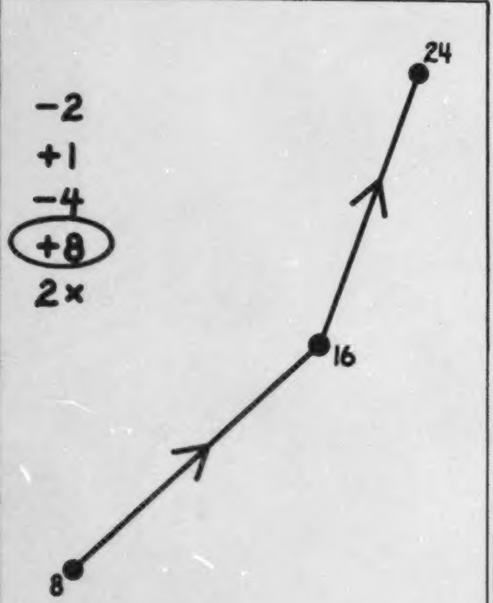
+2
-1
-3
[✓ +6]
+10

Other solutions are possible using +2, -1, or -3.



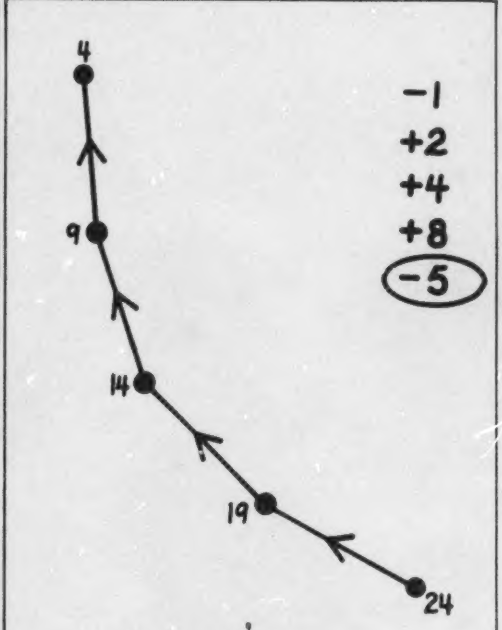
-5
+2
2×
-4
[✓ +10]

Other solutions are possible using -5, +2, or -4.



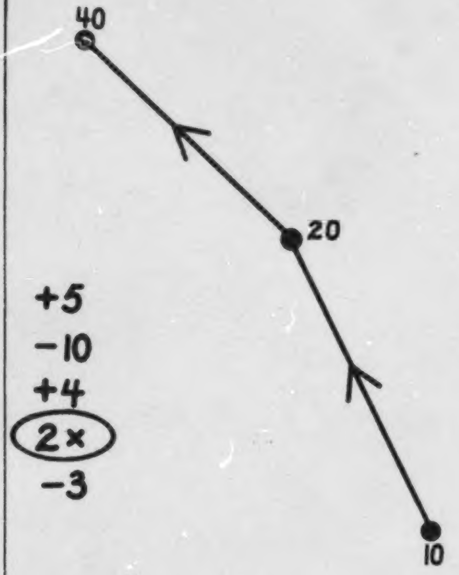
-2
+1
-4
[✓ +8]
2×

Other solutions are possible using -2, +1, or -4.



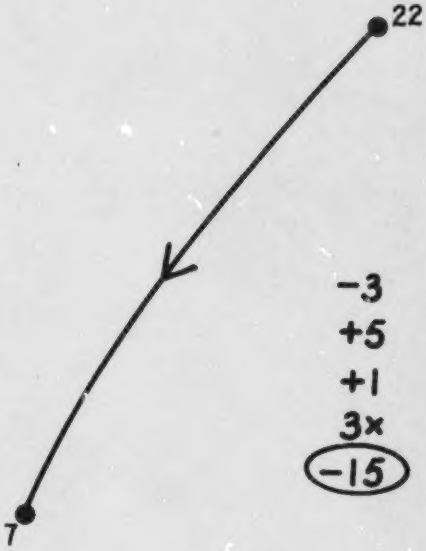
-1
+2
+4
+8
[✓ -5]

Other solutions are possible using -1, +2, or +4.


40
20
10
+5
−10
+4
2×
−3

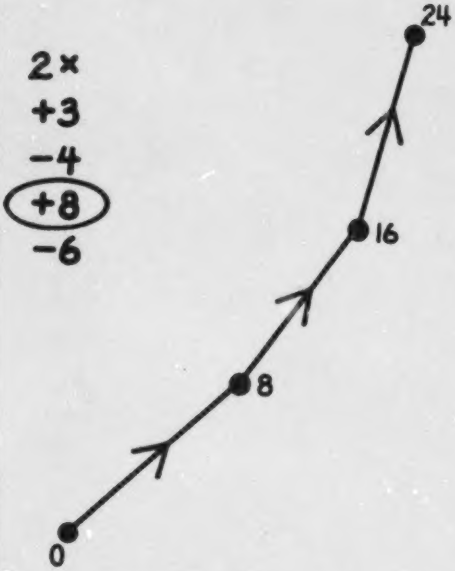
10

Other solutions are possible using
+5, −10, or −3.



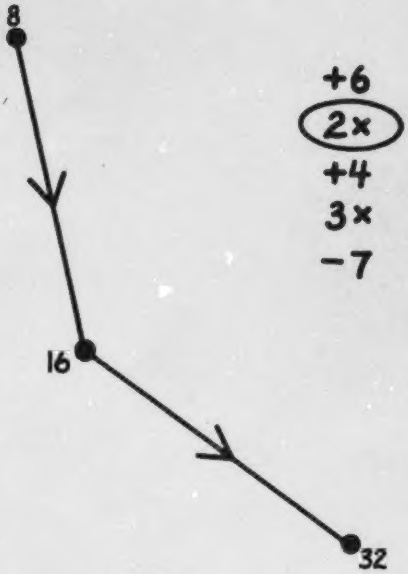
11

Other solutions are possible using
−3, +5, or +1.


24
16
8
0
2×
+3
−4
+8
−6

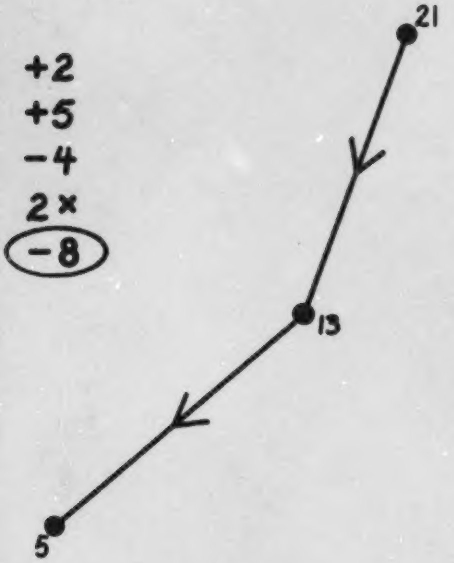
12

Other solutions are possible using
+3, −4, or −6.


8
16
32
+6
2×
+4
3×
−7

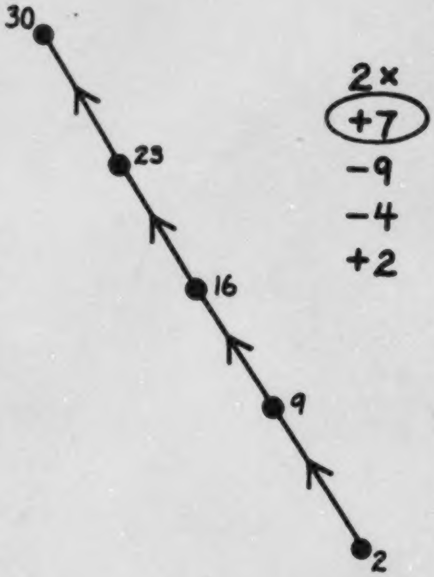
13

Other solutions are possible using
+6 or +4.


+2
+5
−4
2×
−8
21
13
5

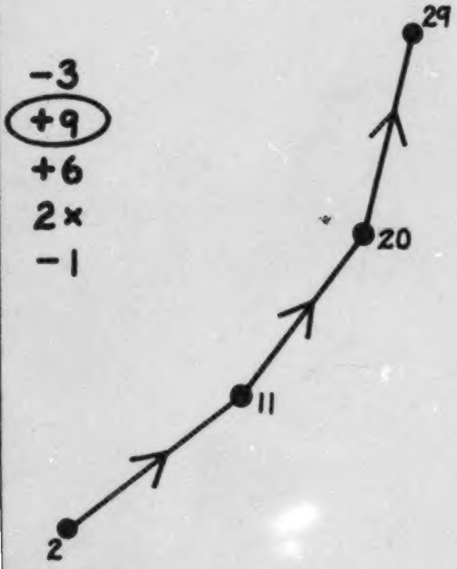
14

Other solutions are possible using + 2 or - 4.


30
23
16
9
2
2×
+7
−9
−4
+2

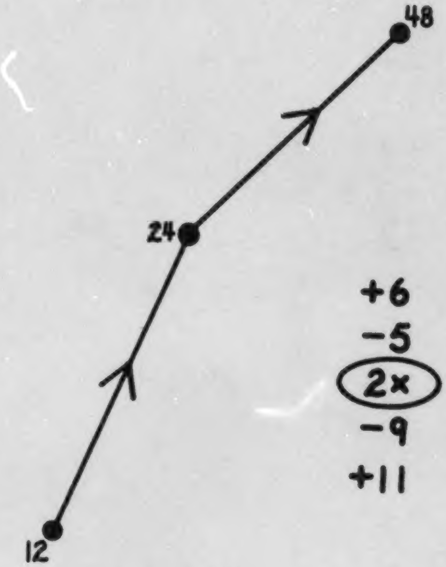
15

Other solutions are possible using - 4 or + 2.


−3
+9
+6
2×
−1
29
20
11
2

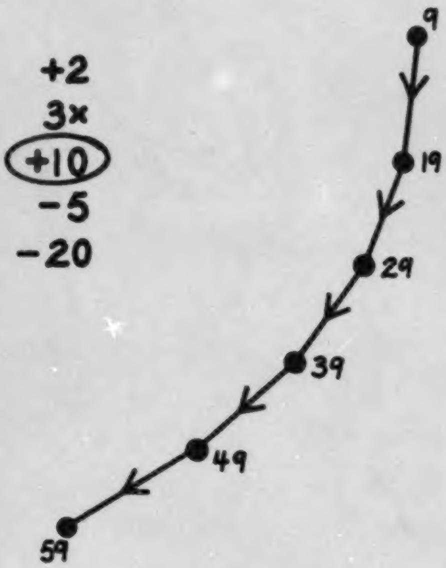
16

Other solutions are possible using - 3 or - 1.


48
24
12
+6
−5
2×
−9
+11

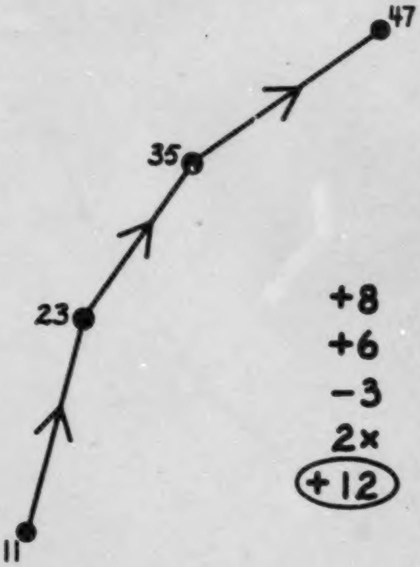
17

Other solutions are possible using + 6 or - 9.


+2
3×
+10
−5
−20
9
19
29
39
49
59

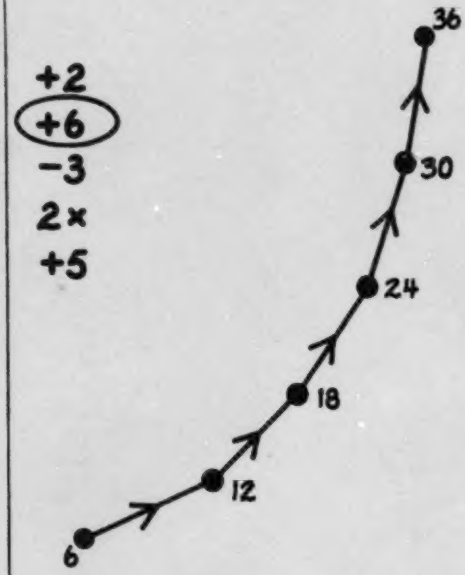
18

Other solutions are possible using
+ 2 or − 5.


47
35
23
11
+8
+6
−3
2×
+12

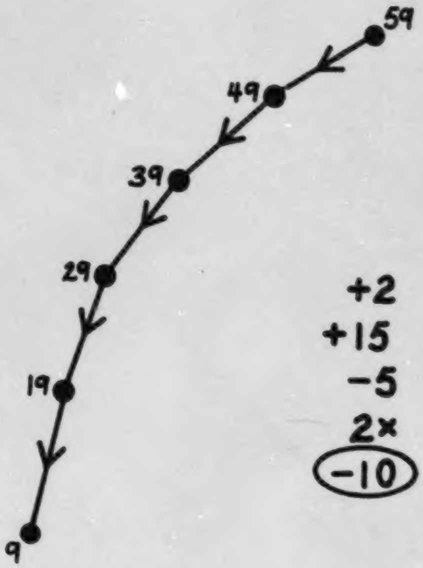
19

Other solutions are possible using
+ 6 or − 3.


+2
+6
−3
2×
+5
36
30
24
18
12
6

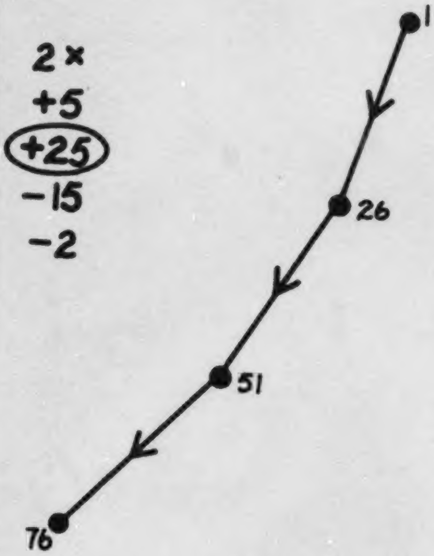
20

Other solutions are possible using
+ 2, − 3, or + 5.


59
49
39
29
19
9
+2
+15
−5
2×
−10

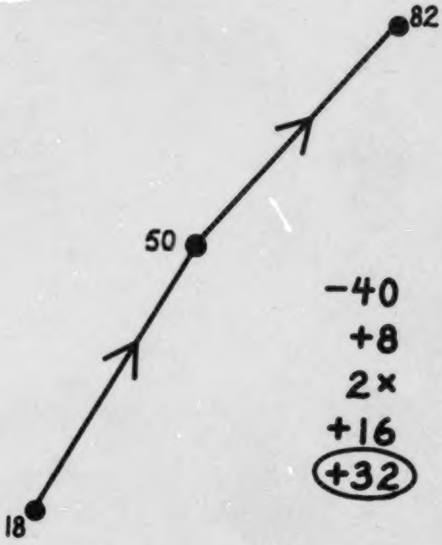
21

Other solutions are possible using
+ 2 or − 5.


2×
+5
+25
−15
−2
1
26
51
76

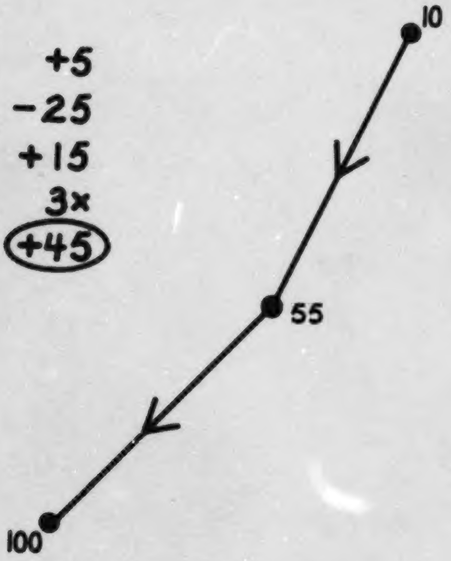
22

Other solutions are possible using
+5 or −15.


82
50
−40
+8
2×
+16
+32
18

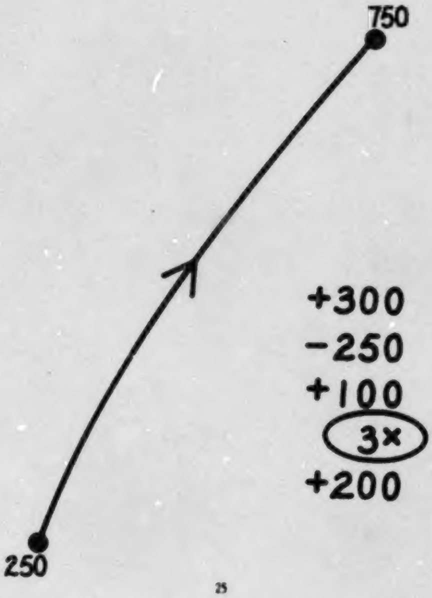
23

Other solutions are possible using
+8 or +16.


10
+5
−25
+15
3×
+45
55
100

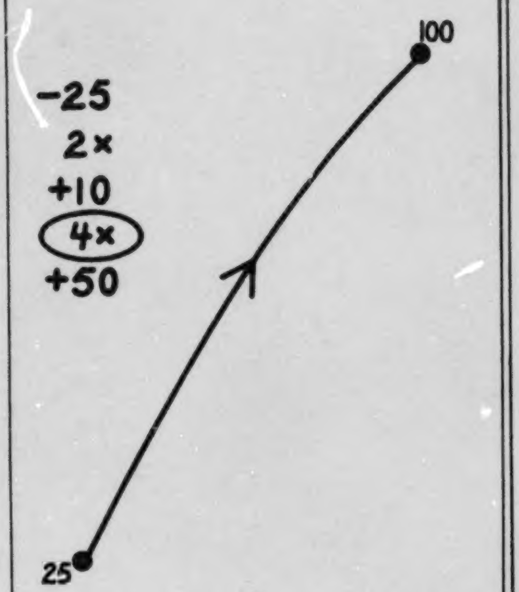
24

Other solutions are possible using
+5 or +15.


750
+300
−250
+100
3×
+200
250

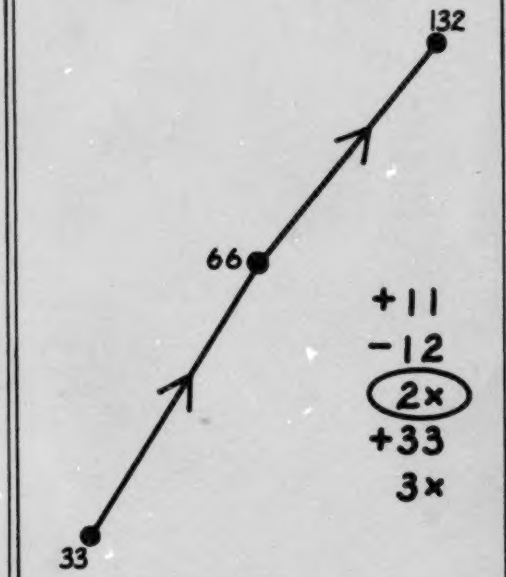
25

Other solutions are possible using
−250 or +100.


100
-25
2×
+10
4×
+50
25

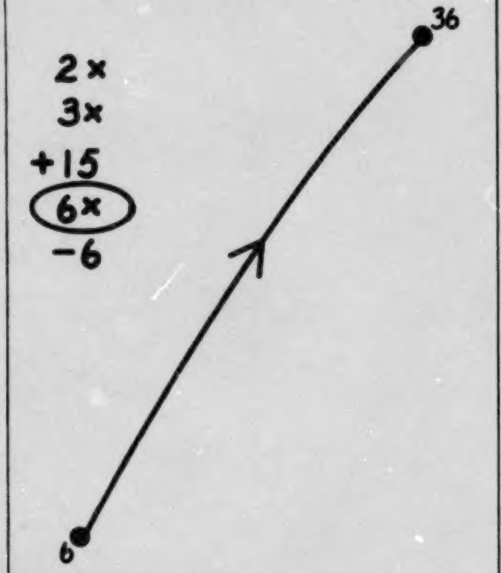
26

Other solutions are possible using - 25 or 2 x.


132
66
+11
-12
2×
+33
3×
33

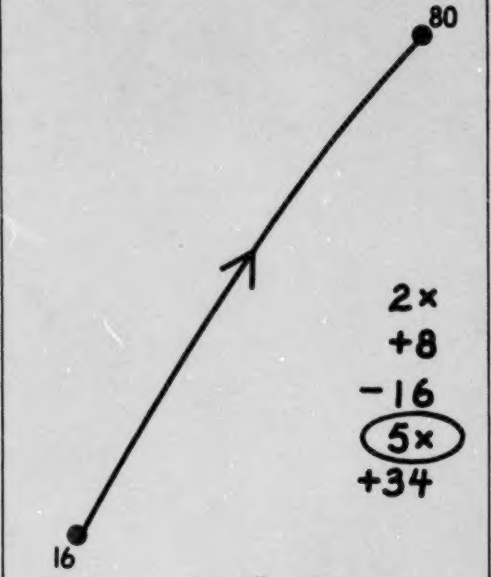
27

Other solutions are possible using + 11 or + 33.



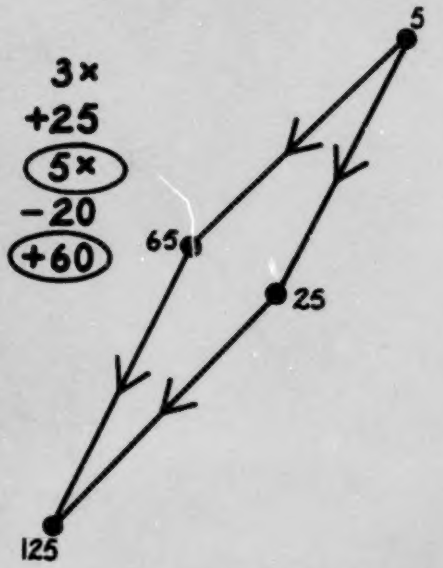
28

Other solutions are possible using + 15, or - 6.


80
2×
+8
-16
5×
+34
16

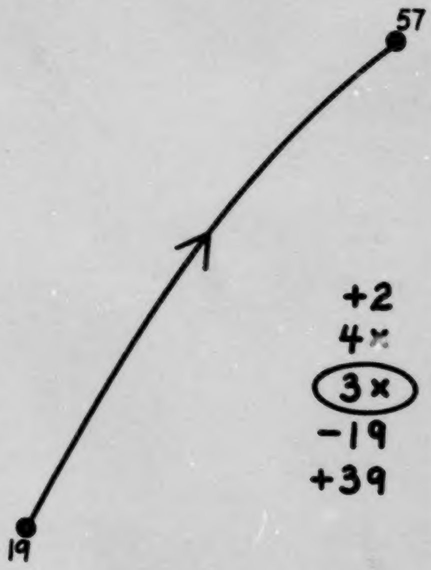
29

Other solutions are possible using + 8 or - 16.


5
3×
+25
5×
−20
+60
65
25
125

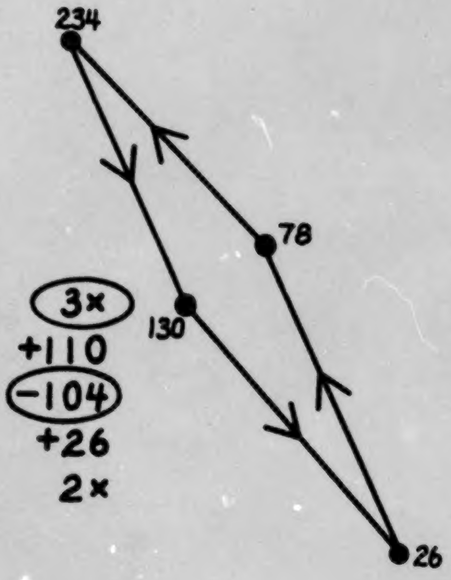
30

Another solution is possible using - 20.


57
+2
4×
3×
−19
+39
19

31

Other solutions are possible using + 2 or - 19.


234
78
3×
+110
−104
+26
2×
130
26

32

Another solution is possible using + 26.

W3 The Empire State Building

CAPSULE LESSON SUMMARY

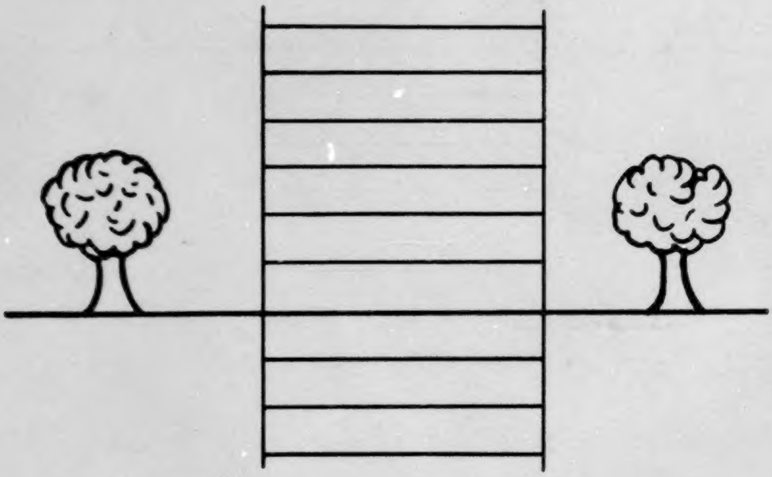
Present a story giving a situation in which to add positive and negative numbers. Within the same setting, express numbers using only +10, -10, +5, -5, +1, and -1. Label the dots in a -5 arrow road which contains some negative numbers.

MATERIALS

Teacher: Colored chalk
Student: None

DESCRIPTION OF LESSON

Draw this picture on the board.



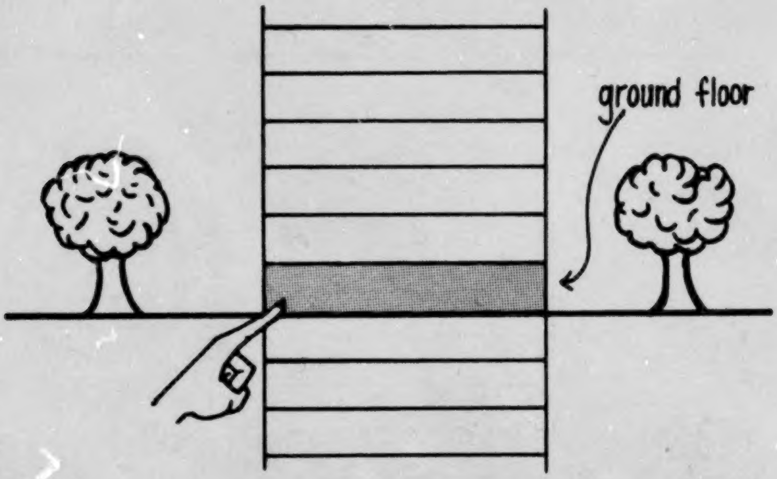
Ask your students what they learned about the Empire State Building in second grade. If necessary, tell the students that the Empire State Building is in New York City and has 102 floors above the ground. For many years it was the tallest building in the world.

W 3

Explain that the picture on the board shows only the floors of the Empire State Building which are near ground level, because it has too many floors to picture all of them.

T: Where is the ground floor in the picture?

The student who volunteers should point to this rectangle (see illustration).



NOTE: Some students may be somewhat confused by the use of the word "floor" in this lesson. In this case, "floor" refers to a story or level of the building rather than just the supporting surface at each level.

Discuss briefly the need to have names for each floor. Indicate that the ground floor is mostly open and has no offices. Often the ground floor has only a lobby, elevators, and stairs.

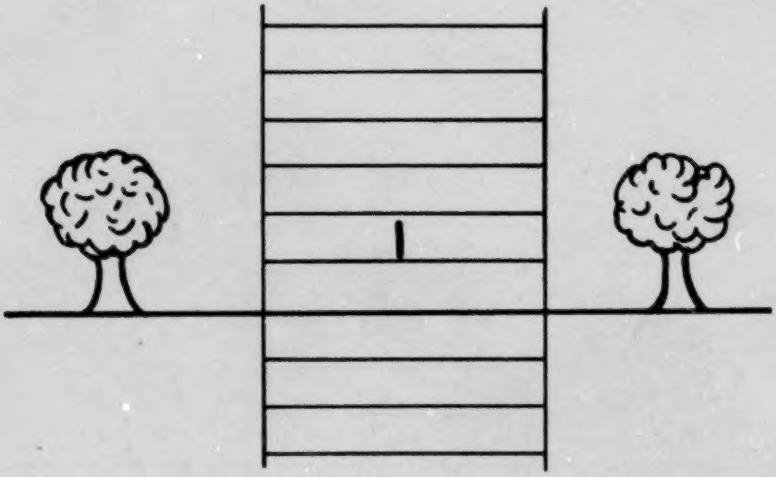
T: Who can show us where the first floor is?

The student who volunteers should point to the floor immediately above the ground floor.

T: Where is the second floor? . . . the fifth floor? Let's assign a number to each of these floors. What number should we give the first floor?

S: 1.

Write "1" inside the rectangle for the first floor.



Label all the floors above the first floor with students' assistance.

T: In many buildings, the ground floor has a letter name. Sometimes the ground floor is called "G" and sometimes it is called "L". Why?

S: "G" indicates ground level and "L" indicates lobby.

T: Let's give our ground floor a number name instead of a letter name. What number shall we assign to the ground floor?

S: 0.

Label the ground floor "0" on the board.

W 3

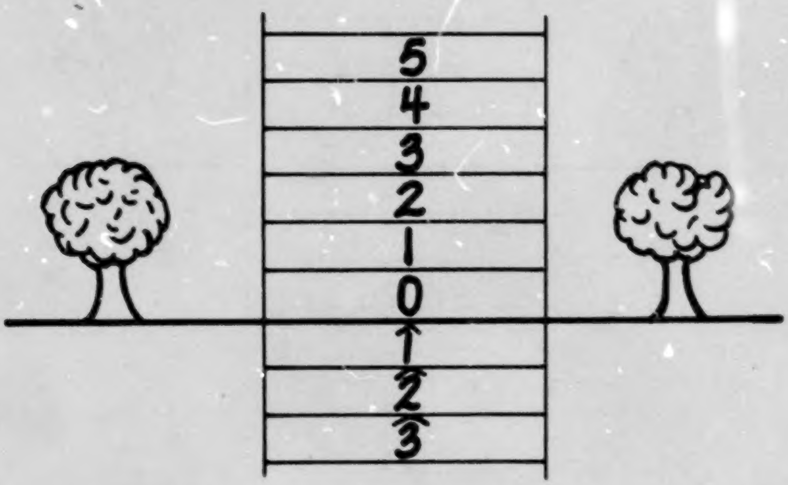
T: In many buildings the floors below the ground are called "B1", "B2", "B3", "B4", and so on. Why?

S: "B" stands for basement.

T: What numbers shall we assign to the floors below ground level?

S: Negative numbers.

Ask the class to assist you in labeling the three floors below ground level shown in the picture.



T: If we are on floor 2 and want to go to floor $\hat{3}$[*], should we go up or down?

S: Down.

T: ...down how many floors?

S: Five.

---

[*] "$\hat{3}$" is read "negative three."

Using the picture on the board, show that floor $\hat{3}$ is five floors below floor 2.

T: If we are on floor $\hat{3}$ and want to go to floor 1, should we go up or down?

S: Up.

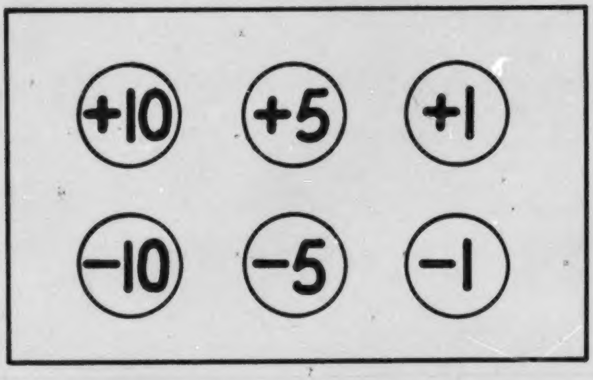
T: ...up how many floors?

S: Four.

Using the picture on the board, show that floor 1 is four floors above floor $\hat{3}$.

T: There is a boy named Larry who sells sandwiches at lunchtime in the Empire State Building. Larry has regular customers on five floors and he goes to their offices everyday on the elevator. One day when Larry gets into the elevator, he discovers that there is a new control panel. The new panel looks like this.

Draw this picture on the board.



T: If Larry is on floor 0 and his first customer is on floor 26, which buttons should Larry press?

W 3

There are many possibilities. The following dialogue supposes someone has suggested pressing (+10) twice and (+1) six times.

T: Let's see if that will get Larry from floor 0 to floor 26. If he starts at floor 0 and presses (+10), which floor will Larry arrive on?

S: On the 10th floor.

T: If he presses (+10) again, which floor will he arrive on?

S: On the 20th floor.

T: If Larry is on floor 20 and presses (+1) six times, where will he be when the elevator stops?

S: On the 26th floor.

Write this number story on the board.

$$0+10+10+1+1+1+1+1+1=26$$

T: This is one way Larry could get from floor 0 to floor 26. Is there another way?

Consider each suggestion and decide collectively if it will get Larry to floor 26. Take advantage of any opportunity to use multiplication facts in a natural way; for example,

S: Larry can press (+5) five times and then (+1) one time.

T: Let's see if that will get him to the 26th floor. What number is 5 × 5?

S: 25.

T: If Larry presses (+5) five times, he will go up 25 floors.
25 + 1 = ... ?

S: 26.

T: So Larry could use this method also.

Write this number story on the board.

$$0 + 5 + 5 + 5 + 5 + 5 + 1 = 26$$

T: Larry's customer on floor 26 is getting very hungry; Larry decides to press (+10) , (+10) , (+5) , and (+1) .

Record the appropriate number story on the board if it isn't already there.

$$0 + 10 + 10 + 5 + 1 = 26$$

T: Now that Larry is on floor 26 he needs to deliver a sandwich on floor 36. Do you have any suggestions for Larry?

Consider each suggestion and decide collectively if it will get him to the 36th floor. You may need to remind the students that Larry is already on floor 26, so he does not need to go up thirty-six floors to get to floor 36.

If no one suggests pressing (+10) once, give the hint that Larry only needs to press one button to go from floor 26 to floor 36.

W 3

When someone suggests pressing (+10) once, write this number story on the board.

$$26 + 10 = 36$$

T: The next customer is on floor 45. What could Larry do now?

S: Press (+1) four times and then press (+5) once.

T: Why does that work?

S: Because 36 + 4 = 40 and 40 + 5 = 45.

S: Press (+10) and then (-1) .

T: Why does that work?

S: 36 + 10 = 46 and 46 - 1 = 45.

If no one suggests pressing (+10) and (-1) , ask the students how Larry could get to floor 45 by pressing only two buttons (same or different).

When someone suggests pressing (+10) and (-1) , record the appropriate number story on the board.

$$36 + 10 - 1 = 45$$

or

$$36 - 1 + 10 = 45$$

T: Now that Larry is on floor 45, how can he get to floor 60?

S: He could press (+5) three times.

S: He could press (+10) and (+5).

S: He could press (+1) fifteen times.

There are many possibilities; check any that is offered. Record one student's solution on the board. For example,

$$45 + 10 + 5 = 60$$

T: Larry's last customer is on floor 79. How can Larry go from floor 60 to floor 79?

S: By pressing (+10), (+10), and (-1).

Again there are many possibilities; check any that is offered.

If no one suggests pressing (+10), (+10), (-1), ask the students how Larry could get to floor 79 by pressing only three buttons (not necessarily different).

When a student suggests pressing (+10), (+10), and (-1), record the appropriate number story on the board.

$$60 + 10 + 10 - 1 = 79^{*}$$

---

* The order in which +10, +10, and -1 occur in this number sentence may vary depending upon how one of your students gives the solution.

T: Larry has delivered sandwiches to all of his customers, so he is ready to leave. How can he get back to floor 0?

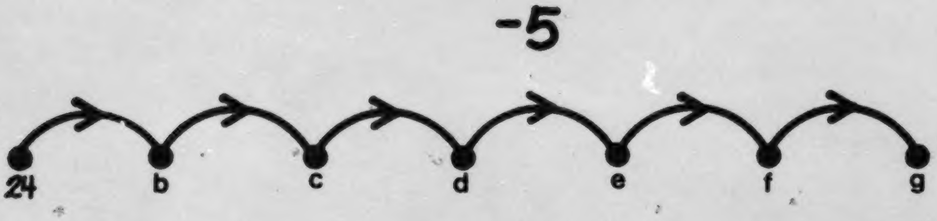
Allow the students to make many suggestions. To press the fewest buttons in order to go from floor 79 to floor 0, Larry should press (-10) eight times and (+1) once.

$$79-10-10-10-10-10-10-10-10+1=0$$

Erase the board except the drawing of the Empire State Building.

Exercise 2

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Suppose the next day, after visiting a friend on floor 24, Larry gets on the elevator and presses the button (-5). On which floor should the elevator stop?

S: On floor 19.

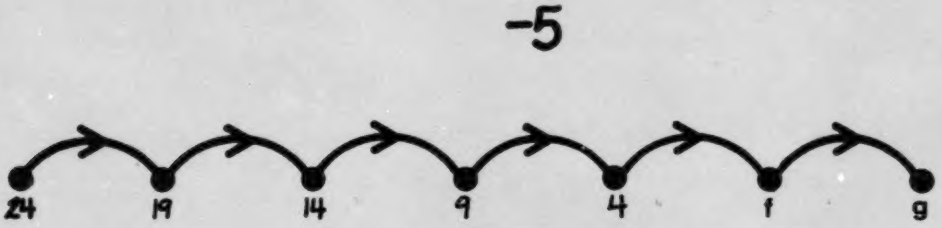
Label b "19".

T: The elevator does stop but the door doesn't open. After a moment, it continues to go down as if Larry had pressed (-5) again. This keeps happening (trace the arrow road with your finger) and Larry wonders where he will be able to get off!

Which is the next floor (point to c) that the elevator will stop momentarily at?

S: Floor 14.

Label c "14" and then continue in the same manner until b, c, d, and e are labeled.



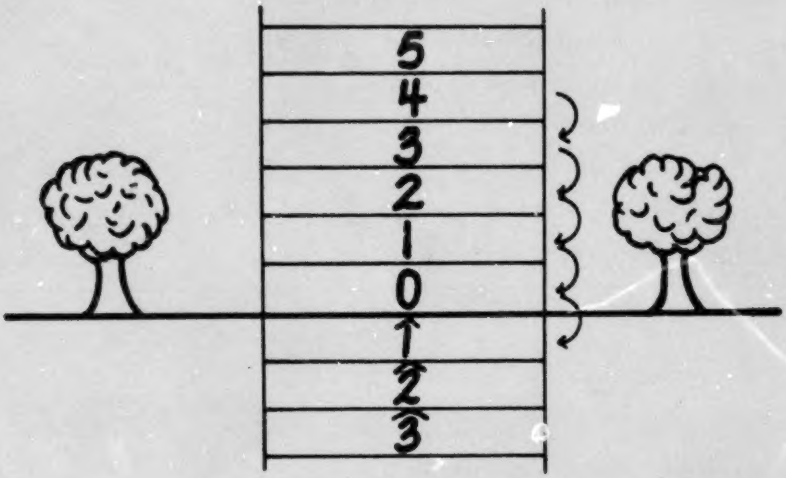
Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: The elevator moves down another five floors. Which floor does it come to (point to f)?

S: Floor $\hat{1}$.

W 3

Refer to the picture of the Empire State Building from Exercise 1 and count down five floors from floor 4.



T (tracing the arrow from 4 to f): $4 - 5 = \hat{1}$.

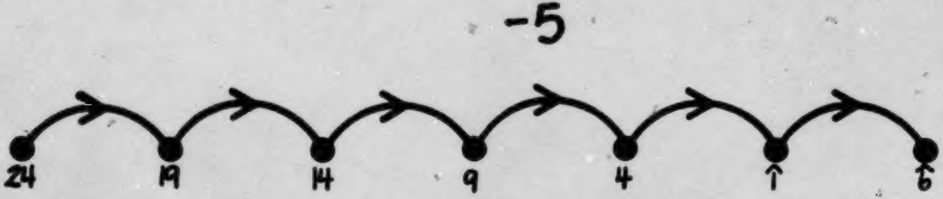
Label f "$\hat{1}$".

T: The elevator continues going down five more floors (point to g) before it completely stops and the door opens. On which floor is Larry able to get off?

S: On floor $\hat{6}$.

T (tracing the arrow $\hat{1}$ to g): $\hat{1} - 5 = \hat{6}$.*

Label g "$\hat{6}$".



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*Read: negative one minus five equals negative six.

W4 FESTIVAL OF PROBLEMS #1 (Lesson 1)

CAPSULE LESSON SUMMARY

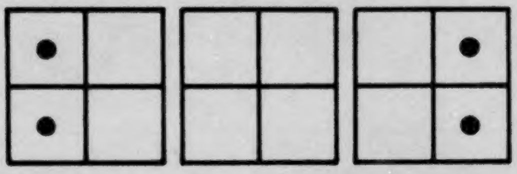
Introduce a Minicomputer game called "Minicomputer Tug-of-War". Begin the Workbook FESTIVAL OF PROBLEMS #1.

MATERIALS

Teacher: Minicomputer set; colored chalk
Student: Workbook FESTIVAL OF PROBLEMS #1

DESCRIPTION OF LESSON

Display three Minicomputer boards. Position two red and two blue checkers on the Minicomputer as in the illustration below.



Divide the class into two teams (the RED team and the BLUE team).

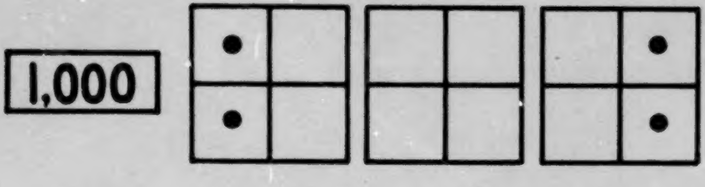
T: We are going to play a game called "Minicomputer Tug-of-War". The BLUE team will play with the blue checkers and the RED team will play with the red checkers.

The number of the RED team is the number which is on the Minicomputer with red checkers. What is the starting number of the RED team?

S: 1,000.

W 4

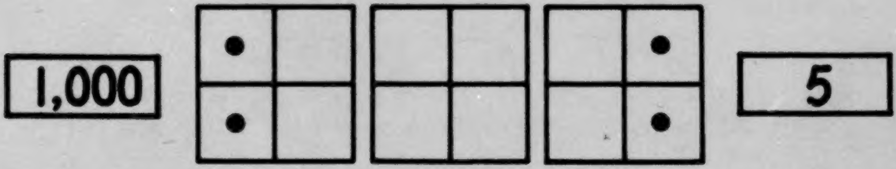
Draw a red rectangle to the left of the Minicomputer and record "1,000" inside it.



T: The number of the BLUE team is the number which is on the Minicomputer with the blue checkers. What is the starting number of the BLUE team?

S: 5.

Draw a blue rectangle to the right of the Minicomputer and record "5" inside it.



T: Teams alternate turns. Players on the BLUE team must move one of their checkers and make their number larger. Players on the RED team must move one of their checkers and make their number smaller. If the RED team always makes its number smaller and the BLUE team always makes its number larger, eventually the red and the blue numbers will tie or pass each other. If the RED team makes the red number the same as or smaller than the blue number, the RED team loses. If the BLUE team makes the blue number the same as or larger than the red number, the BLUE team loses.

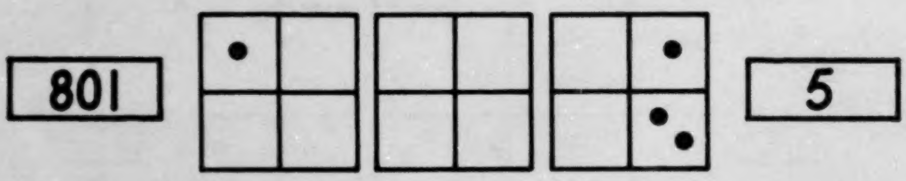
Begin playing the game.

NOTE : We suggest that during this first playing of the Minicomputer Tug-of-War game you call on volunteers from each team rather than asking the students to play in some order. This will speed up the game and allow students who are unsure of the rules to become more familiar with the game before they have to play.

A description of a possible game is given here.

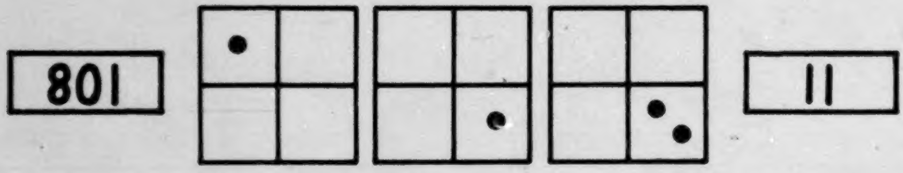
T: The RED team plays first. You must make the red number smaller without tying or passing the blue number.

A student from the RED team moves a checker from the 200-square to the 1-square. The student determines that the red number is now 801 and changes the number inside the red box to 801.



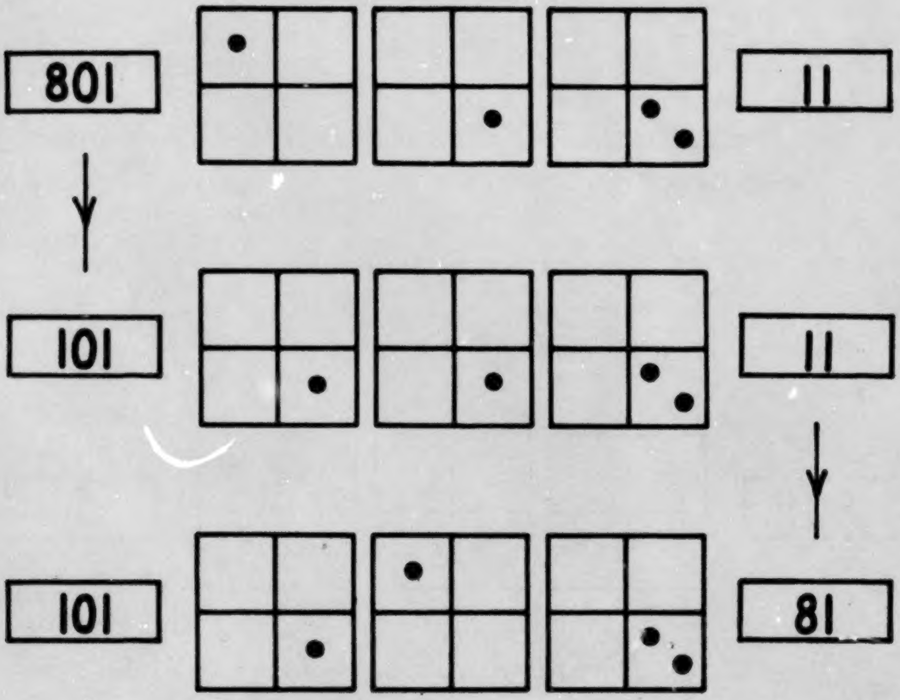
T: The BLUE team plays next. You must make the blue number larger without tying or passing the red number.

A student from the BLUE team moves a checker from the 4-square to the 10-square. The student determines that the blue number is now 11 and changes the number inside the blue box to 11.



T: 11 is less than 801, so the blue number is still less than the red number.

The game continues as shown below.



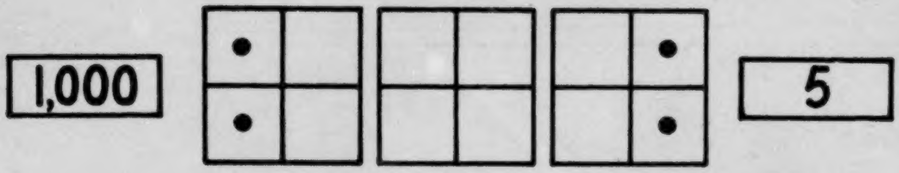
It is the RED team's turn to play, but there are no volunteers and some of the students claim it is impossible for the RED team to play without tying or passing the blue number.

T: The RED team has to move one of its checkers and make the red number smaller without tying or passing the blue number. Is this possible?

S: We can't move the red checker on the 1-square because that would make the red number larger.

S: We can't move the checker on the 100-square either because the smallest backward move we can make would be from 100 to 80 and then the number in red would be 81.

Conclude that the RED team cannot move without losing and congratulate the BLUE team. Return the checkers to their original positions, change the numbers in the rectangles, and play the game again.



NOTE: When you are playing the game with the students playing in order, avoid forcing a student to make a losing play. Whenever a student declines to play because he/she cannot find any moves which will prevent his/her team from losing, ask if any other member of the team is able to play. If none of the team's members are able to play, conclude that the team has lost the game.

Distribute copies of the Workbook FESTIVAL OF PROBLEMS #1. Allow approximately 30 minutes for individual work and then collect the workbooks for your review.

W5 FESTIVAL OF PROBLEMS #1 (Lesson 2)

CAPSULE LESSON SUMMARY

Continue working in FESTIVAL OF PROBLEMS #1. (This is the second of two lessons using this workbook.)

MATERIALS

Teacher: None
Student: Workbook FESTIVAL OF PROBLEMS #1; pencil; colored pencils

DESCRIPTION OF LESSON

Your review of the students' workbooks may have indicated that a short collective discussion is needed about a particular page in the workbook. If this is the case, solve a problem similar to the one on that page collectively before distributing the students' copies of the Workbook FESTIVAL OF PROBLEMS #1.

Allow the students to work on the workbook for 30 to 40 minutes. Spend any remaining time playing the String Game.*

---

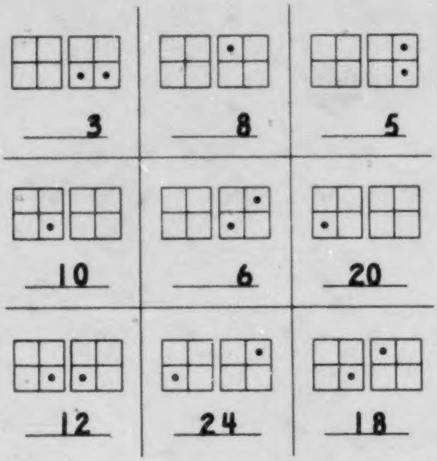
* See the appendix to the Languages of Strings and Arrows strand for a description of the String Game.

FESTIVAL OF PROBLEMS

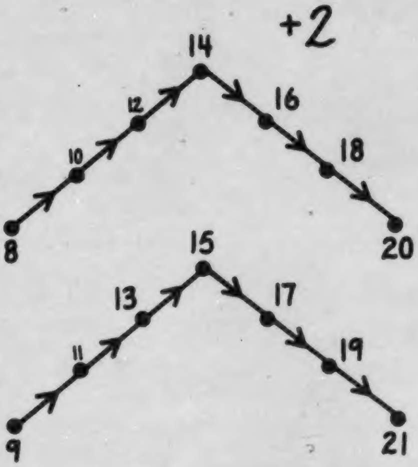
#1

Festival of Problems #1

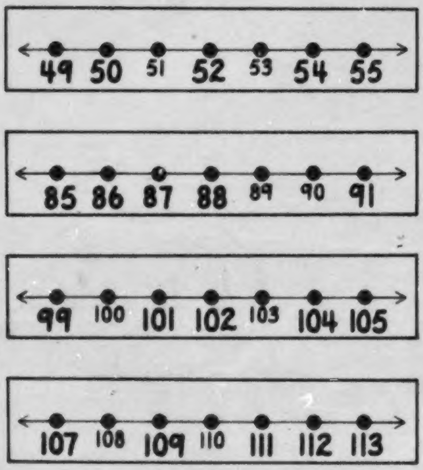
What number is on the Minicomputer ?


3
8
5
10
6
20
12
24
18

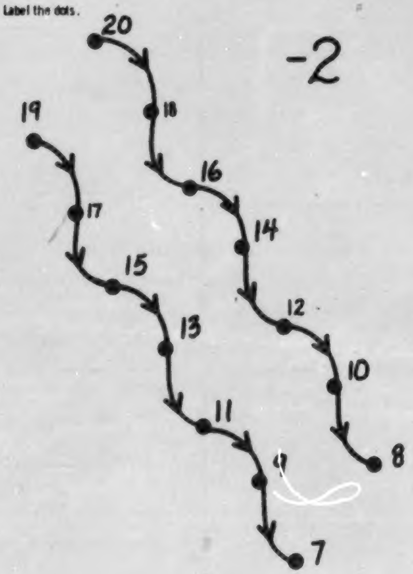
Label the dots.


+2
8
10
12
14
16
18
20
9
11
13
15
17
19
21

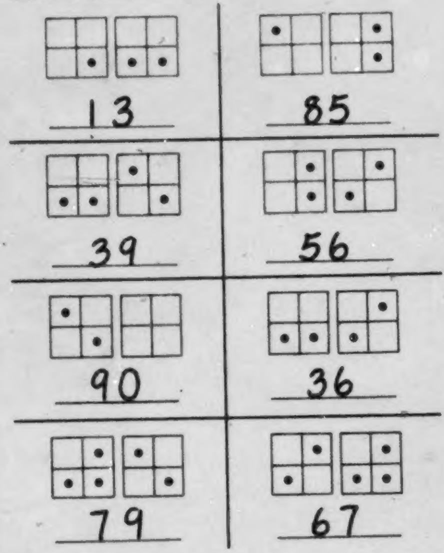
Label the dots on these number lines.


49 50 51 52 53 54 55
85 86 87 88 89 90 91
99 100 101 102 103 104 105
107 108 109 110 111 112 113

Label the dots.


-2
20
18
16
14
12
10
8
19
17
15
13
11
9
7

What number is on the Minicomputer ?



13

85

39

56

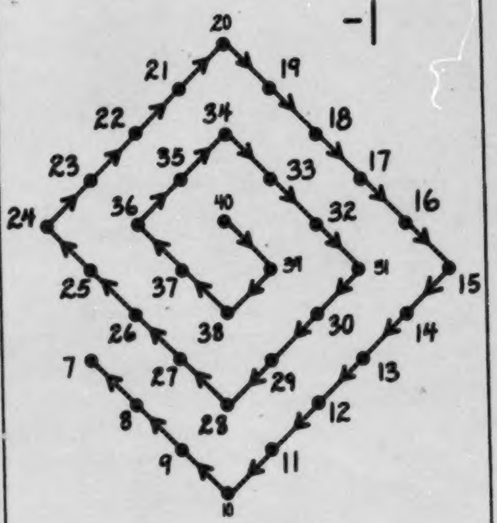
90

36

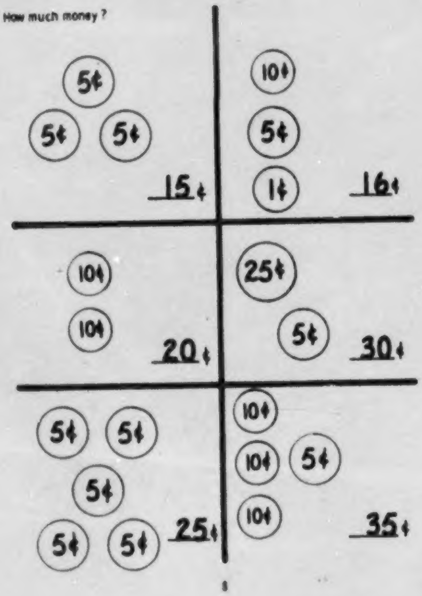
79

67

Label the dots.


-1
20
19
18
17
16
15
14
13
12
11
10
9
8
7
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40

How much money ?


5¢
5¢
5¢
10¢
5¢
1¢
10¢
10¢
25¢
5¢
5¢
5¢
5¢
5¢
5¢
10¢
10¢
5¢
10¢

15¢

16¢

20¢

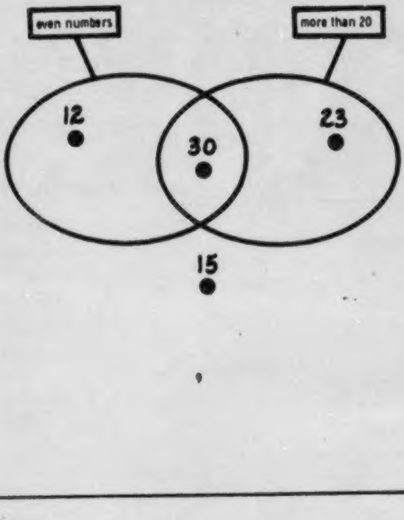
30¢

25¢

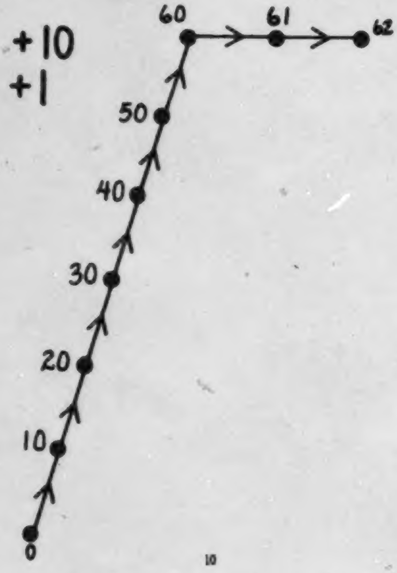
35¢

Label the dots in this picture with these numbers :

12 15 23 30


even numbers
more than 20
12
30
23
15

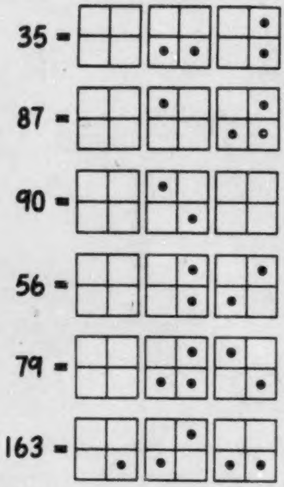
Build a red-blue road from 0 to 62.


+10
+1
0
10
20
30
40
50
60
61
62

10

Many solutions are possible; however, a shortest road has eight arrows.

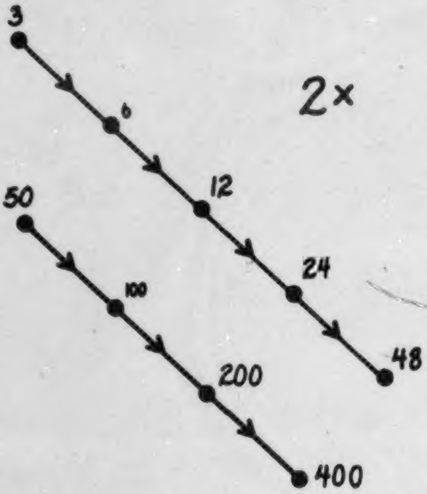
Put these numbers on the Minicomputer.


35 =
87 =
90 =
56 =
79 =
163 =

11

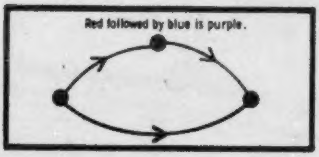
Many configurations for these numbers are possible although only standard configurations are given here.

Label the dots.

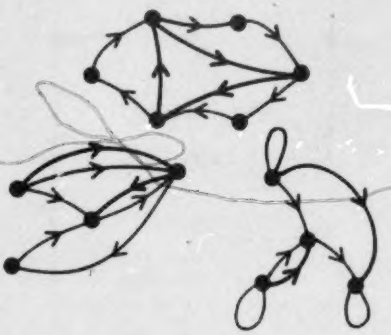

3
6
12
24
48
2×
50
100
200
400

12

Red followed by blue is purple.



Draw the missing purple arrows.



13

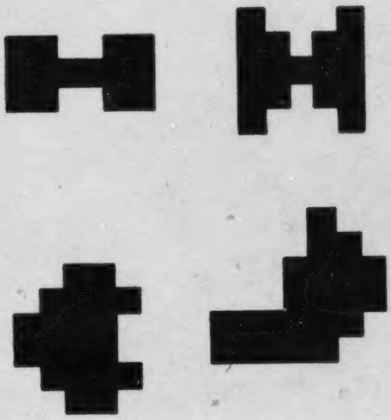
Complete the addition calculations.

$$\begin{array}{r} 25 \\ +15 \\ \hline 40 \end{array} \qquad \begin{array}{r} 52 \\ +64 \\ \hline 116 \end{array}$$

$$\begin{array}{r} 350 \\ +240 \\ \hline 590 \end{array} \qquad \begin{array}{r} 416 \\ +322 \\ \hline 738 \end{array}$$

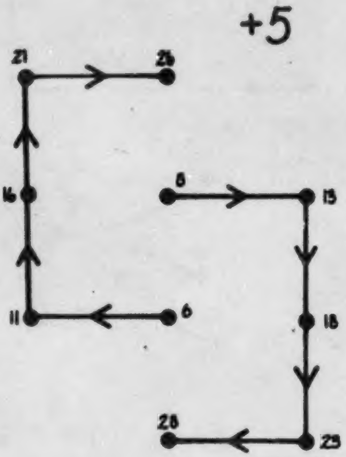
$$\begin{array}{r} 512 \\ +329 \\ \hline 841 \end{array} \qquad \begin{array}{r} 321 \\ +284 \\ \hline 605 \end{array}$$

Color each shape one-half red and one-half blue.

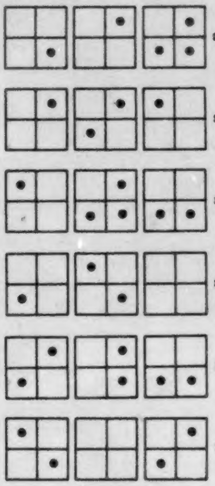


Other colorings are possible.

Draw all the +5 arrows. One is done for you.



What number is on the Minicomputer ?



= 147

= 468

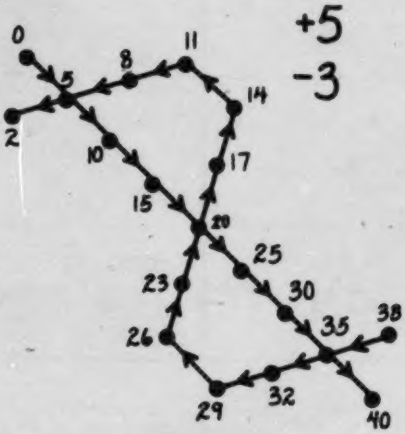
= 873

= 290

= 653

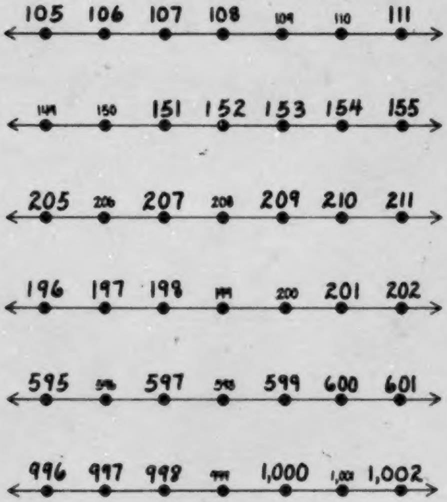
= 906

Label the dots.


+5
-3
0
5
8
11
14
2
10
17
15
20
25
23
30
26
35
38
29
32
40

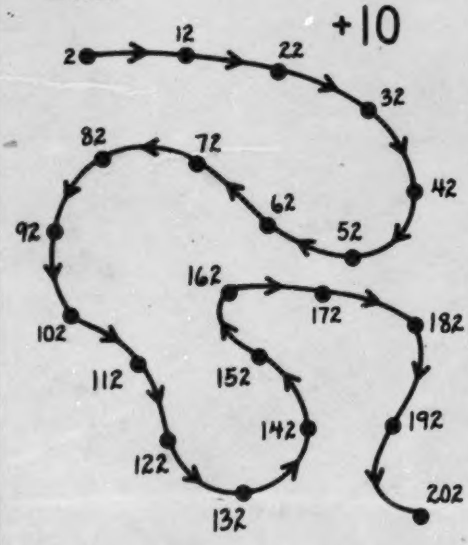
18

Label the dots on these number lines.


105 106 107 108 109 110 111
149 150 151 152 153 154 155
205 206 207 208 209 210 211
196 197 198 199 200 201 202
595 596 597 598 599 600 601
996 997 998 999 1,000 1,001 1,002

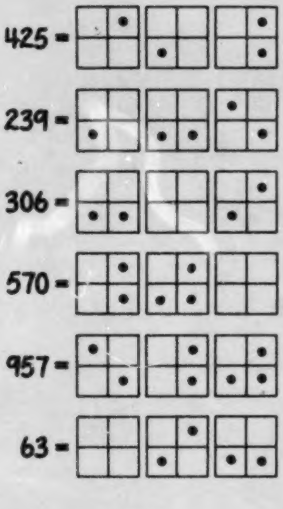
19

Label the dots.


+10
2
12
22
32
82
72
42
62
92
52
162
172
182
102
152
112
142
192
122
202
132

20

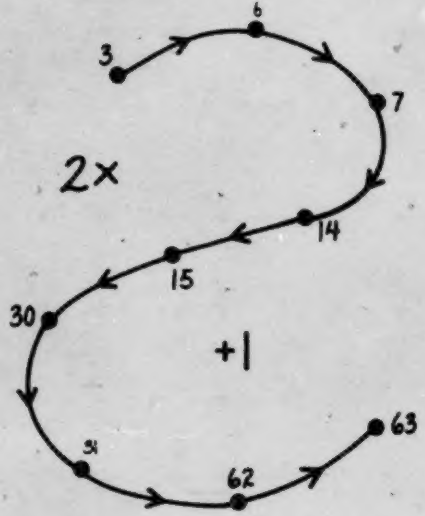
Put these numbers on the Minicomputer.


425 =
239 =
306 =
570 =
957 =
63 =

21

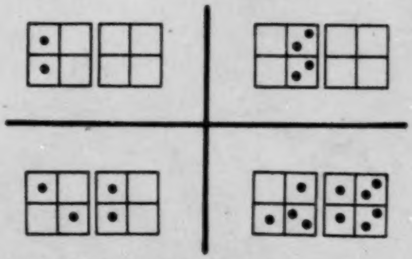
Many configurations for these numbers are possible although only standard configurations are given here.

Label the dots.

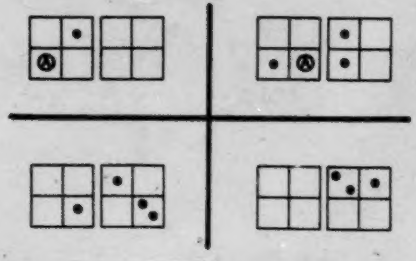


3
6
7
2x
14
15
30
+1
31
62
63

Put 100 on the Minicomputer in four different ways.



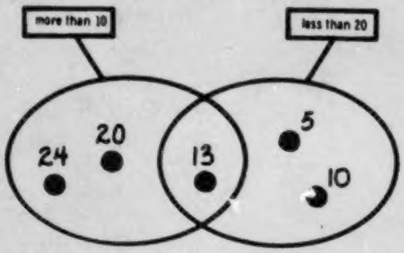
Put 20 on the Minicomputer in four different ways.



Many solutions are possible.

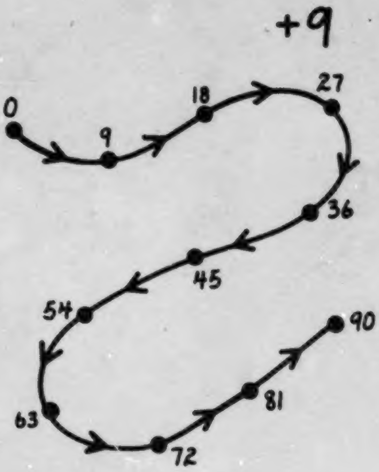
Put these numbers in this string picture.

5 10 13 20 24



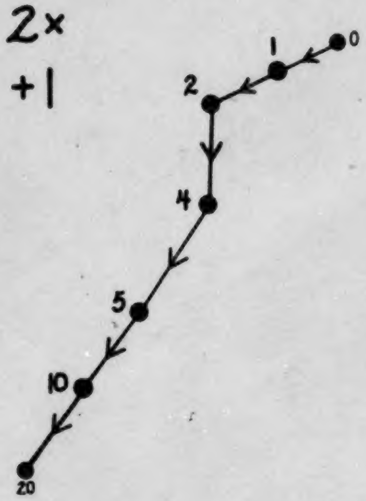
more than 10
less than 20
24
20
13
5
10

Label the dots.



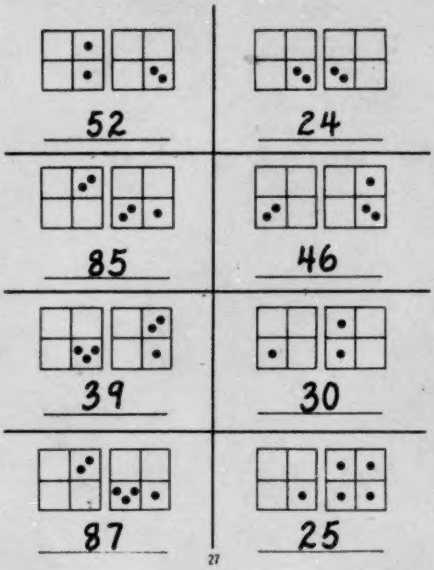
+9
0
9
18
27
36
45
54
63
72
81
90

Build a red - blue road from 0 to 20.


2x
+1
0
1
2
4
5
10
20

Other solutions are possible; however, a shortest road has six arrows.

What number is on the Minicomputer ?

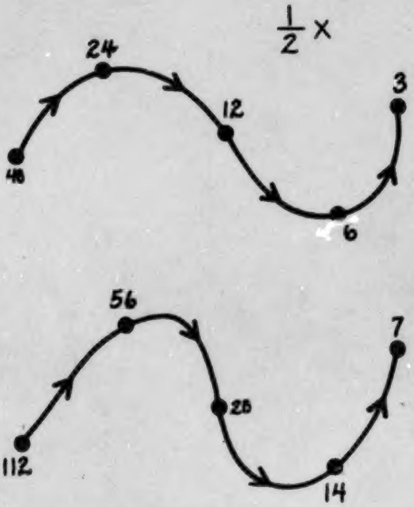

52
24
85
46
39
30
87
25

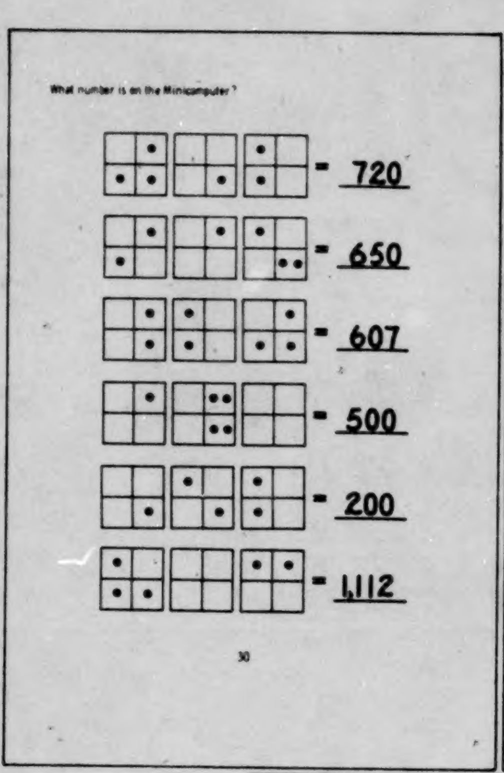
Show ways to make 50¢. One is done for you.

Quarters	Dimes	Nickels
25¢	10¢	5¢ 5¢ 5¢
25¢	10¢ 10¢	5¢
25¢		5¢ 5¢ 5¢ 5¢ 5¢
	10¢ 10¢ 10¢ 10¢ 10¢	
	10¢ 10¢ 10¢ 10¢	5¢ 5¢
	10¢ 10¢ 10¢	5¢ 5¢ 5¢ 5¢

Other solutions are two dimes and six nickels, one dime and eight nickels, and ten nickels.

Label the dots.


$\frac{1}{2}$ x
48
24
12
6
3
112
56
28
14
7



What number is on the Minicomputer?

= 720

= 650

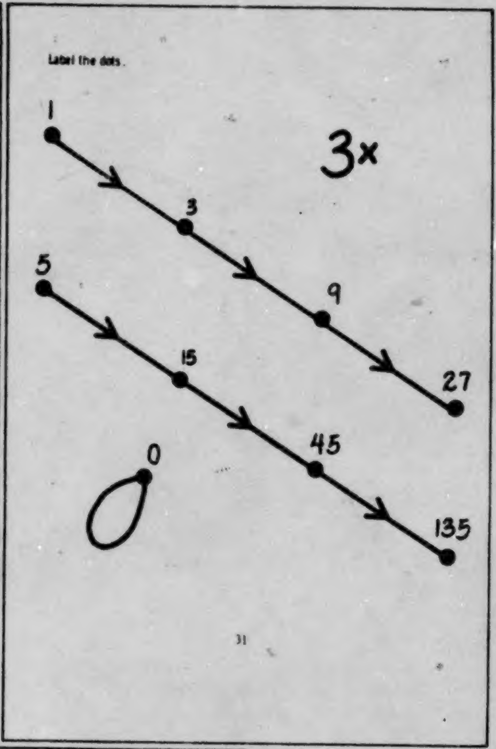
= 607

= 500

= 200

= 1,112

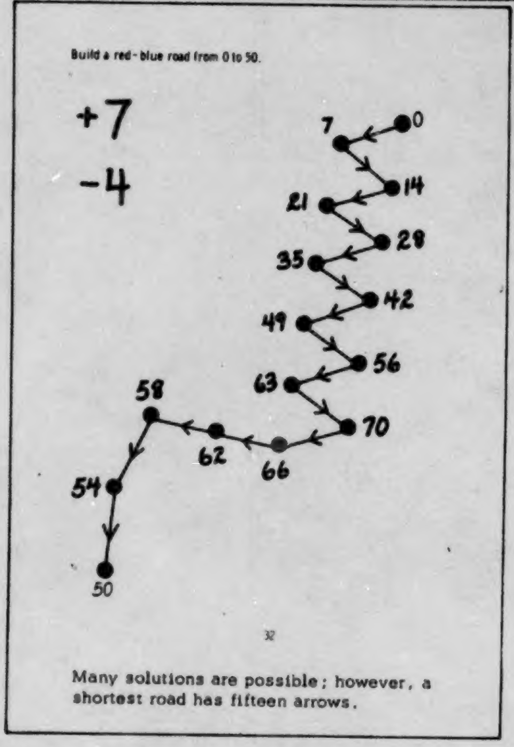
30



Label the dots.

3x
1
3
9
27
5
15
45
135
0

31



Build a red-blue road from 0 to 50.

+7
-4
0
7
14
21
28
35
42
49
56
63
70
66
62
58
54
50

32

Many solutions are possible; however, a shortest road has fifteen arrows.

W6 FISHING FOR NUMBERS (Lesson 1)

CAPSULE LESSON SUMMARY

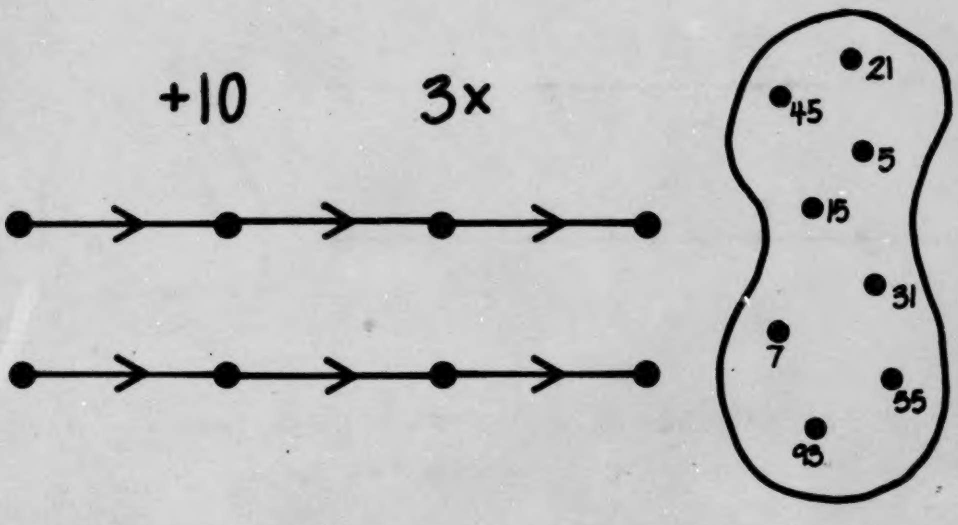
Put the numbers in a string picture in their proper places in an arrow picture. Begin the Workbook FISHING FOR NUMBERS.

MATERIALS

Teacher: Colored chalk
Student: Workbook FISHING FOR NUMBERS; pencil

DESCRIPTION OF LESSON

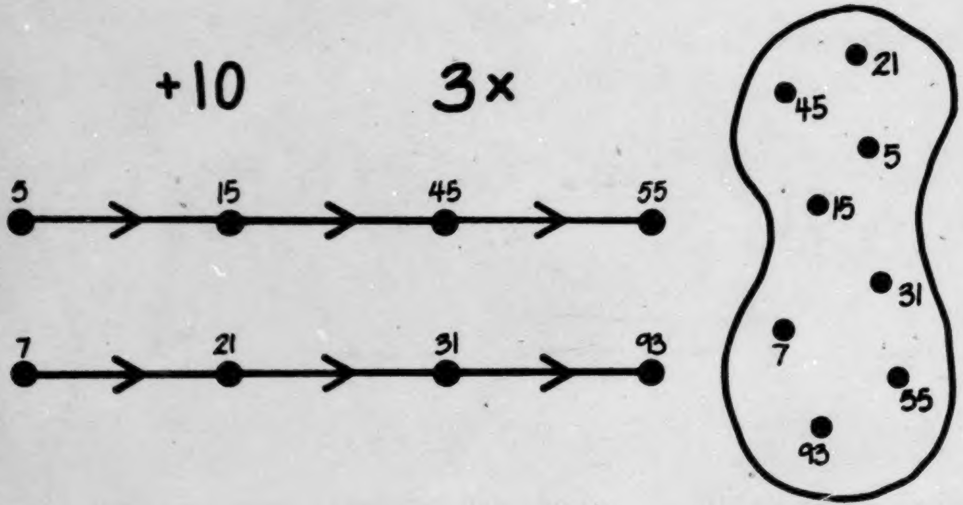
Draw these pictures on the board.



Remind your students of the "Fishing for Numbers" activities they did in the preceding year. If necessary, tell the students that such activities involve putting all of the numbers shown in a string in their proper places in an arrow picture.

Ask someone to suggest a number for the starting dot of one of the arrow roads. Pursue this choice and see where it leads. If it leads to an incorrect situation, let the students discover this and discuss why it is wrong and what they should do. A student may suggest that the smallest number must go at the beginning of one of the roads; if so, ask the student to convince the rest of the class. The students might suggest crossing out the numbers inside the string after they have been placed in the arrow picture. This procedure is questionable since in a trial-and-error method many mistakes may be made and new starts may be necessary. In the course of solving this problem collectively, the opportunity to discuss various strategies should arise, but allow ample opportunity for students to discover some good strategies themselves.

The completed arrow picture is given below.



Distribute copies of the Workbook FISHING FOR NUMBERS and allow about thirty minutes for individual work. At the end of the lesson, collect the workbooks for your review.

W7 FISHING FOR NUMBERS (Lesson 2)

CAPSULE LESSON SUMMARY

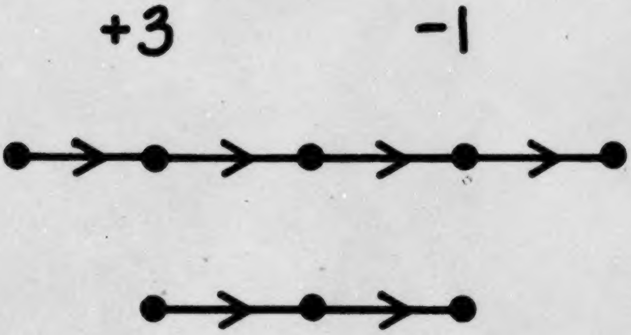
Put the numbers in a string picture in their proper places in an arrow picture. Continue working in the Workbook FISHING FOR NUMBERS. (This is the second of two lessons using this workbook.)

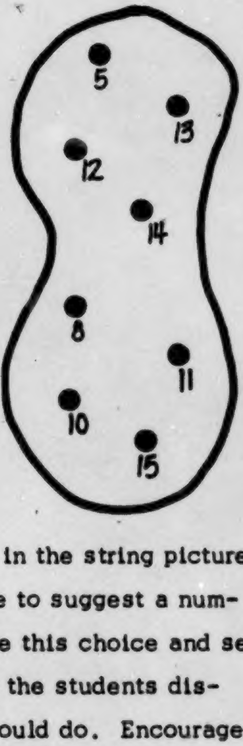
MATERIALS

Teacher: Colored chalk
Student: Workbook FISHING FOR NUMBERS; pencil

DESCRIPTION OF LESSON

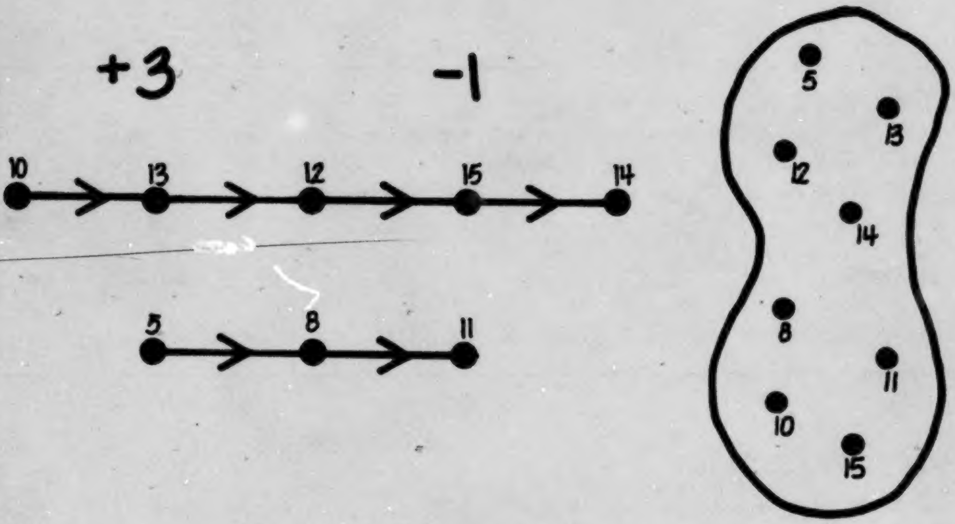
Draw these pictures on the board.





Tell the class that the problem is to put all the numbers in the string picture in their proper places in the arrow picture. Ask someone to suggest a number for the starting dot of one of the arrow roads. Pursue this choice and see where it leads. If it leads to an incorrect situation, let the students discover this and discuss why it is wrong and what they should do. Encourage the strategy of finding where the smallest and largest numbers are in each of these arrow roads. Continue until all the dots in the arrow picture are labeled. (See the next illustration.)

W 7

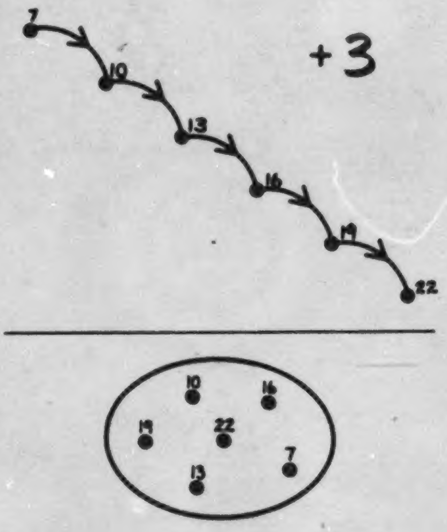

+3
-1
10
13
12
15
14
5
8
11
5
13
12
14
8
11
10
15

Distribute the students' copies of the Workbook FISHING FOR NUMBERS and allow about thirty minutes for individual work. At the end of the lesson, collect the workbooks for your review.

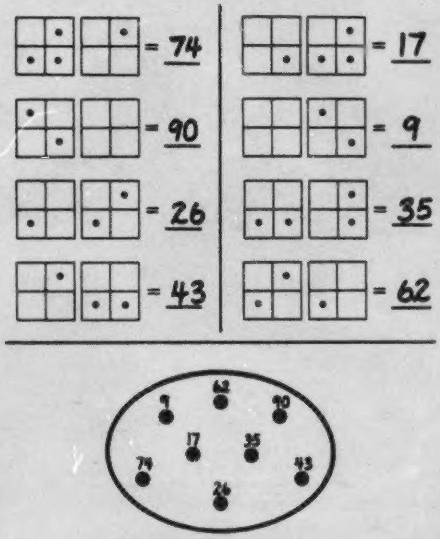
FISHING FOR NUMBERS

(Part III)

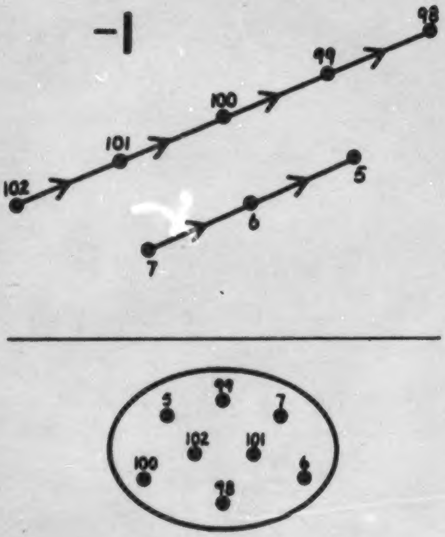
Fishing for Numbers (Part III)


+3
7
10
13
16
19
22
10
16
19
22
7
13

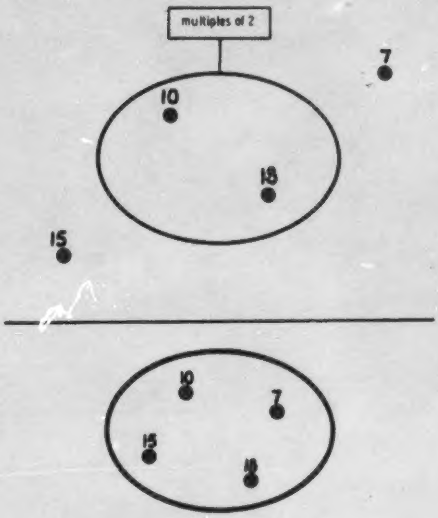
2


= 74
= 90
= 26
= 43
= 17
= 9
= 35
= 62
9
62
90
17
35
74
43
26

3

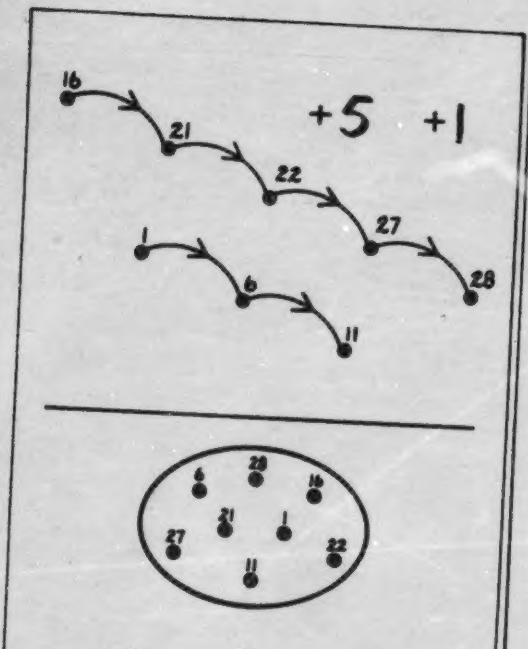

-1
98
99
100
101
102
5
6
7
5
99
7
102
101
100
6
98

4


multiples of 2
7
10
18
15
10
7
15
18

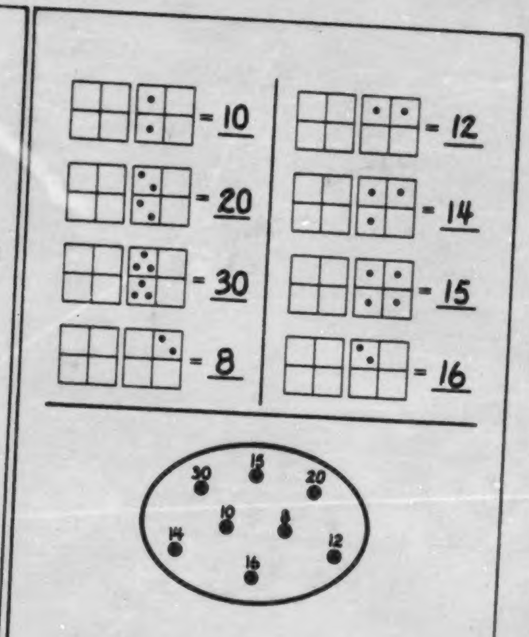
5

"10" and "18" may be interchanged;
"7" and "15" may be interchanged.



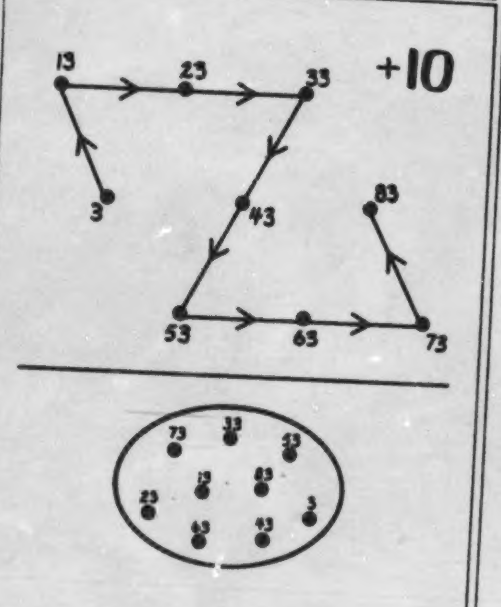
16
21
22
27
28
1
6
11
+5
+1
6
28
16
21
1
27
22
11

6



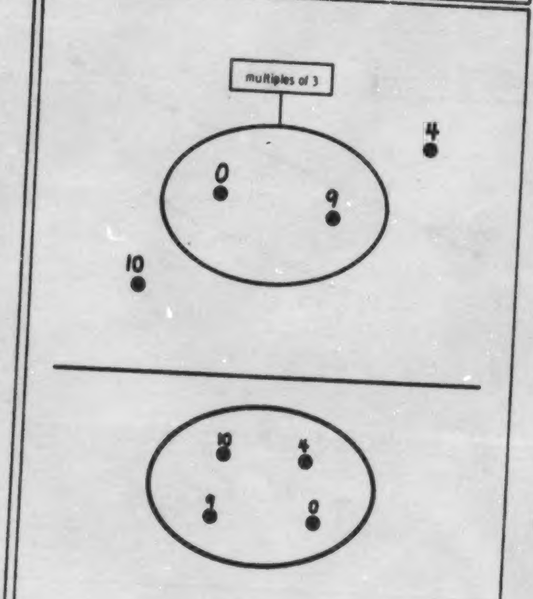
= 10
= 20
= 30
= 8
= 12
= 14
= 15
= 16
30
15
20
10
8
14
12
16

7



13
23
33
+10
3
43
83
53
63
73
73
33
13
13
83
23
3
63
43

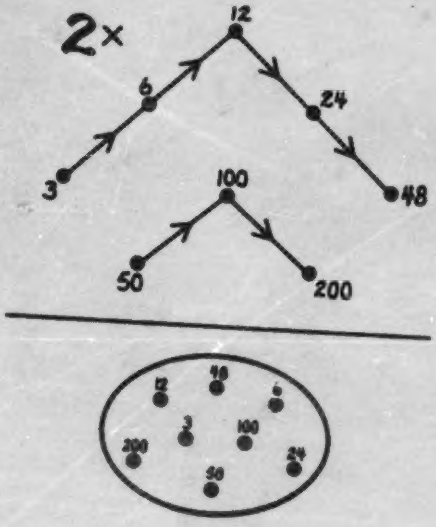
8



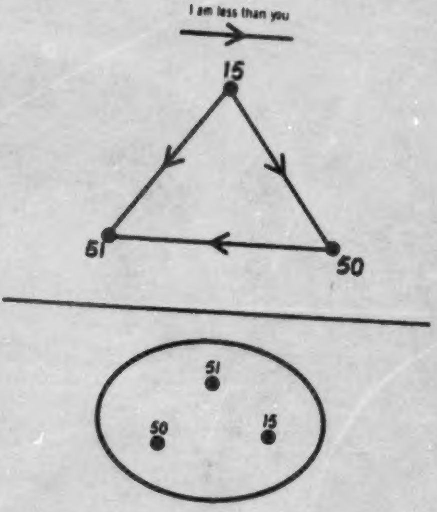
multiples of 3
4
0
9
10
10
4
9
0

9

"0" and "9" may be interchanged;
"4" and "10" may be interchanged.


2×
3
6
12
24
48
50
100
200
12
48
6
3
100
200
24
50

10


I am less than you
15
51
50
51
50
15

11

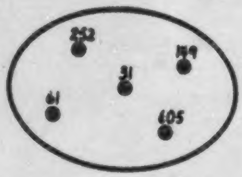
45
+16
61

14
9
+8
31

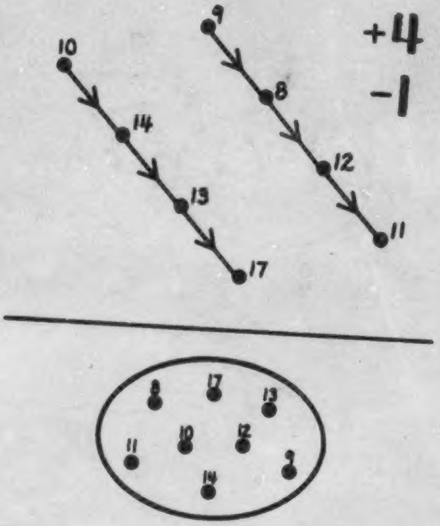
287
+318
605

70
+79
149

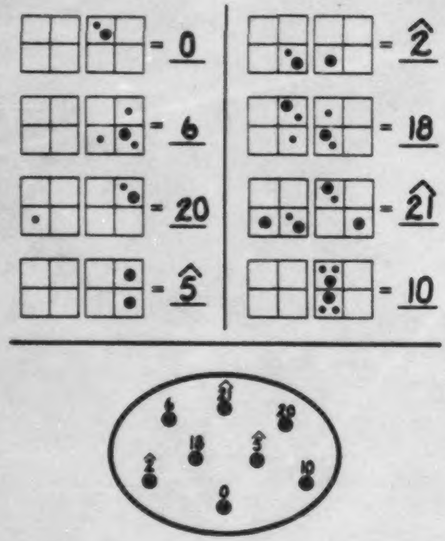
153
+99
252


252
149
31
61
605

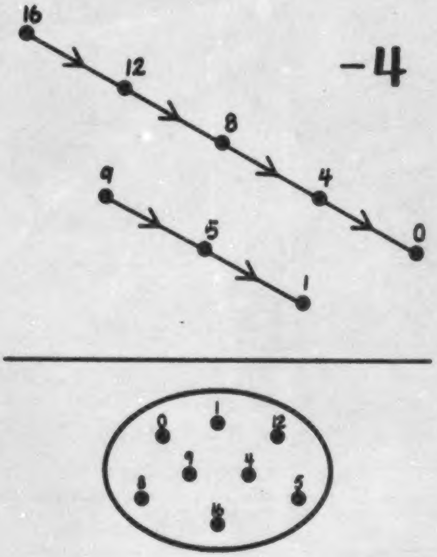
12


10
14
13
17
9
8
12
11
+4
-1
8
17
13
11
10
12
9
14

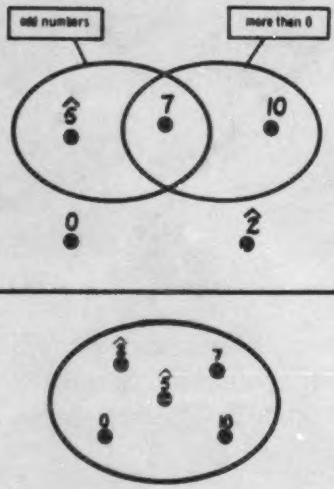
13


= 0
= 6
= 20
= 5̂
= 2̂
= 18
= 2̂1
= 10
6
2̂1
20
18
3̂
2̂
10
0

14

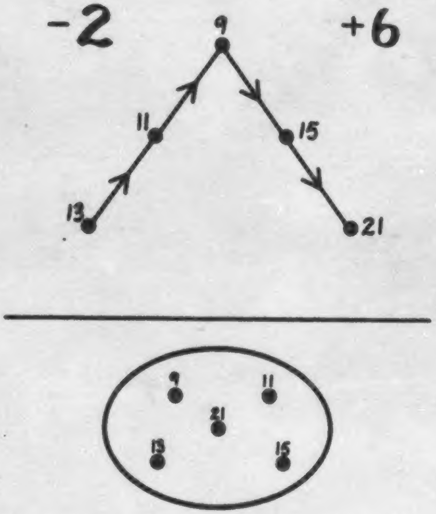

16
12
8
4
0
9
5
1
-4
0
1
12
9
4
8
5
16

15

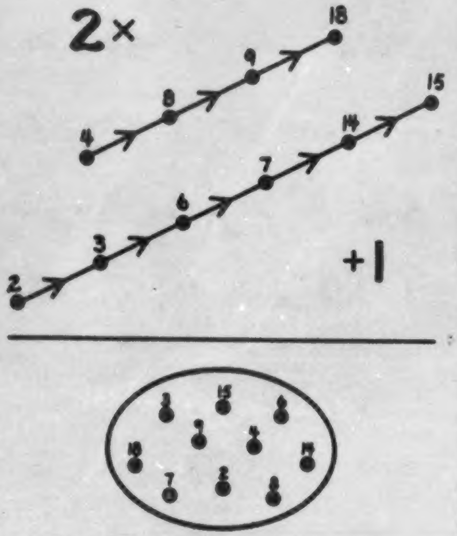

odd numbers
more than 0
5̂
7
10
0
2̂
7
0
10

16

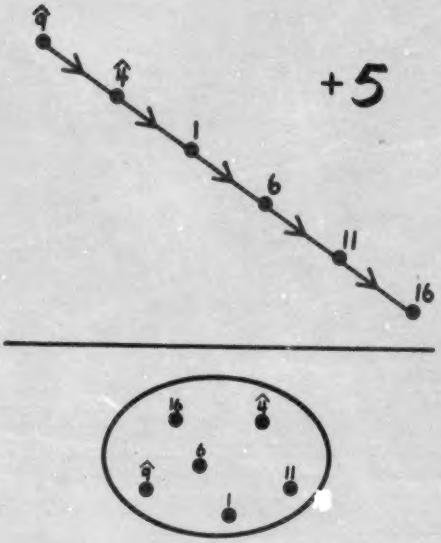
"0" and "2̂" may be interchanged.


-2
9
+6
11
15
13
21
9
11
21
13
15

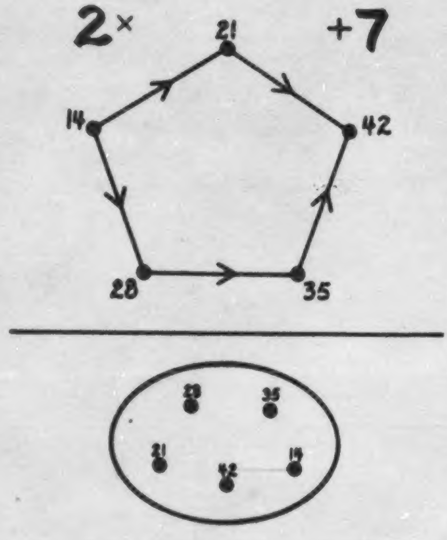
17


2×
+1
2
3
6
7
14
15
4
8
9
18

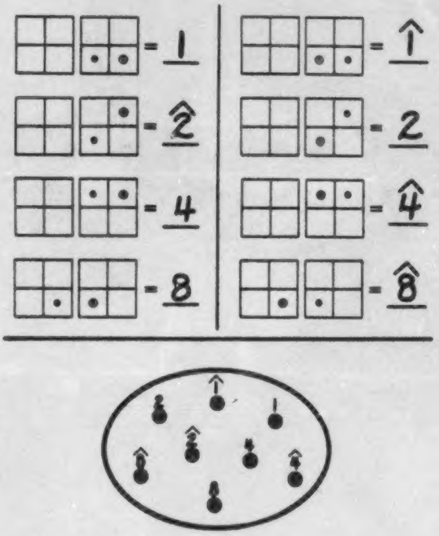
18



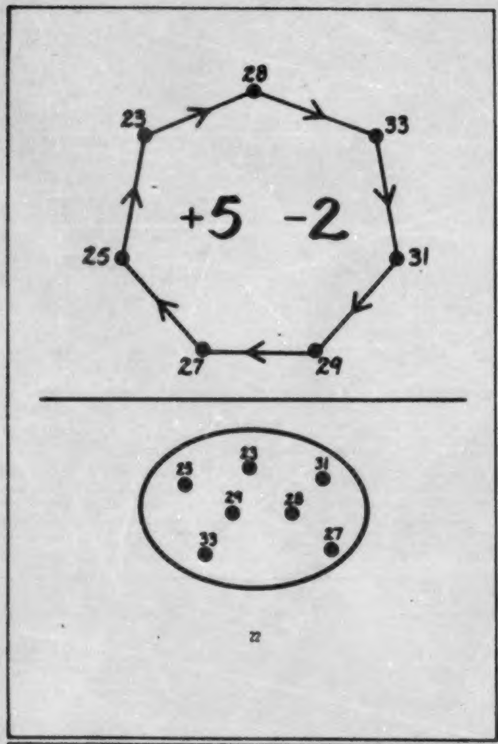
19


2×
+7
21
14
42
28
35

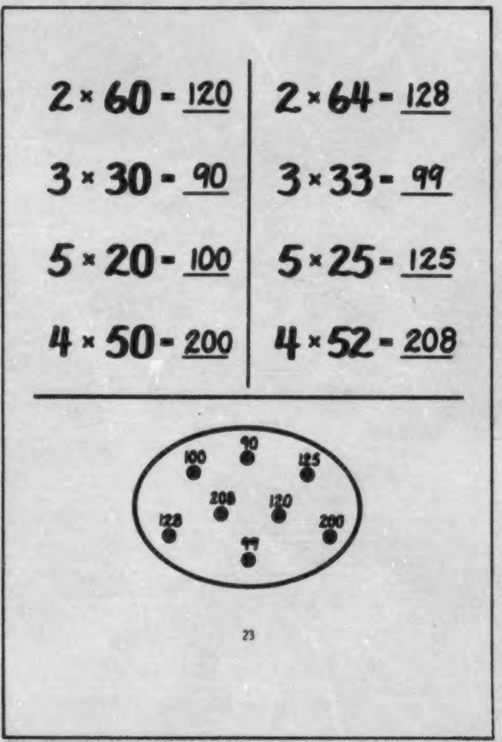
20



21


28
23
33
+5 -2
25
31
27
29
25
23
31
29
28
33
27

22

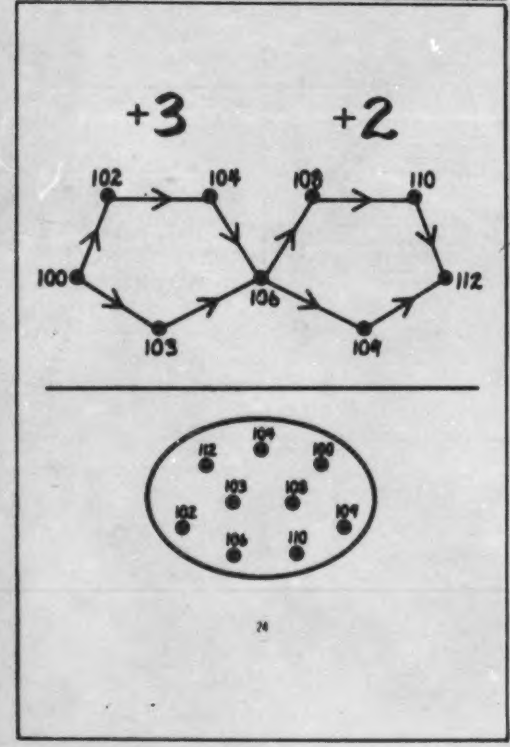


2 × 60 = 120
3 × 30 = 90
5 × 20 = 100
4 × 50 = 200

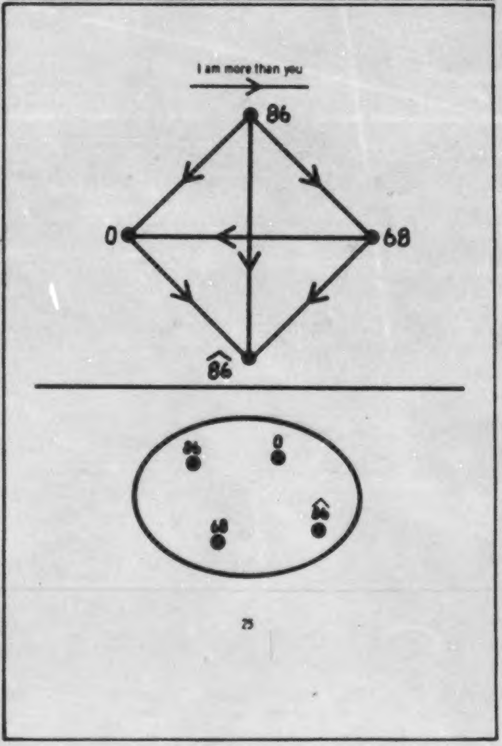
2 × 64 = 128
3 × 33 = 99
5 × 25 = 125
4 × 52 = 208

100
90
125
208
120
128
200
99

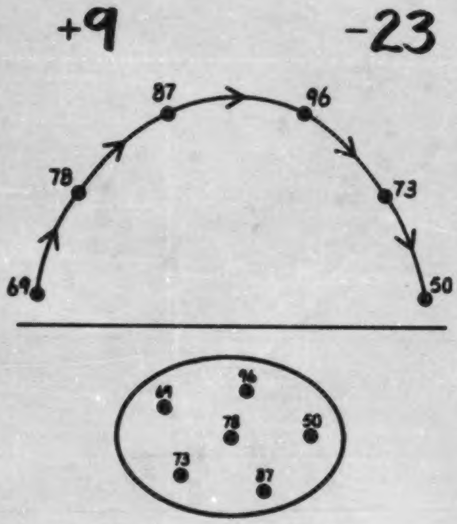
23


+3
+2
102
104
108
110
100
106
112
103
104
104
112
100
103
108
102
109
106
110

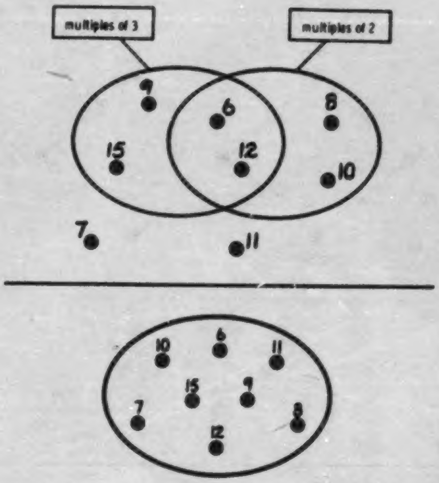
24


I am more than you
86
0
68
$\widehat{86}$
86
0
68
$\widehat{86}$

25

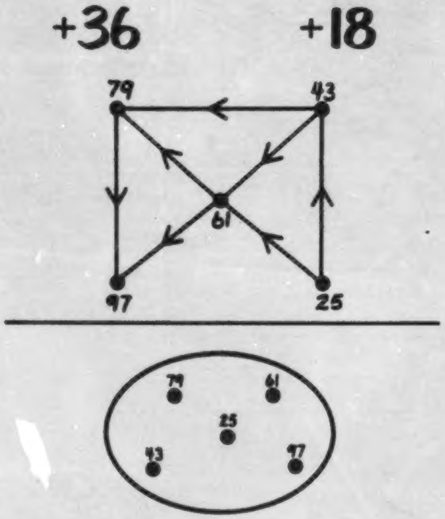

+9
-23
69
78
87
96
73
50
69
96
78
50
73
87

26

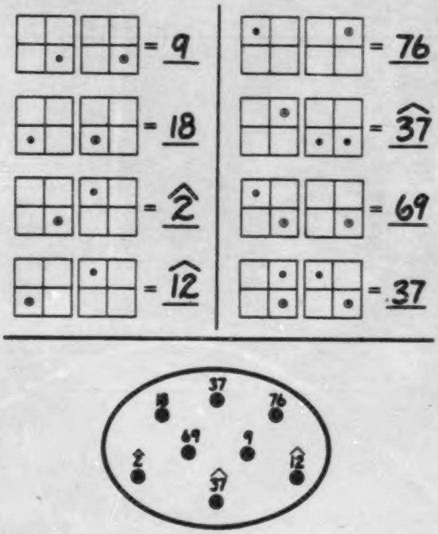

multiples of 3
multiples of 2
9
6
8
15
12
10
7
11
10
6
11
15
9
7
8
12

27

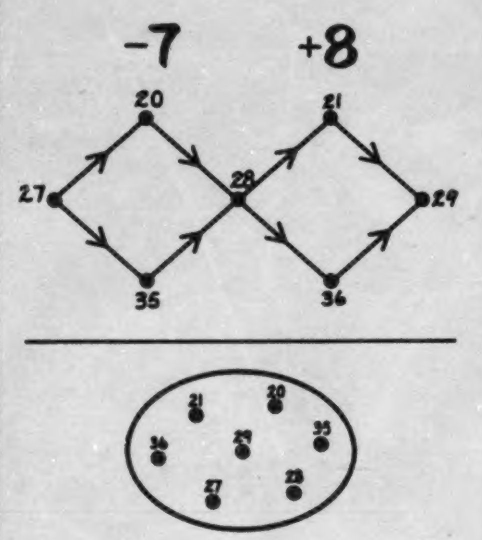
"9" and "15", "6" and "12", "8" and "10", "7" and "11" may be interchanged.


+36
+18
79
43
61
97
25
79
61
25
43
97

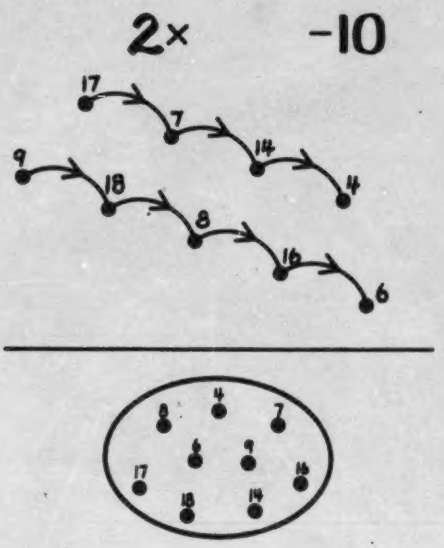
28


= 9
= 18
= $\widehat{2}$
= $\widehat{12}$
= 76
= $\widehat{37}$
= 69
= 37
37
18
76
69
9
$\widehat{2}$
$\widehat{12}$
$\widehat{37}$

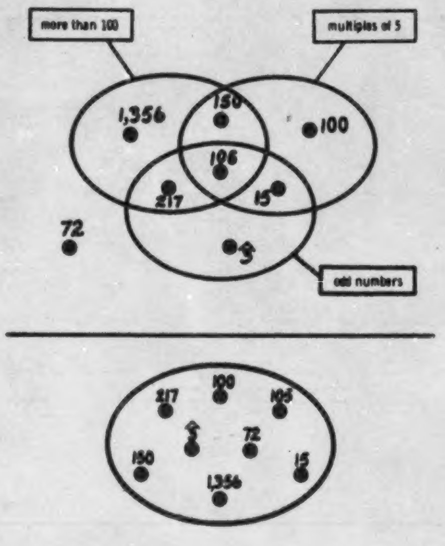
29


-7
+8

30


2×
-10

31


more than 100
multiples of 5
odd numbers
1,356
150
100
105
217
15
72
5
217
100
105
5
72
150
15
1,356

32

W8 A Spinner Game #1

CAPSULE LESSON SUMMARY

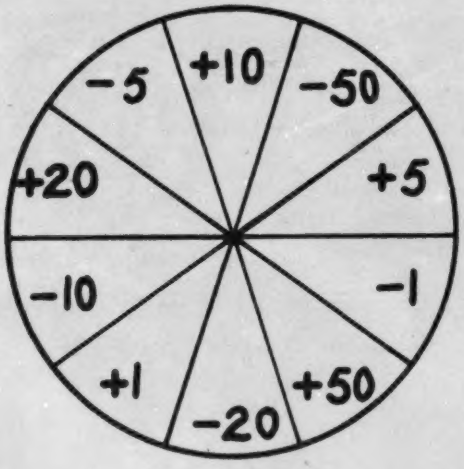
Using a spinner face with ten numerical relations, find some of the possible and impossible scores one can get by starting at 0, spinning the spinner four times, and following the number relations on which the spinner lands.

MATERIALS

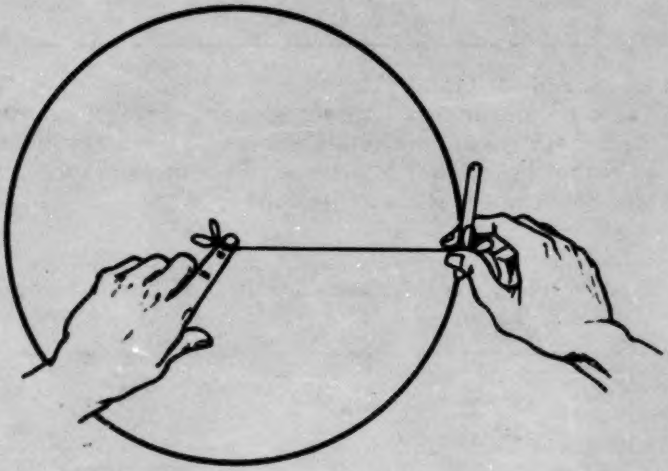
Teacher: A piece of chalk; a string 45 cm long (optional)
Student: Paper; pencil

DESCRIPTION OF LESSON

Before the lesson begins, draw this spinner face very carefully on the board. The circle should have a radius of about 40 cm.



One way to draw this circle is to use a piece of string about 45 cm long with loops tied at both ends. Put a piece of chalk through one loop and secure it tightly; put an index finger through the other loop and draw the circle as shown in the next illustration.



T: Have you seen anything like this before?

Let the students tell about any spinners that they might be acquainted with through games they play at home or at school. If necessary, mention that spinners usually have arrows attached at their centers.

T: Some children were playing a game with a spinner which looks like this one. Each child would spin the spinner four times and then starting at 0 follow the results of the four spins to get the score. Let's pretend we are playing this game. Where might the arrow point after the first spin?

A possible dialogue is given here.

S: +50.

T: These children always start at 0 and then record each of their spins.

Begin forming a number story on the board.

$$0 + 50$$

T: Where might the arrow point after the second spin?

S: +10.

Record "+10" in the number story forming on the board.

$$0 + 50 + 10$$

T: How many points would we have now?

S: 60.

T: Where might the arrow point after the third spin?

S: -1.

Record "-1" in the number story forming on the board.

$$0 + 50 + 10 - 1$$

T: How many points would we have now?

S: 59.

T: Where might the arrow point after the fourth spin?

S: +1.

Record "+1" in the number story forming on the board.

$$0 + 50 + 10 - 1 + 1$$

T: We would have spun four times, so our turn would be over. What would our score be?

S: 60.

Record "=60" to complete the number story.

$$0 + 50 + 10 - 1 + 1 = 60$$

T: What is another final score we could get if we played this spinner game?

Encourage the students to offer many suggestions and to explain how to obtain each score. Record the results of four spins and the final scores on the board as they are suggested. When necessary, help students to subtract a larger number from a smaller number by referring to the number line.

During the discussion ask the following questions and any others that you think are appropriate.

T: What is the highest score we could get playing this game? (200)
How could we get this score? ($0 + 50 + 50 + 50 + 50 = 200$)
What is the lowest score we could get? ($\widehat{200}$)
How could we get this score? ($0 - 50 - 50 - 50 - 50 = \widehat{200}$)
Is it possible to get a score of 0? (Yes)
Is there more than one way to get a score of 0? (Yes)
What is one of them? ($0 + 50 - 20 - 20 - 10 = 0$)

T: What is the next to the highest score we could get? (170)
How could we get this score? (0 + 50 + 50 + 50 + 20 = 170)
What is the next to the lowest score we could get? ($\widehat{170}$)
How could we get this score? (0 - 50 - 50 - 50 - 20 = $\widehat{170}$)

Continue this activity until you have a list of ten to fifteen scores on the board.

T: One child told me that his score was 115. Is this possible?

S: Yes.

Ask the students to tell you at least one of the two possible ways to get a score of 115.

$$0 + 50 + 50 + 10 + 5 = 115$$

$$0 + 50 + 50 + 20 - 5 = 115$$

T: Another child told me that her score was 150. Is this possible?

It is impossible to have a score of 150 after four spins. Encourage the students to attempt by trial and error to find a way to score 150 until several students observe that it is not possible in this game. Perhaps someone will be able to explain why a score of 150 is not possible, but do not insist upon an explanation.

NOTE: It is possible to get a score of 150 after three spins but not after four spins. It is possible to get a score of 100 after two spins but it is not possible to increase that score by 50 in two more spins. This is for your information only.

W 8

Draw a number line on the board.

Point to the list of scores recorded on the board.

T: I would like to put each of these scores on a number line in order. What is the largest number in this list?

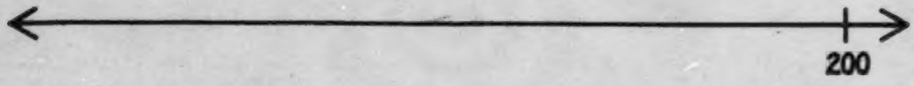
S: 200.

T: Where shall we put 200 on our number line?

The students very likely will indicate that 200 should be on the right.

NOTE: 200 could be assigned to any point on this number line but the students' previous experiences usually have involved number lines with the numbers increasing as you move to the right.

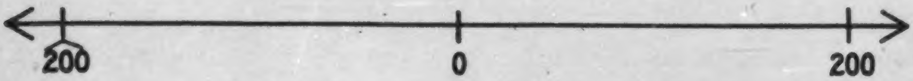
Ask a student to put 200 on the number line.



T: What is the smallest number in our list?

S: $\widehat{200}$.

Ask for a volunteer to put $\widehat{200}$ on the number line. Since $\widehat{200}$ is the smallest number in the discussion, it would be best if the student locates $\widehat{200}$ to the far left on the number line on the board. Draw and label a mark for 0 halfway between $\widehat{200}$ and 200 yourself.



Invite students to locate on the number line the other numbers in your list. Students may not locate the numbers exactly in the right place, but insist that the numbers be in the correct order. When $\widehat{170}$ is located on the number line, emphasize that $\widehat{170}$ is more than $\widehat{200}$.

T: There are many, many possible scores. We have found only some of them. On a piece of paper write down other possibilities.

Students should record some of the possible scores and how to get each of them after four spins. Walk around the room and look at the students' work. When a student finds a possible score which has not been discussed before, challenge the class to find a way to get that score. A possible dialogue is given.

T: Mark has found a way to get a score of 20. How could we get a score of 20?

S: 0 + 5 + 5 + 5 + 5 = 20.

T: Is that the way you got 20, Mark?

Mark: No, I calculated "0 + 50 - 10 - 10 - 10".

S: I did it another way; 0 + 20 - 10 + 20 - 10 = 20.

Continue this activity for a few minutes. Erase everything on the board except the spinner face.

W8

T: There are several numbers between 0 and 100 which are impossible to get as scores when you play this spinner game*. Try to find some of them.

As numbers are suggested record them on the board. Challenge the class to try to get those numbers as scores after exactly four spins. If your students discover a way to get any of the suggested numbers as scores, cross them off your list. Continue this activity for a few minutes.

Be careful not to conclude that all the remaining numbers are impossible to get as scores in the spinner game. A more accurate conclusion would be that your students have not found a way to get any of the remaining numbers as scores in the spinner game.

---

*The numbers between 0 and 100 which are impossible to get are:

33
37
48
52
63
67
73
77
78
82
83
84
87
88
92
93
97
99

This is for your information only.

W9 A Spinner Game #2

CAPSULE LESSON SUMMARY

Solve a detective story in which the secret number is a multiple of 11 and the other clues involve a string picture, the Minicomputer, and a spinner game.

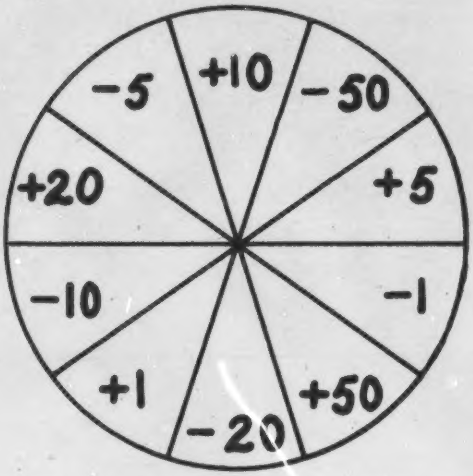
MATERIALS

Teacher: A piece of paper; a string 45 cm long (optional); colored chalk; Minicomputer set; two hand-calculators

Student: Paper; pencil

DESCRIPTION OF LESSON

Before the lesson write a large "55" on a piece of paper and fold the paper over so that the "55" is hidden. Also draw this spinner face very carefully to one side on the board.



T: Two children were playing the Spinner Game. Each child spun the spinner four times and then from the results of the four spins arrived at a final score. One of the children told me that her score was 100. Is this possible? (Yes)

W 9

Ask the students to tell you at least two ways to get a score of 100. A few of the possible ways are given below.

$$0 + 50 + 50 + 1 - 1 = 100$$

$$0 + 50 + 20 + 20 + 10 = 100$$

$$0 + 50 + 50 + 50 - 50 = 100$$

T: The other child told me that his score was 101. Is this possible?

It is impossible to have a score of 101 after four spins. Encourage the students to attempt by trial and error to find a way to score 101 until several students observe it is not possible in this game. Perhaps someone will be able to explain why a score of 101 is not possible, but do not insist upon an explanation.

NOTE: It is possible to get a score of 100 after two spins but is is not possible to increase that score by 1 in two more spins. It is impossible to get a score of 100 after one spin or three spins. This is for your information only.

Hold up the paper you prepared before class.

T: I have written the name of a secret number on this paper. If you are careful detectives, you can discover what this number is.

You may wish to ask a veteran student to explain what a detective story is for the benefit of students new to the program. If your students want to guess the number, you should emphasize that good detectives always look for clues until they know for sure what is the secret number.

First clue

T: Your first clue is that the secret number is a whole number and a multiple of 11. Name some numbers which are multiples of 11.

Let several students suggest multiples of 11. Perhaps one of the students will be able to count by elevens. Give two students hand-calculators. Ask the students with hand-calculators to turn them on and to do exactly what you tell them.

T: We can use the hand-calculators to find many of the multiples of 11. Is 0 a multiple of 11? (Yes)

Record "0" near the top of the board.

T: Push $\boxed{+}\boxed{1}\boxed{1}\boxed{=}$ (read: plus eleven equals). What number is on the display?

S: 11.

Record "11" below "0" on the board.

T: What is the next multiple of 11?

S: 22.

T: Push $\boxed{=}$. What number is on the display?

S: 22.

Ask if someone can predict what the next multiple of 11 is before it is found on the hand-calculators.

W 9

Continue this activity until you have the multiples of 11 from 0 to 44 recorded on the board.

0
11
22
33
44

T: Do you see a pattern?

Let the students comment and then ask the students to continue naming the multiples of 11 until they reach 99.

Allow several students to predict what the next multiple of 11 is before asking the students with calculators to push $\boxed{=}$ again. Record "110" near the top of the board. With the help of your students continue listing the multiples of 11 until the list includes 143.

0
11
22
33
44
55
66
77
88
99

110
121
132
143

T (pointing to the four numbers in the second column): Do you see a pattern to these numbers?

Let the students comment. Continue listing the multiples of 11 with your class until your list includes 198. Allow several students to predict what the next multiple of 11 is before asking the students with hand-calculators to press [=] again. Record 209 on the board.

T: Are these all the multiples of 11?

S: No, we could add 11 to 209, and then add 11 to that number, and then 11 to that number, and so on.

S: The list goes on and on.

T: I will put three dots at the end of our list to indicate that the list of multiples of 11 continues beyond the numbers we have recorded on the board.

0
11
22
33
44
55
66
77
88
99

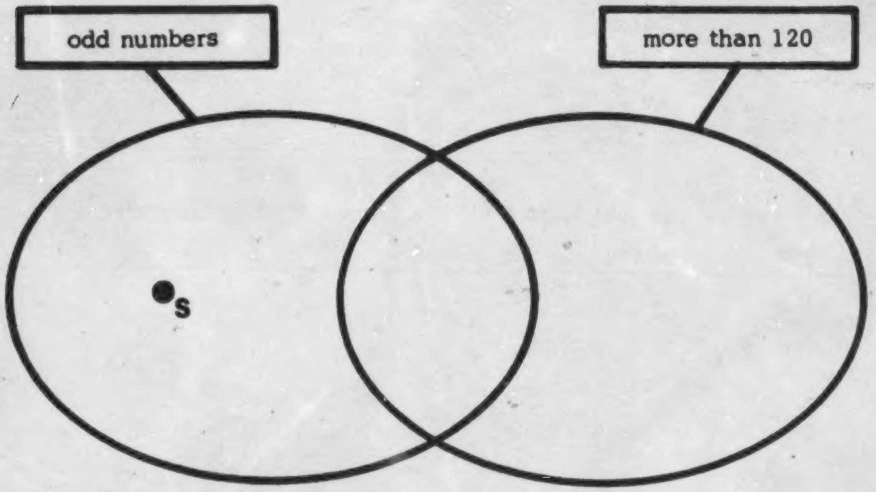
110
121
132
143
154
165
176
187
198
209
⋮

Second clue

T: The secret number is a multiple of 11. Here is another clue.

W 9

Draw this string picture on the board.



T: I have put a dot for the secret number in this string picture. What does this clue tell us about the secret number?

Encourage the students to comment. When your class agrees that the secret number is an odd number and that it is not more than 120, they should indicate which numbers in the list cannot be the secret number. A possible dialogue is given here.

T: Could the secret number be more than 120?

S: No. We can erase all the numbers which are more than 120, because the secret number isn't in the red string.

Ask the class for agreement and then erase all the numbers larger than 120.

T: Should I erase the three dots?

S: Yes, they indicate numbers larger than 120.

Erase the dots.

T: Is the secret number an odd number?

S: Yes, it is in the blue string.

S: We can erase all the even numbers.

T: Which numbers should I erase?

As the students name even numbers in the list, erase them. Finally the list should be reduced to five numbers.

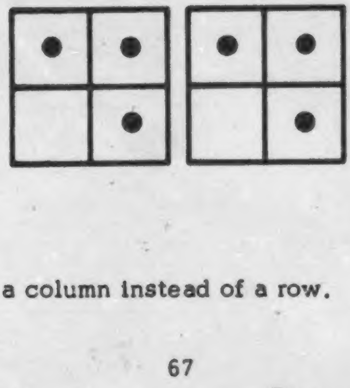
11, 33, 55, 77, 99*

T: One of these five numbers is the secret number, but we don't know which one.

S: We need another clue.

Third clue

Put this configuration on the Minicomputer.



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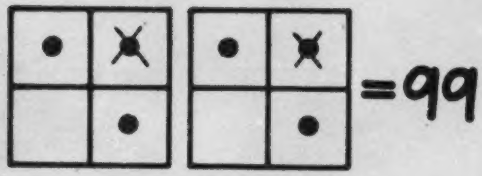
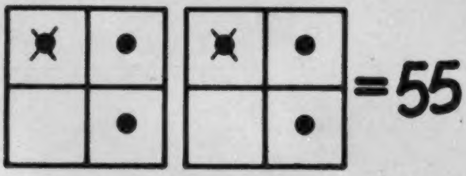
[*] Your list will appear as a column instead of a row.

T: We can put the secret number on the Minicomputer by removing two of these checkers. What could the secret number be?

Let students respond. When a student suggests that one of the numbers in the list could be the secret number, ask the student to show which two checkers should be removed. Your students should determine quickly that 99 and 55 could be the secret number.





Encourage the students to explain why 11, 33, and 77 cannot be the secret number but do not expect precise explanations.

NOTE: For your information only, we present an argument why 11, 33, and 77 cannot be the secret number. 11 cannot be the secret number because the smallest possible number from the third clue results from removing the checker from the 80-square and the checker from the 40-square; that is, the number 23. 33 cannot be the secret number because the next to smallest possible number from the third clue results from removing the checker from the 80-square and the checkers from the 10-square; that is, the number 53. 77 cannot be the secret number because after removing the checker from the 80-square, 63 is on the Minicomputer which is smaller than 77 and a second checker has not yet been removed.

Erase "11", "33", and "77".

Fourth clue

T: The secret number is either 55 or 99. Listen carefully to this clue. The secret number is one of the numbers we could get as a final score if we are playing the spinner game. If you know the secret number, write it on a piece of paper.

Look at many of the students' answers. Ask how you could get a final score of 55. Two of the possibilities are: 0 + 50 + 20 - 10 - 5 and 0 + 20 + 20 + 10 + 5.

Ask why a score of 99 is impossible. Your class might conclude that 99 is possible after three spins but not after four spins.

Reveal that 55 is the secret number.

W10 Road Map

CAPSULE LESSON SUMMARY

Find the lengths of routes from one place to another on a road map. In doing so, add two digit and three digit numbers.

MATERIALS

Teacher: None
Student: Workbook ROAD MAP; colored pencils

DESCRIPTION OF LESSON

Distribute copies of the Workbook ROAD MAP. Ask the students to look at page 2 and to tell you the names of many of the places on the map. Perhaps one of the students will recognize that the map is of Yellowstone and Grand Teton National Parks. If necessary, tell the students this yourself. Encourage the students to share what they know about Yellowstone and Grand Teton National Parks and the places indicated on the map. Tell the students that the map tells how far it is in kilometers from one place to another.

Ask the students to look at the cover and to tell you about the picture.

T: This is a picture of a signpost somewhere in Yellowstone National Park. This signpost indicates that Old Faithful is 27 km to our left and that Lake is 32 km to our right. Where might we see this signpost? Look again at the map on page 2.

S: In West Thumb.

T: There are several routes indicated on the map on page 2. The solid blue line indicates a route between which two cities?

S: Between Old Faithful and Mammoth Hot Spring.

T: It goes through which other city?

S : Madison Junction.

T : How many kilometers is it from Old Faithful to Mammoth Hot Springs? How could we find the length of the solid blue route?

S : We could add 55 and 25.

T : The distance from Mammoth Hot Springs to Madison Junction is 55 km and from Madison Junction to Old Faithful is 25 km. We can find the distance from Mammoth Hot Springs to Old Faithful by calculating "55 + 25".

Write this addition problem on the board.

$$\begin{array}{r} 55 \\ +25 \\ \hline \end{array}$$

T : Write this addition problem inside the solid blue rectangle on page 3 and then solve it.

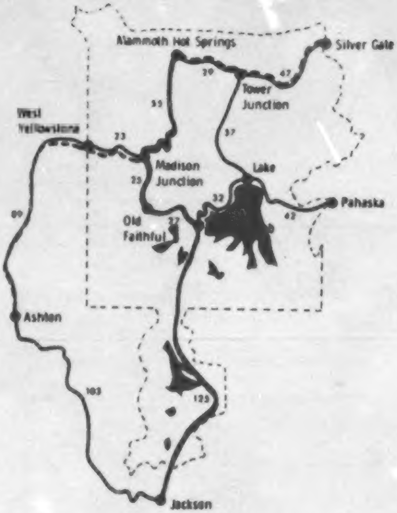
When most of the students have solved this problem, invite one of them to do the addition at the board.

$$\begin{array}{r} 1 \\ 55 \\ +25 \\ \hline 80 \end{array}$$

Conclude that the length of the solid blue route is 80 km and then ask the students to complete the sentence inside the solid blue rectangle on page 3.

Repeat this activity for the route indicated by a solid red line. Emphasize that the columns of numbers in an addition problem should be lined up carefully so that all the ones' digits are in the same column, all the ten's digits are in the same column, and all the hundred's digits are in the same column.

Ask the students to continue working independently. Encourage them to read all of the instructions on every page. Some of the students may need your help because of the workbook's reading level.

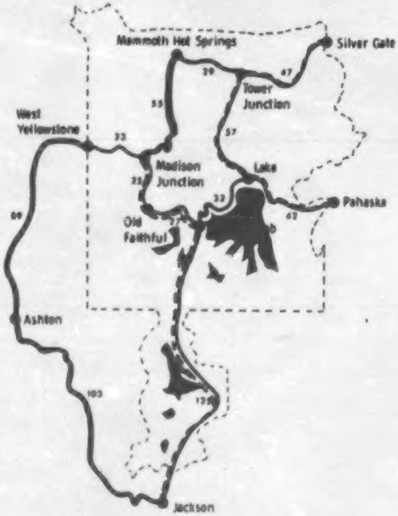

Mammoth Hot Springs
Silver Gate
29
47
Tower Junction
53
West Yellowstone
23
37
Madison Junction
Lake
23
32
Pahaska
89
Old Faithful
27
42
Ashton
103
125
Jackson

The solid blue route is 80 km long.

The solid red route is 152 km long.

The blue dashed route is 76 km long.

The red dashed route is 112 km long.


Mammoth Hot Springs
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Tower Junction
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West Yellowstone
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37
Madison Junction
Lake
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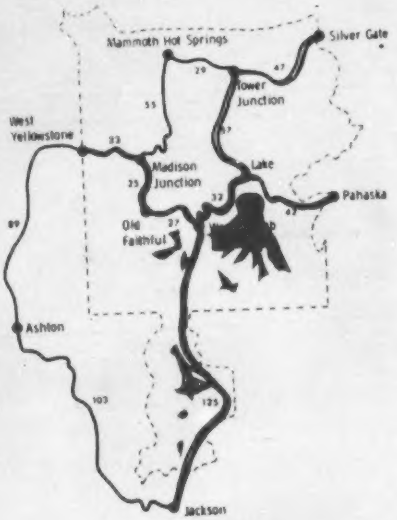
32
Pahaska
89
Old Faithful
27
42
Ashton
103
125
Jackson

The solid blue route is 192 km long.

The solid red route is 84 km long.

The blue dashed route is 146 km long.

The red dashed route is 177 km long.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Tower Junction
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West Yellowstone
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37
Madison Junction
Lake
25
32
Pahaska
89
Old Faithful
27
W b
42
Ashton
103
125
Jackson

6

Find the shortest route between these places.

Lake to Tower Junction: 57 km
Tower Junction to Silver Gate: 47 km

Lake to Tower Junction to Silver Gate: 104 km
Color this route red.

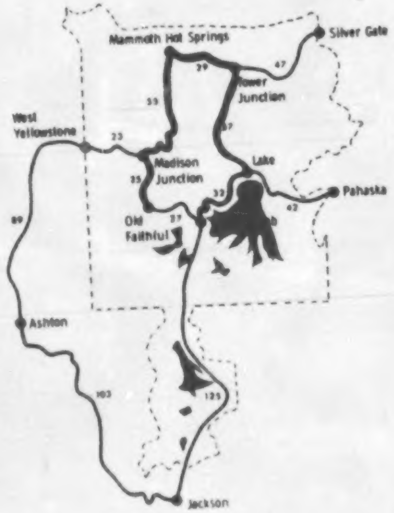
West Thumb to Old Faithful: 27 km
Old Faithful to Madison Junction: 25 km
Madison Junction to West Yellowstone: 23 km

West Thumb to Old Faithful to
Madison Junction to West Yellowstone: 75 km
Color this route blue.

Pahaska to Lake: 42 km
Lake to West Thumb: 32 km
West Thumb to Jackson: 125 km

Pahaska to Lake to West Thumb to Jackson: 199 km
Color this route green.

7


Mammoth Hot Springs
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West Yellowstone
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37
Madison Junction
Lake
25
32
Pahaska
89
Old Faithful
27
b
42
Ashton
103
125
Jackson

8

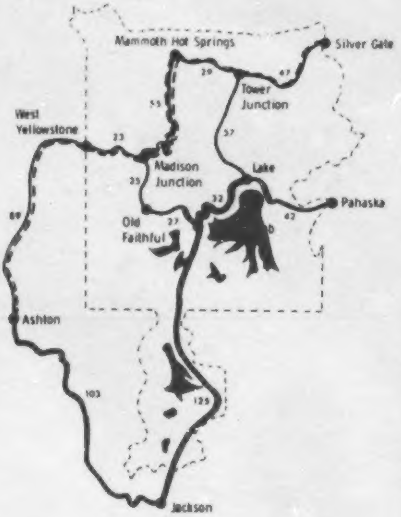
Start at Old Faithful
Plan a trip which goes to exactly three other places. Color your route red.
How long is this trip? 109 km

Many other solutions are possible.

Plan a trip from Mammoth Hot Springs to West Thumb.
Color your route blue.
How long is this trip? 118 km

Other solutions are possible.

9


Mammoth Hot Springs
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West Yellowstone
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Madison Junction
Lake
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Pahaska
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89
Old Faithful
27
b
Ashton
103
125
Jackson

10

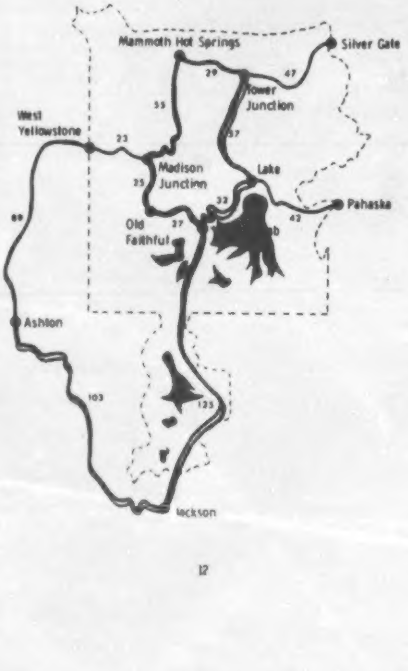
The solid blue route is 101 km long.

The solid red route is 260 km long.

The blue dashed route is 167 km long.

The red dashed route is 131 km long.

11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Pahaska
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Old Faithful
27
b
Ashton
103
125
Jackson

12

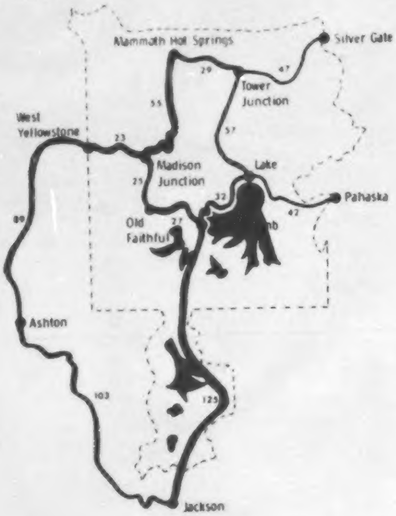
Start at Lake. Plan a trip which is less than 200 km long. Go to as many places as you can. Color your route red. How long is this trip? 193 km

Many solutions are possible; however, this solution goes to as many places as possible.

Start at Tower Junction. Plan a trip which is more than 400 km long. Go to as few places as you can. Color your route blue. How long is this trip? 406 km

Other solutions are possible. The only way to visit fewer places would be to go to one town more than once on the trip.

13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Tower Junction
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West Yellowstone
23
57
Madison Junction
Lake
25
32
Pahaska
42
89
Old Faithful
27
Ashton
103
125
Jackson

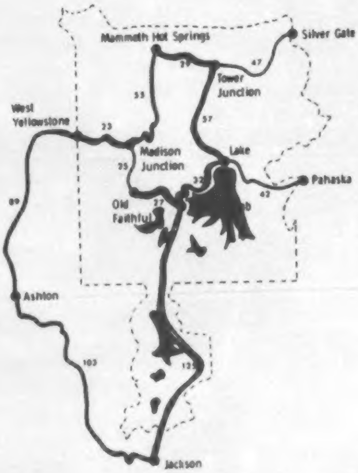
Plan three different trips from Mammoth Hot Springs to Jackson.
Color each route a different color.

What color is the shortest route? Blue
How long is this trip? 232 km

What color is the longest route? Green
How long is this trip? 270 km

Your longest trip is 38 km longer than your shortest trip.

Other solutions are possible.


Mammoth Hot Springs
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23
57
Madison Junction
Lake
25
32
Pahaska
42
89
Old Faithful
27
Ashton
103
125
Jackson

Plan a very long trip which starts at Old Faithful, goes to at least six other places, and then returns to Old Faithful.
Color your route red.
How long is this trip? 567 km

Other solutions are possible.

W11 ROLLERSKATING 37 (Lesson 1)

CAPSULE LESSON SUMMARY

Each whole number is on one of three +3 arrow roads; locate 37. Do a similar activity to locate 37 on one of five +5 arrow roads. Given one +4 arrow road, draw three other +4 arrow roads so that every whole number belongs on one of them. Then locate 37.

MATERIALS

Teacher:	Workbook Poster #1 for UPG-III; thick black marker or crayon; one copy of the Story-workbook ROLLERSKATING 37
Student:	Story-workbook ROLLERSKATING 37; pencil; colored pencils

DESCRIPTION OF LESSON

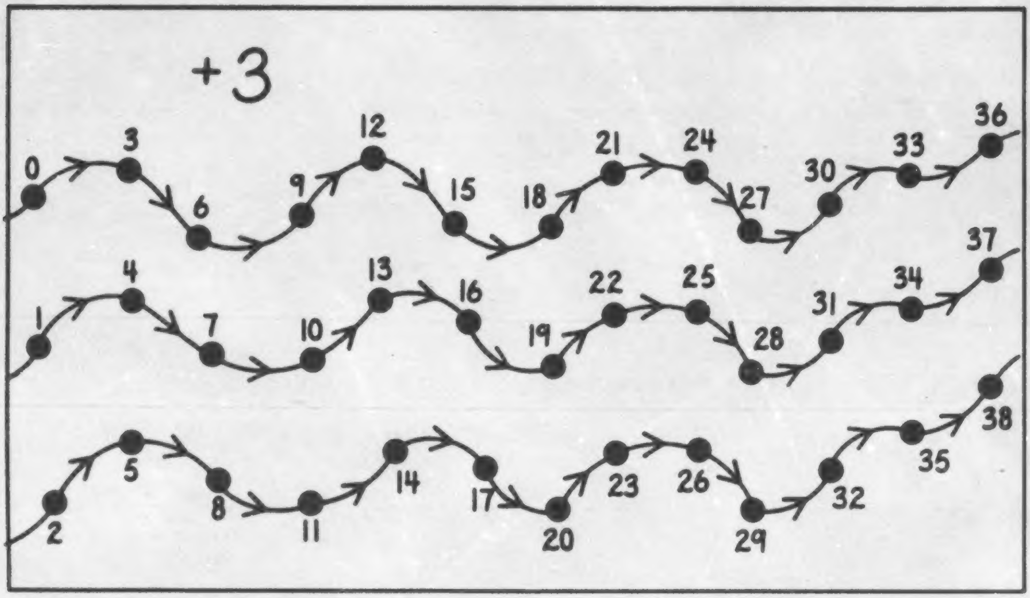
Distribute copies of the Story-workbook ROLLERSKATING 37. Briefly discuss the picture on the cover.

Pages 3-5

Read or ask students to read these pages aloud. While the students are searching for 37 on pages 4 and 5, display Workbook Poster #1. When several students think they have found 37, ask one of them to label the dot for 37 on the poster and to explain why that dot is for 37. The last dot on the right on the second arrow road is for 37.

Ask the students to label all the dots on pages 4 and 5. Invite the students who finish quickly to label the dots on the poster with a thick black marker or crayon so the numerals can be easily seen by all the students.
(See the illustration on the next page.)

W 11


+3
0
3
6
9
12
15
18
21
24
27
30
33
36
1
4
7
10
13
16
19
22
25
28
31
34
37
2
5
8
11
14
17
20
23
26
29
32
35
38

T: How many +3 arrow roads are on this poster? (Three)
What patterns do you see?

There are several patterns the students can observe in this arrow picture, but these two patterns should be emphasized:

- each of the whole numbers between 0 and 38 is represented by a dot on the poster;
- each number in the second road and in the third road is 1 more than the number directly above it.

T: What can you tell me about the numbers in the first arrow road?

S: All the numbers in the first arrow road are multiples of 3.

T: Since these arrow roads extend beyond the numbers which are on this poster, all of the multiples of 3 are in the first arrow road. What can you tell me about the numbers in the second +3 arrow road?

S: None of them is a multiple of 3.

T: How much more is each of the numbers in the second arrow road than the closest multiple of 3? For example, let's look at 13. Which multiple of 3 is the closest to 13? (12)
How much more is 13 than 12? (1 more)
So 13 is 1 more than a multiple of 3.
How much more is 22 than the closest multiple of 3? (1 more)

Continue this activity with a few more of the numbers in the second arrow road and then conclude that all the numbers in the second arrow road are 1 more than a multiple of 3.

T: Since these arrow roads extend in both directions, all the numbers which are 1 more than a multiple of 3 are in the second arrow road. What can you tell me about the numbers in the third arrow road?

S: They are all 2 more than a multiple of 3.

T: Let's look at 17 as an example. Is 17 two more than a multiple of 3?

S: Yes, 15 is a multiple of 3 and 15 + 2 = 17.

T: What is the closest multiple of 3 to 17? (18)
How much less is 17 than 18? (1 less)

Continue this activity with a few more numbers in the third arrow road and conclude that all the numbers in the third arrow road are 2 more (1 less) than a multiple of 3.

Pages 6 and 7

After page 6 has been read aloud, let the students predict on which road 37 is skating. Ask the students to label all the dots on pages 6 and 7 and to circle the dot for 37. The students who finish quickly should continue on pages 8 and 9. When many students have completed labeling the dots on pages 6 and 7, continue the collective lesson.

T: What patterns do you see on pages 6 and 7?

Students should suggest many patterns, including the following:

- each of the whole numbers is on one of these +5 arrow roads;
- all the multiples of 5 are in the first +5 arrow road;
- all the numbers which are 1 more than a multiple of 5 are in the second arrow road;
- all the numbers which are 2 more than a multiple of 5 are in the third arrow road;
- all the numbers which are 3 more (2 less) than a multiple of 5 are in the fourth arrow road;
- all the numbers which are 4 more (1 less) than a multiple of 5 are in the fifth arrow road.

If none of the students mention that the final digits of the numbers in each arrow road are alternating between two numbers (0,5; 1,6; 2,7; 3,8; and 4,9), mention this yourself.

Pages 8 and 9

Read or ask students to read pages 8 and 9. Some of the students will have already determined that 37 is not on the same arrow road as 28. Ask the students to label the dots in this arrow road and to draw the three missing +4 arrow roads. Allow a few minutes for this activity and then collect the story-workbooks for use in W12.

W12 ROLLERSKATING 37 (Lesson 2)

CAPSULE LESSON SUMMARY

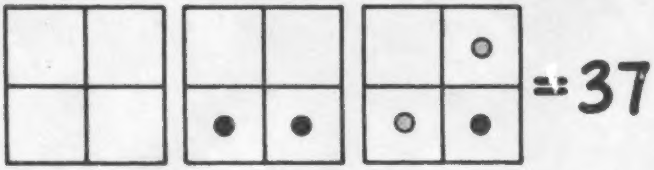
Display 37 on the Minicomputer in a variety of ways. Remove some checkers from a given configuration on the Minicomputer so that the number will be 37.

MATERIALS

Teacher: Minicomputer set[*]
Student: Story-workbook ROLLERSKATING 37; pencil

DESCRIPTION OF LESSON

Ask a student to put 37 on the Minicomputer. Accept any correct configuration; for example,



T: What number is on the Minicomputer with red checkers? (21)

. . . with blue checkers? (10)

. . . with yellow checkers? (6)

Write this number story on the board.

$$21 + 10 + 6 = 37$$

Ask for volunteers to show 37 in other ways. Continue recording number stories about the configurations on the board. Ask for a configuration using negative checkers if none have been displayed. A few of the possible configurations for 37 using negative checkers are shown on the next page.

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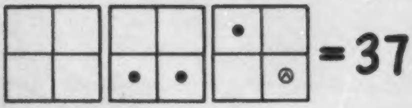
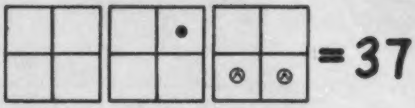
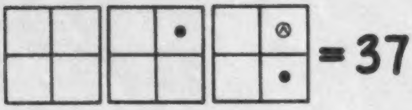
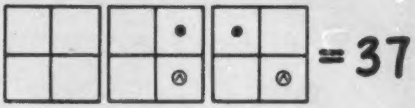
[*] Two sets of checkers are needed for this lesson. You will need to borrow a set from another teacher.

W 12



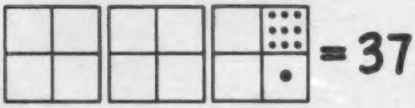


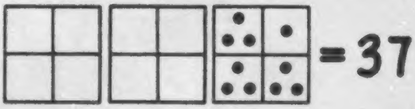


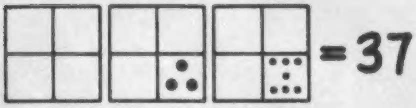


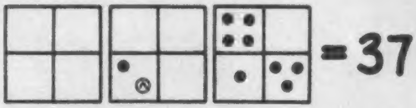
T: I would like to put 37 on the Minicomputer using exactly ten checkers. Is this possible? (Yes)

Invite students to put 37 on the Minicomputer using exactly ten checkers. A few of the possible configurations are shown here.









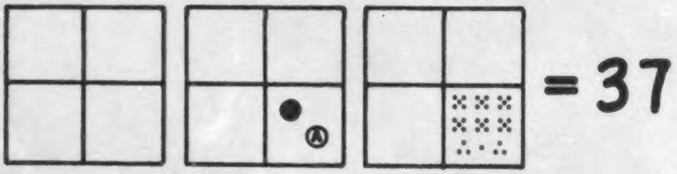
T: What is the largest number of checkers we could use to put 37 on the Minicomputer?

A student very likely will respond with the largest number of regular checkers.

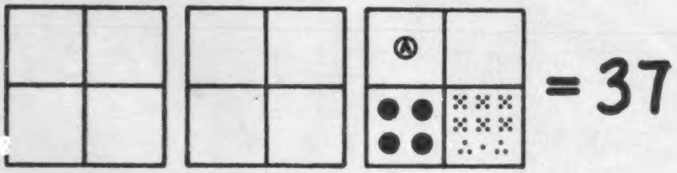
S: Thirty-seven, we could put thirty-seven checkers on the 1-square.

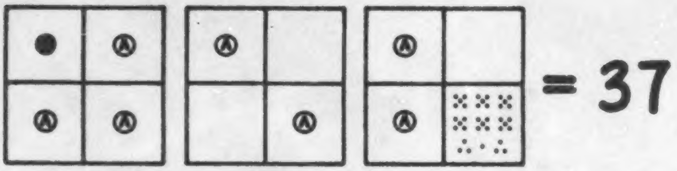
T: Could we put 37 on the Minicomputer using more than thirty-seven checkers? (Yes)

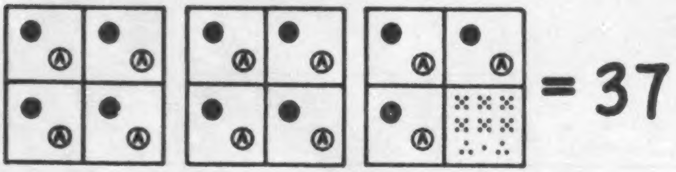
Invite a student to put 37 on the Minicomputer using more than thirty-seven checkers. (See the illustration on the next page.)



Ask for other volunteers to put 37 on the Minicomputer using even more checkers. Several of the many possible configurations are shown here.







T: What is the largest number of checkers we could use to put 37 on the Minicomputer?

S: There is no largest number; we can always put more checkers on the Minicomputer.

Conclude with your class that if negative checkers can be used, then there is no largest number of checkers which can be used to put 37 on the Minicomputer.

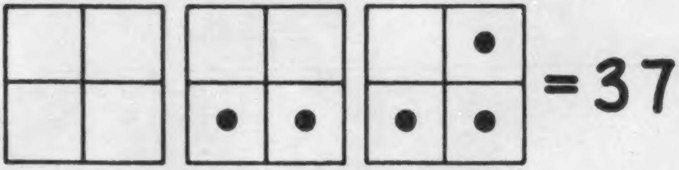
W 12

Remove the checkers from the Minicomputer.

T: What is the smallest number of checkers we could use to put 37 on the Minicomputer?

A possible dialogue is given here.

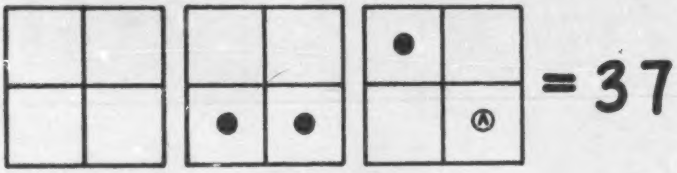
S: I can put 37 on the Minicomputer using five checkers.

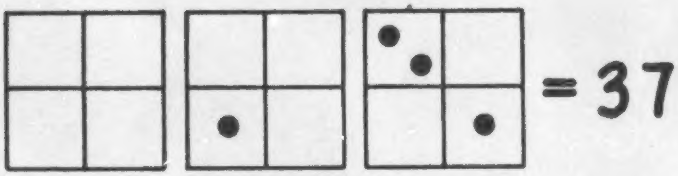


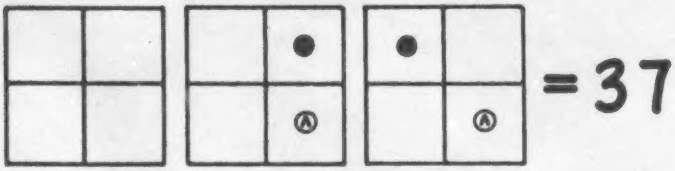
T: Could we put 37 on the Minicomputer using less than five checkers?

S: I can put 37 on the Minicomputer using four checkers.

Ask the student to put 37 on the Minicomputer using four checkers. A few of the possible configurations are shown here.



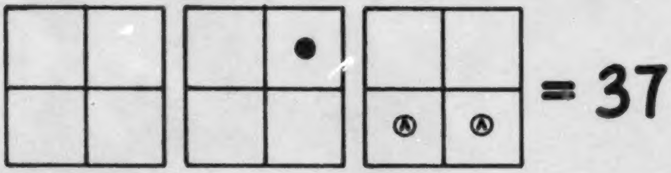


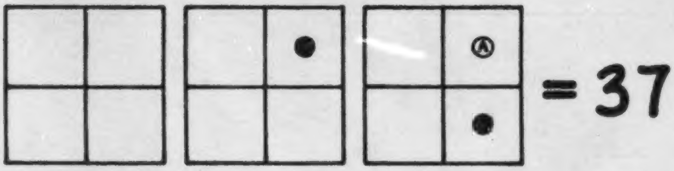


T: Could we put 37 on the Minicomputer using less than four checkers?

S: I can put 37 on the Minicomputer using three checkers.

Ask the student to put 37 on the Minicomputer using three checkers. There are two possible configurations.





T: Could we put 37 on the Minicomputer using less than three checkers?
(No)
Why not?

S: We need one checker to put 40 on the Minicomputer and at least two checkers to put $\hat{3}$ on the Minicomputer.

Conclude with your class that you need at least three checkers to put 37 on the Minicomputer.

Erase the board and remove the checkers from the Minicomputer. Distribute the students' copies of the Story-workbook ROLLERSKATING 37 and ask the students to turn to page 10. Briefly read through pages 10 to 16 with your students and discuss the various problems to be solved; then ask your students to complete the rest of the book independently beginning on page 10.

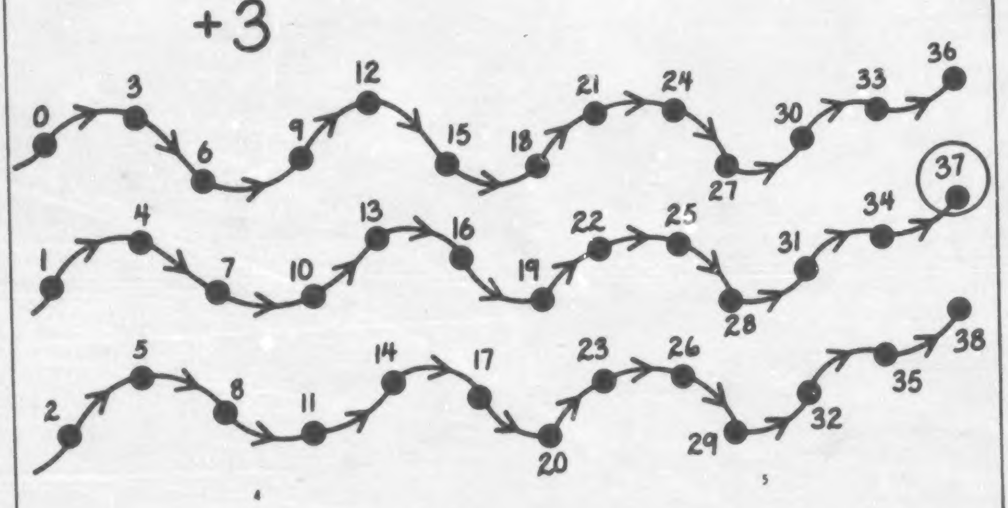
ROLLERSKATING

37

Rollerskating 37

Some minutes later, 37 came by again. This time 37 was rollerskating with some friends. They were skating in three rows.

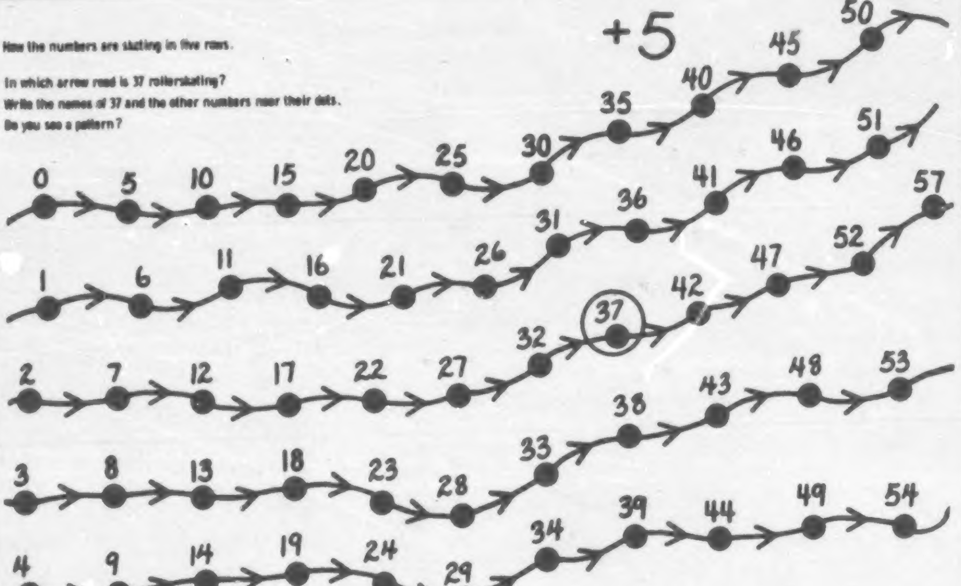
Do you see 37 in these arrow roads?
Point to the dot for 37 and then write "37" near it.
Write the names of 37's friends near their dots.



+3
0
3
6
9
12
15
18
21
24
27
30
33
36
1
4
7
10
13
16
19
22
25
28
31
34
37
2
5
8
11
14
17
20
23
26
29
32
35
38
4
5

Now the numbers are skating in five rows.

In which arrow road is 37 rollerskating?
Write the names of 37 and the other numbers near their dots.
Do you see a pattern?



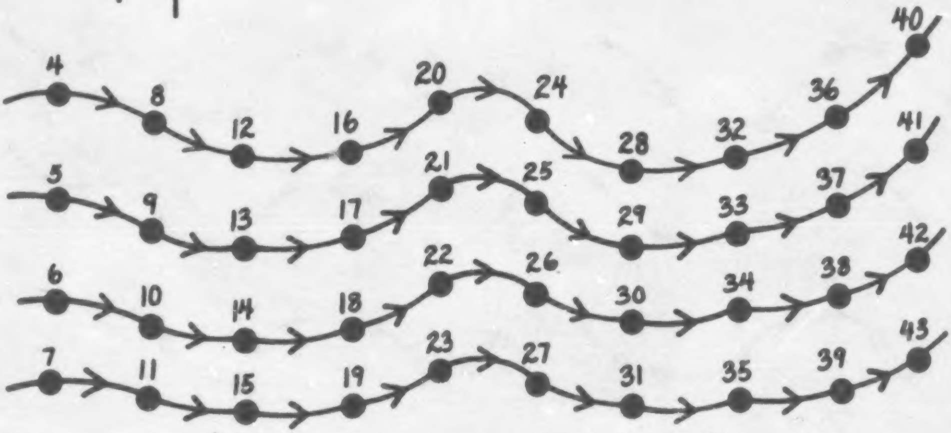
+5
0
5
10
15
20
25
30
35
40
45
50
1
6
11
16
21
26
31
36
41
46
51
2
7
12
17
22
27
32
37
42
47
52
57
3
8
13
18
23
28
33
38
43
48
53
4
9
14
19
24
29
34
39
44
49
54
6
7

The number friends are in a new rollerskating show that they call "+4".

They are skating in four rows but only one arrow road is drawn for you. Is 37 in the same arrow road as 28?

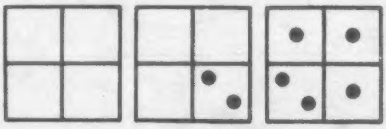
Draw the three missing arrow roads. 37 should be in one of them. Make a big dot for 37. Write the names of the numbers near their dots.

+4


4
8
12
16
20
24
28
32
36
40
5
9
13
17
21
25
29
33
37
41
6
10
14
18
22
26
30
34
38
42
7
11
15
19
23
27
31
35
39
43

The new arrow roads may be drawn in a different order and may include more or fewer numbers than appear here.

37 was very tired at the end of this busy day and fell asleep on the Minicomputer. Do you recognize 37?



Write a number story to show how you see 37 on the Minicomputer.

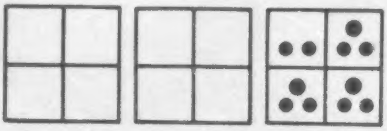
20 + 17 = 37

10 + 10 + 8 + 4 + 2 + 2 + 1 = 37

20 + 10 + 7 = 37

Other number stories are possible.

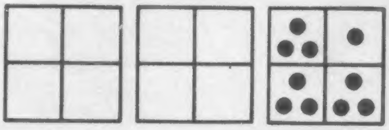
During the night, 37 had a dream and fell off the Minicomputer. 37 was half-asleep and had trouble getting back on the Minicomputer. Some checkers are missing. Can you draw them?



Many solutions are possible.

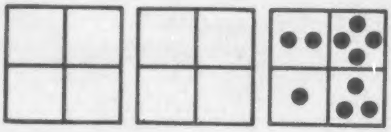
Later, 37 dreamed about sleeping on the Minicomputer with ten checkers. Do you think this is possible?

Can you show 37 with ten checkers?

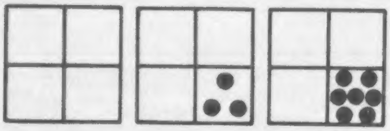


Many solutions are possible.

Can you show 37 with ten checkers in another way?

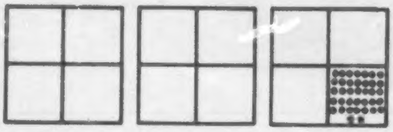


. . . in still another way?



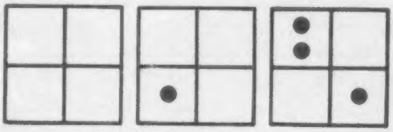
Many solutions are possible.

Put 37 on this Minicomputer using as many checkers as possible.



More checkers could be used if negative checkers are included.

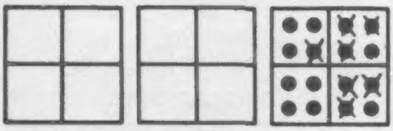
Put 37 on this Minicomputer using as few checkers as possible.



Be careful! You can show 37 with less than five checkers.

Another solution with four checkers is possible when negative checkers are included (30 + 8 + $\hat{1}$). There also is a solution with three checkers when negative checkers are included (40 + $\hat{3}$).

The number 37 had a nightmare. 37 dreamed of being on the Minicomputer in this way.



Other solutions are possible.

"This is not me!" cried 37. "There are too many checkers. Help me take some of them away."

Cross off the extra checkers and show 37 sleeping peacefully again.
Write a number story to show that you have left 37 on the Minicomputer.

$$8+2+8+2+8+2+4+2+1=37$$

$$10+10+10+7=37$$

$$24+4+8+1=37$$

Other solutions are possible.

This is the end of the story of the number 37, who is proud to sign this book in many different ways.

$40-3$

$(5\times7)+2$

$26+11$

$(2\times10)+17$

$50-13$

$2+5+6+7+8+9$

Invent some other signatures for 37 and write them below.

$$30+7$$

$$(3\times10)+7$$

$$40+\hat{3}$$

$$(4\times10)-3$$

$$18+19$$

Many solutions are possible.

W13 FESTIVAL OF PROBLEMS #2 (Lesson 1)

CAPSULE LESSON SUMMARY

Find the area of shapes on a grid board if each little square has area 4 cm^2. Draw shapes of a given area. Begin the Workbook FESTIVAL OF PROBLEMS #2.

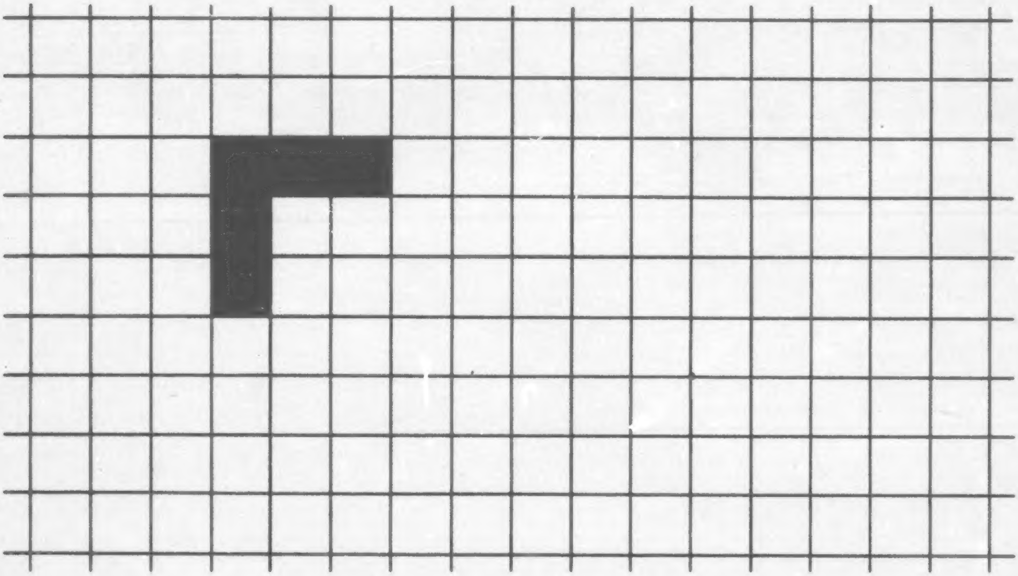
MATERIALS

Teacher:	Grid board; colored chalk
Student:	Workbook FESTIVAL OF PROBLEMS #2; colored pencils; pencil

DESCRIPTION OF LESSON

Call on several students to count by fours starting at 0. Let each student continue to 40 or until an error is made.

Display a grid board and color this shape in red.



T: The red shape has how many little squares in it?

S: Five.

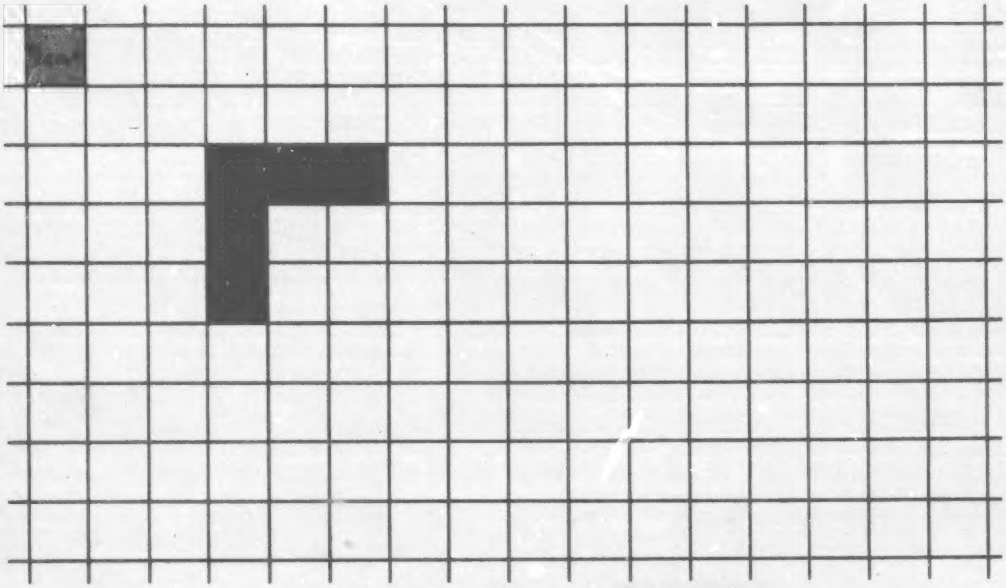
759

W 13

T: Let's pretend that the area of each little square is 4 cm^2 (read: four square centimeters).

NOTE: A square of area 4 cm^2 measures 2 cm by 2 cm. This is for your information only.

Indicate the area of a little square on the grid board.



T: What is the area of the red shape?

S: 20.

T: 20 what?

S: 20 cm^2 (read: twenty square centimeters).

T: How do you know?

S: I added "4 + 4 + 4 + 4 + 4".

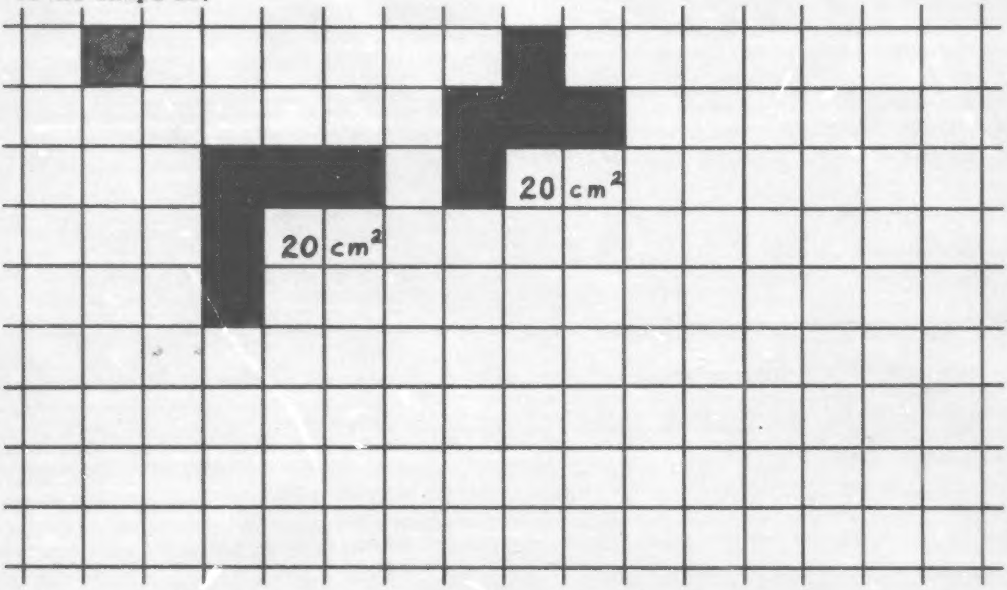
S: 5 x 4 = 20.

S: I counted by fours: 4, 8, 12, 16, 20.

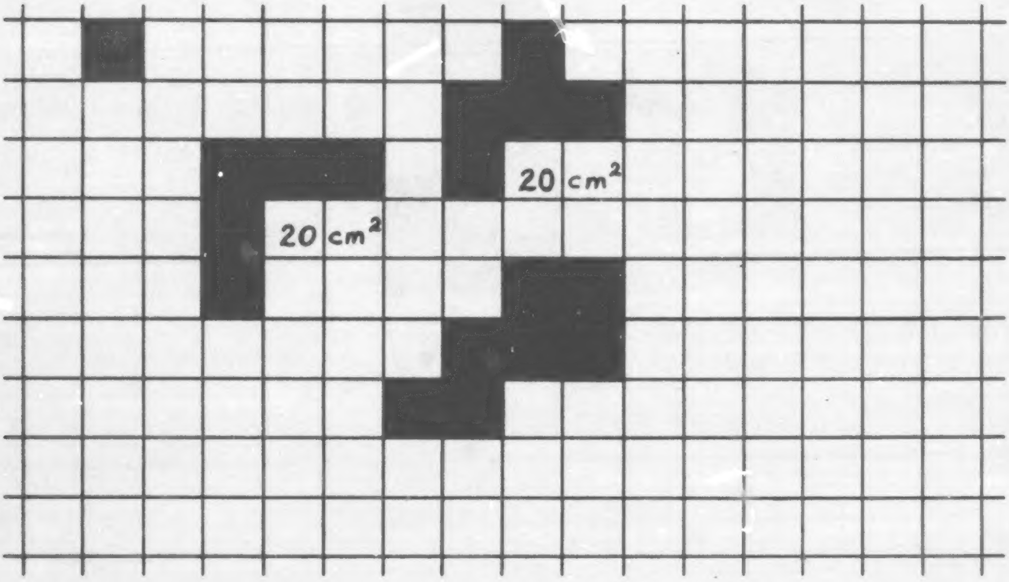
Count by fours as you point to the squares of the red shape.

T: Who can color another shape whose area is 20 cm²?

NOTE: It is possible that a student will color a shape whose area is difficult to determine; in which case, reply that you are not sure exactly what the area of the shape is.



Color this shape on the grid (see the green shape below) and ask for its area.



W 13

S: The area of the green shape is 28 cm^2.

T: How do you know?

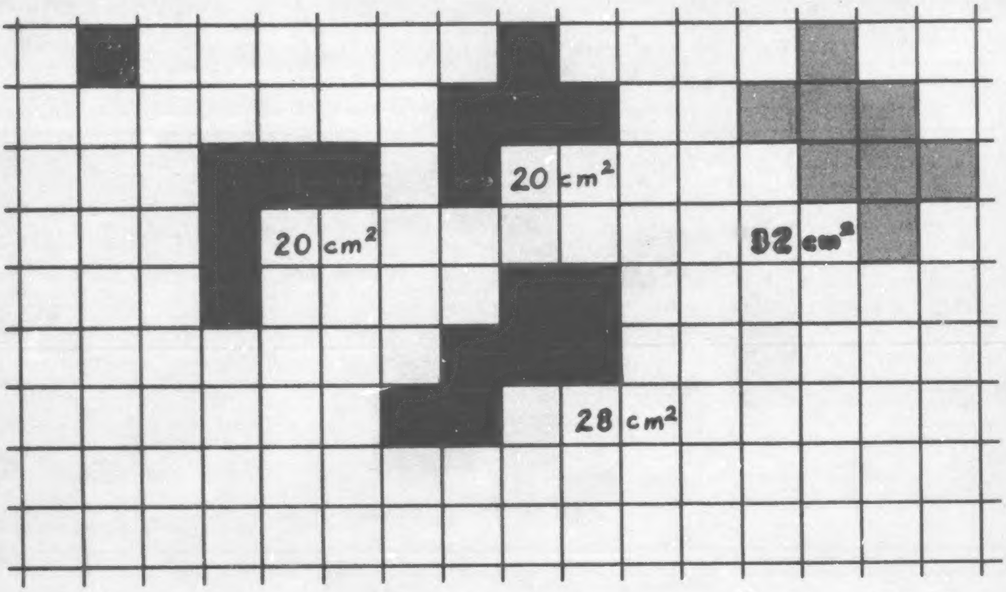
S: It has seven little squares; 7 x 4 = 28.

S: I counted by fours.

Ask the class to count by fours as you point to the little squares in the green shape. Record "28 cm^2" on the grid.

T: Who can color a shape with area 32 cm^2?

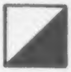
After a student has colored a shape, check with your class that the area of the shape is 32 cm^2 by counting by fours. If the shape does not have that area, erase it and ask for another volunteer. Any shape which has eight little squares is correct. Perhaps a student will use half-squares in the shape. One possible shape is shown in yellow below.

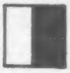


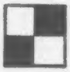
Repeat this activity asking for:

- a shape with area 16 cm^2; and
- a shape with area 18 cm^2.

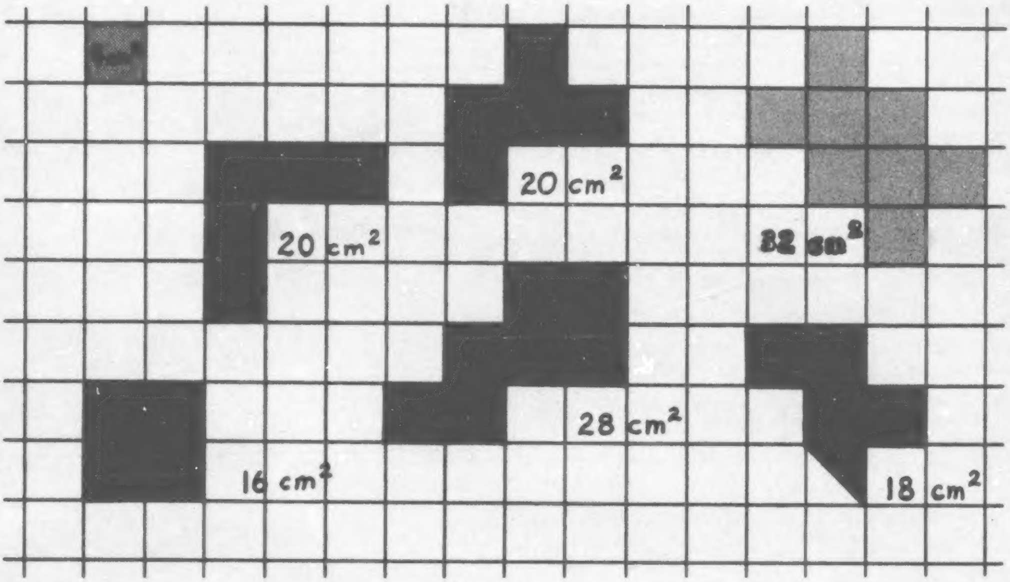
To draw a shape with area 18 cm^2, at least one little square needs to be cut in half. A student might halve a little square in one of these ways.







Your picture might look similar to this one.



If your students have difficulty finding a shape with area 18 cm^2, guide them to a solution as in the following dialogue.

T: We have a shape with area 16 cm^2 (point to it) and a shape with area 20 cm^2 (point to it). Now we want a shape with area 18 cm^2.

How many little squares does the red shape (20 cm^2) have?

S: Five.

T: How many little squares does the orange shape (16 cm^2) have?

S: Four.

T: How many little squares would a shape with area 18 cm^2 have?

S: Four and a half.

T: How do you know?

S: Because four 4's are 16 and $\frac{1}{2} \times 4$ (read: one-half of four) is 2 more; $16 + 2 = 18$.

Let that student color a shape with area 18 cm^2 on the board.

Distribute copies of the Workbook FESTIVAL OF PROBLEMS #2. Allow about 30 minutes for individual work and then collect the workbooks for use in W14.

W14 FESTIVAL OF PROBLEMS #2 (Lesson 2)

CAPSULE LESSON SUMMARY

Put numbers on the Minicomputer using exactly two checkers. Continue working in the Workbook FESTIVAL OF PROBLEMS #2. (This is the second of two lessons using this workbook.)

MATERIALS

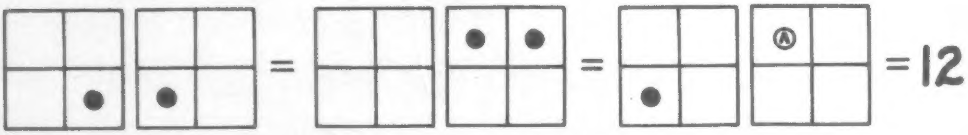
Teacher:	Minicomputer set
Student:	Workbook FESTIVAL OF PROBLEMS #2; colored pencils; pencil

DESCRIPTION OF LESSON

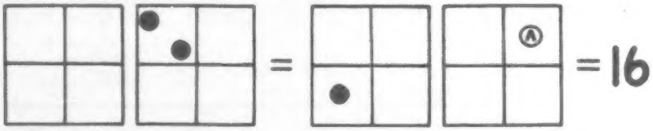
Display two Minicomputer boards.

T: Who can put 12 on this Minicomputer using exactly two checkers? Is there another way to put 12 on this Minicomputer using exactly two checkers?

Continue until your students have found all three ways to put 12 on the Minicomputer using exactly two checkers. If necessary, suggest these configurations yourself.



Repeat this activity with 16. There are two ways to put 16 on the Minicomputer using exactly two checkers.



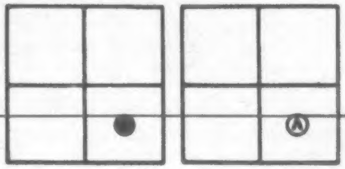
W 14

Put a regular (positive) checker on the 10-square.

T: What number is this?

S: 10.

Put a negative checker on the 1-square.

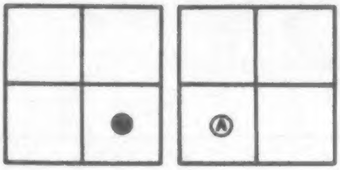


T: What number is this?

S: 9.

T (pointing to the apporpriate checkers): 10 - 1 = 9.

Move the negative checker to the 2-square.

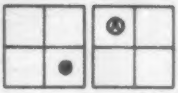


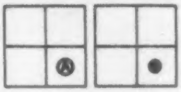
T: What number is this?

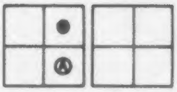
S: 8.

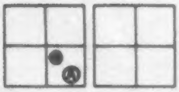
T (pointing to the appropriate checkers): 10 - 2 = 8.

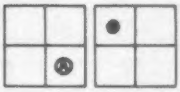
Continue this activity with these configurations.

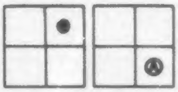

$= 2$

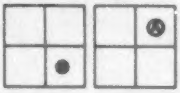

$= \hat{9}$

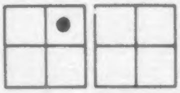

$= 30$

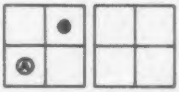

$= 0$

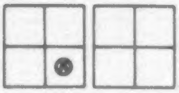

$= \hat{2}$

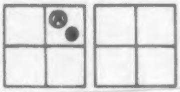

$= 39$

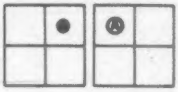

$= 6$


$= 40$


$= 20$


$= \widehat{10}$


$= 0$


$= 32$

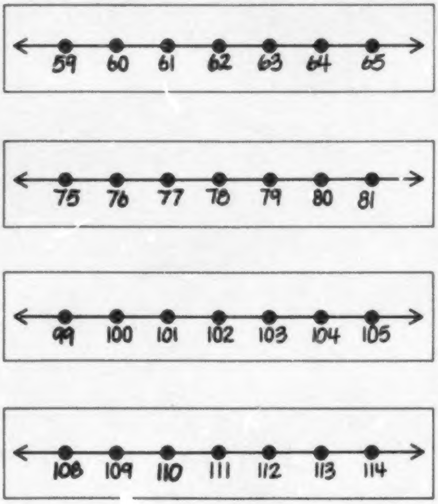
Distribute the students' copies of the Workbook FESTIVAL OF PROBLEMS #2. At the end of the lesson, collect the workbooks for your review.

FESTIVAL OF PROBLEMS

#2

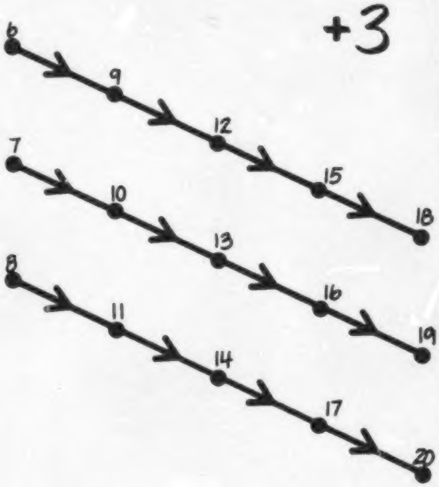
Festival of Problems #2

Label the dots on these number lines.


59 60 61 62 63 64 65
75 76 77 78 79 80 81
99 100 101 102 103 104 105
108 109 110 111 112 113 114

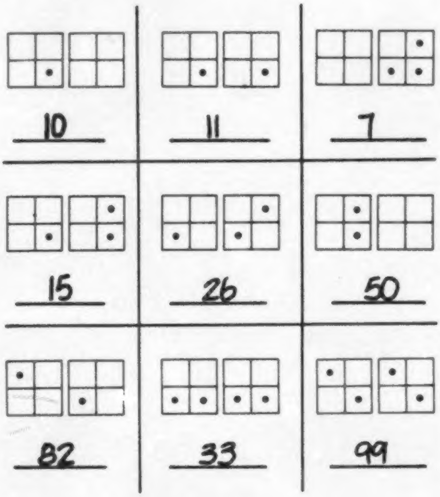
2

Label the dots.


+3
6 9 12 15 18
7 10 13 16 19
8 11 14 17 20

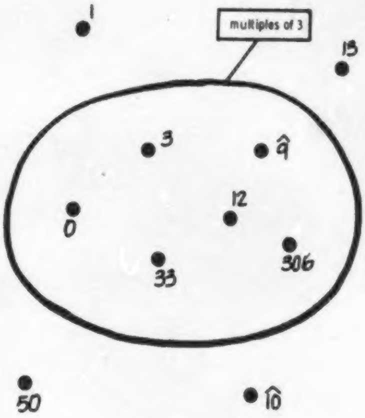
3

What number is on the Minicomputer?


10
11
7
15
26
50
82
33
99

4

Put at least ten numbers in this string picture.


multiples of 3
1
15
3
9̂
0
12
33
306
50
1̂0

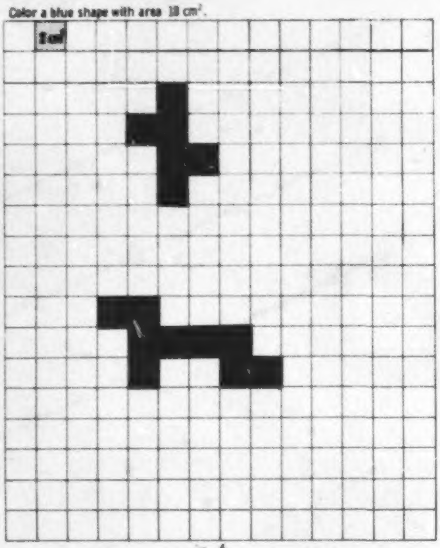
5

Many solutions are possible.

Each little square has area 2 cm².

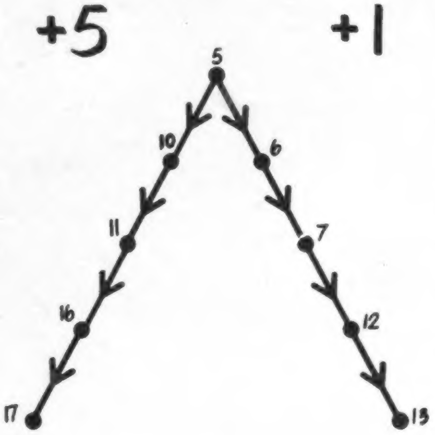
Color a red shape with area 12 cm².

Color a blue shape with area 18 cm².

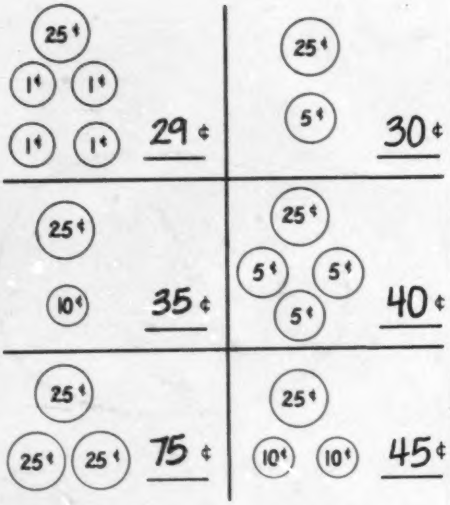


Many solutions are possible.

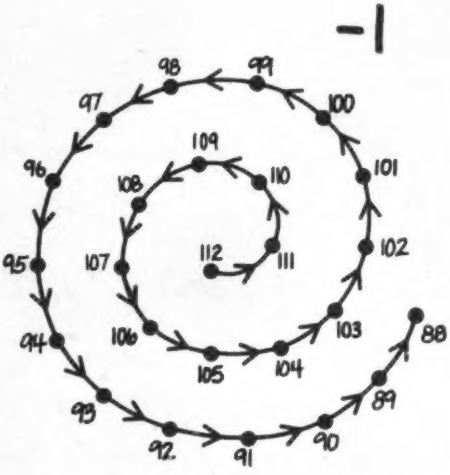
Label the dots.


+5
+1
5
10
6
11
7
16
12
17
13

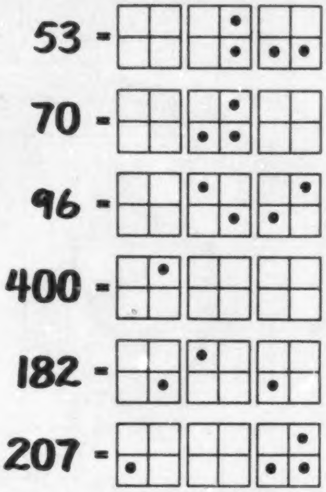
How much money?


25¢
1¢
1¢
1¢
1¢
29¢
25¢
5¢
30¢
25¢
10¢
35¢
25¢
5¢
5¢
5¢
40¢
25¢
25¢
25¢
75¢
25¢
10¢
10¢
45¢

Label the dots.


-1
98
99
97
100
109
96
110
101
108
95
107
112
111
102
106
103
88
94
105
104
89
93
92
91
90

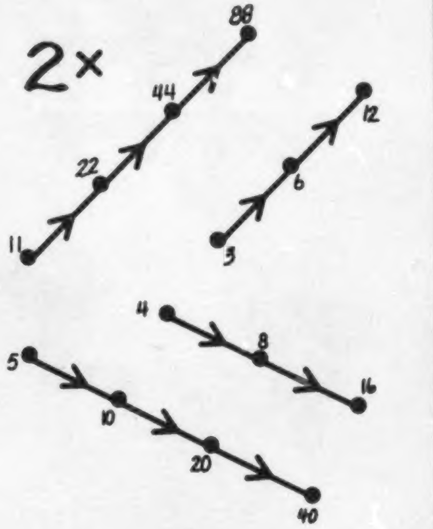
Put these numbers on the Minicomputer.


53 =
70 =
96 =
400 =
182 =
207 =

10

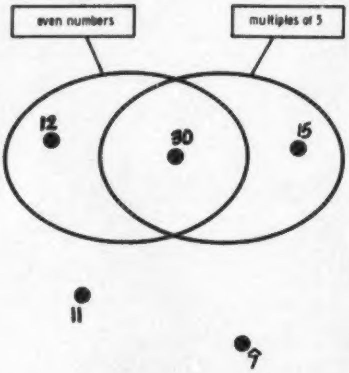
Standard configurations are given here; however, many other solutions are possible.

Label the dots.


2×
11
22
44
88
3
6
12
4
8
16
5
10
20
40

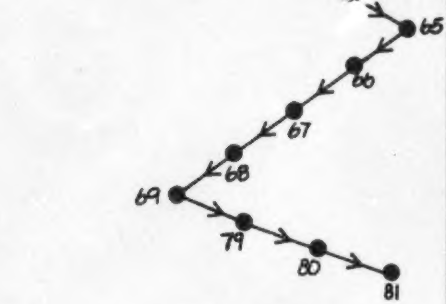
11

Label the dots in this string picture.
Many answers are possible.


even numbers
multiples of 5
12
30
15
11
9

12

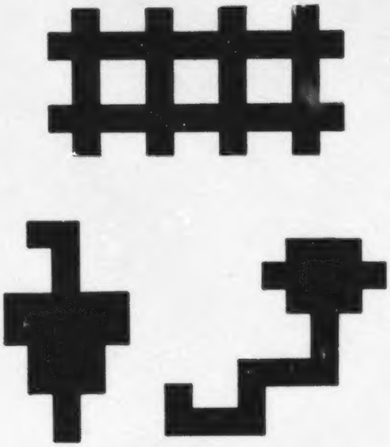
Build a red-blue road from 5 to 81.


+10
+1
5
15
25
35
45
55
65
66
67
68
69
79
80
81

13

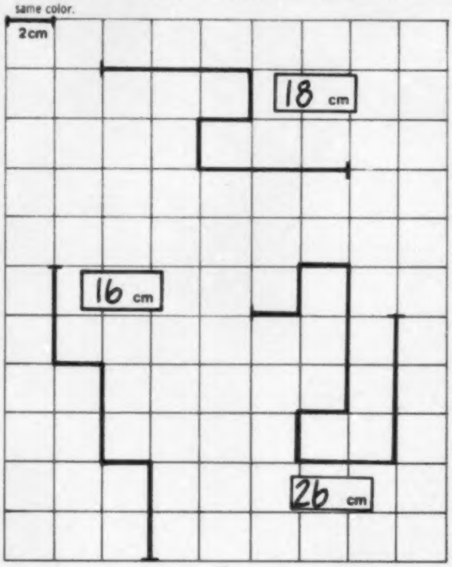
Many solutions are possible; however, the shortest roads have seven red arrows and six blue arrows.

Color each shape one-half red and one-half blue.

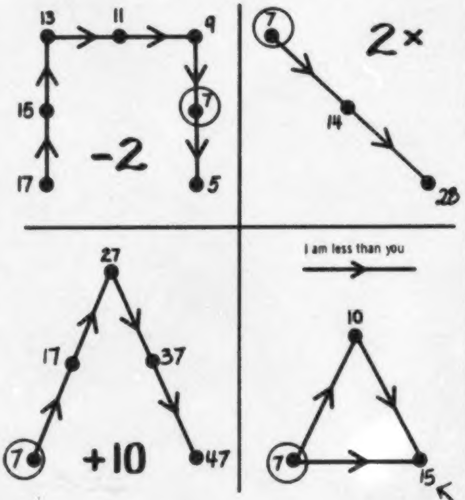


Many other colorings are possible.

Find the length of each zigzag path and put your answer in the box of the same color.

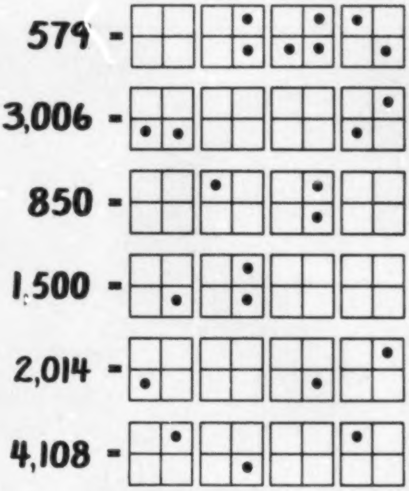

2cm
18 cm
16 cm
26 cm

7 is in each of these arrow pictures.
In each one, circle its dot.


13
11
9
15
7
17
-2
5
7
2×
14
28
I am less than you
27
17
37
7
+10
47
10
7
15

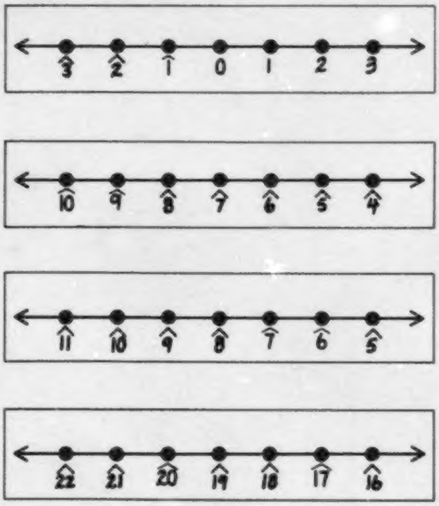
Any number larger than 10 could be here.

Put these numbers on the Minicomputer.


579 =
3,006 =
850 =
1,500 =
2,014 =
4,108 =

Standard configurations are given here; however, many other solutions are possible.

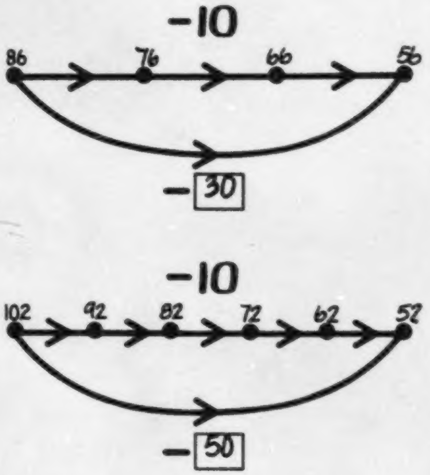
Label the dots on these number lines.


$\hat{3}$ $\hat{2}$ $\hat{1}$ 0 1 2 3
$\hat{10}$ $\hat{9}$ $\hat{8}$ $\hat{7}$ $\hat{6}$ $\hat{5}$ $\hat{4}$
$\hat{11}$ $\hat{10}$ $\hat{9}$ $\hat{8}$ $\hat{7}$ $\hat{6}$ $\hat{5}$
$\hat{22}$ $\hat{21}$ $\hat{20}$ $\hat{19}$ $\hat{18}$ $\hat{17}$ $\hat{16}$

18

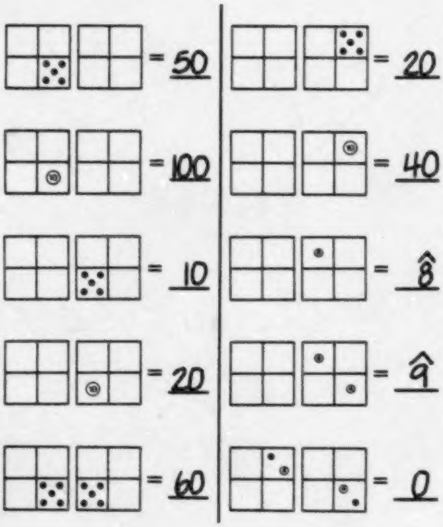
Label the dots.

Fill in the box for each blue arrow.


-10
86
76
66
56
-30
-10
102
92
82
72
62
52
-50

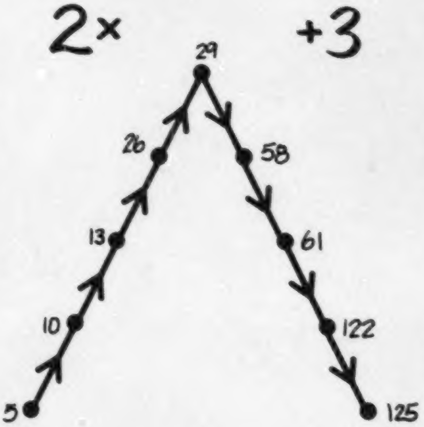
19

What number is on the Minicomputer?


= 50
= 100
= 10
= 20
= 60
= 20
= 40
= $\hat{8}$
= $\hat{9}$
= 0

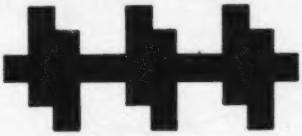
20

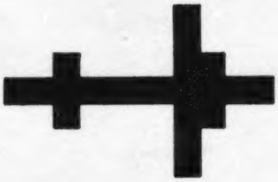
Label the dots.


2×
+3
5
10
13
26
29
58
61
122
125

21

Color each shape one-third red, one-third blue, and one-third green.



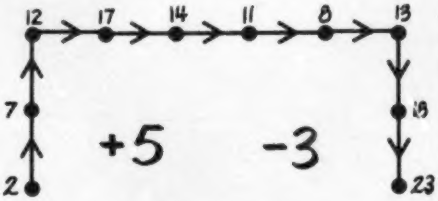


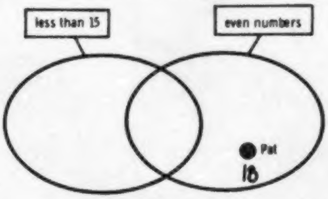
Many other colorings are possible.

Pat is a secret number.

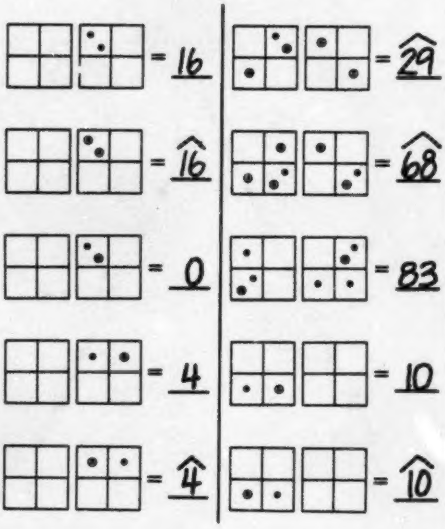
Pat is in this arrow picture and in this string picture.

Who is Pat? 18


12
17
14
11
8
13
7
18
+5
-3
2
23


less than 15
even numbers
Pat
18

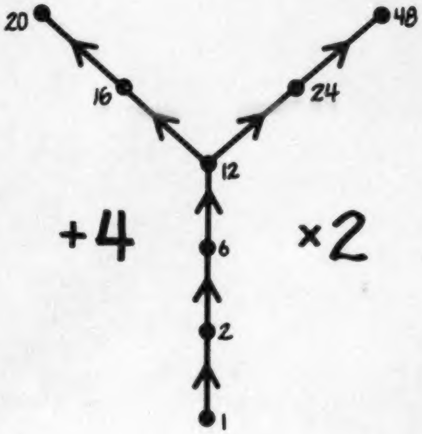
What number is on the Minicomputer?


= 16
= $\widehat{29}$
= $\widehat{16}$
= $\widehat{68}$
= 0
= 83
= 4
= 10
= $\widehat{4}$
= $\widehat{10}$

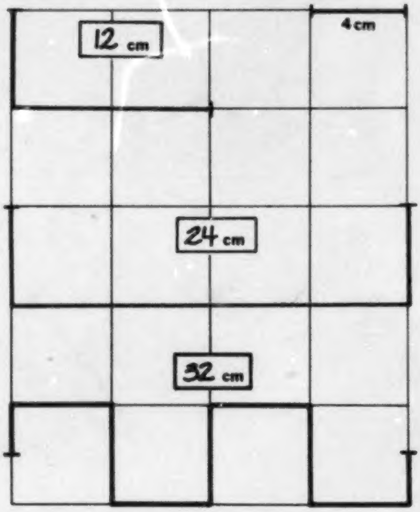
Pam is a secret number.

Pam is a multiple of 5 and in this arrow picture.

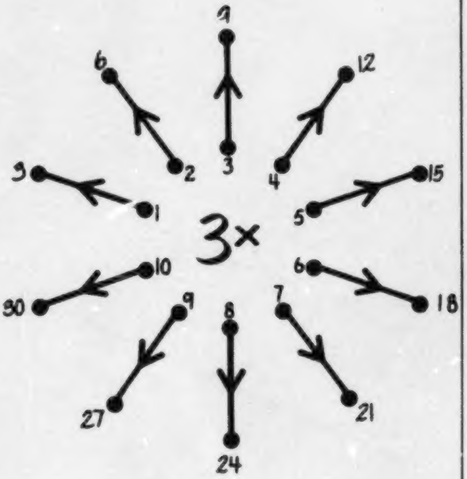
Who is Pam? 20


20
48
16
24
12
+4
6
×2
2
1

Find the length of each zigzag path and put your answer in the box of the same color.


12 cm
4 cm
24 cm
32 cm

Label the dots.

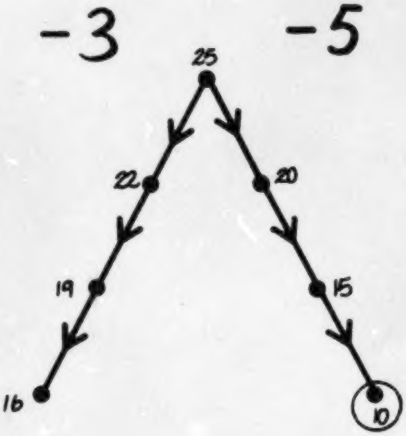

3×
9
3
6
2
12
4
3
1
15
5
30
10
18
6
27
9
24
8
21
7

Complete.

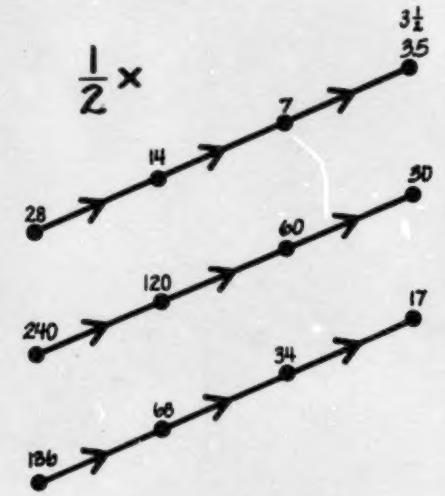
$$\begin{array}{r} 48 \\ +44 \\ \hline 92 \end{array} \qquad \begin{array}{r} 175 \\ +\ 15 \\ \hline 190 \end{array}$$

$$\begin{array}{r} 2{,}371 \\ +3{,}557 \\ \hline 5{,}928 \end{array} \qquad \begin{array}{r} 4{,}351 \\ +\ 649 \\ \hline 5{,}000 \end{array}$$

10 is the smallest number in this arrow picture.
Where is 10? Who are the other numbers?
Label the dots.


-3
-5
25
22
20
19
15
16
10

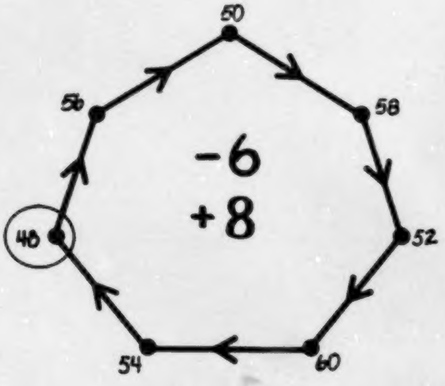
Label the dots.



$\frac{1}{2}\times$
28
14
7
3½
35
240
120
60
30
136
68
34
17

30

48 is the smallest number in the arrow picture.
Where is 48? Who are the other numbers?
Label the dots.



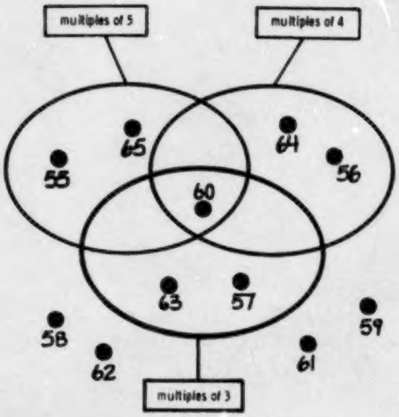
-6
+8
50
56
58
48
52
54
60

31

Put these numbers in the string picture.

55 56 57 58 59 60 61 62 63 64 65

One is done for you.



multiples of 5
multiples of 4
65
55
64
56
60
63
57
58
59
62
61
multiples of 3

32

W15 Who is Flip?

CAPSULE LESSON SUMMARY

Do some mental arithmetic activities which involve the hand-calculator. Solve a detective story in which the clues involve the hand-calculator, an arrow picture, and the Minicomputer.

MATERIALS

Teacher: Two hand-calculators[*]; colored chalk; Minicomputer set
Student: Worksheet W15; pencil

DESCRIPTION OF LESSON

Before the lesson begins, write a large "18" on a piece of paper and fold the paper so the "18" is hidden.

Exercise 1: Hand-calculator mental arithmetic

Give two students each a hand-calculator and ask them to turn the calculators on and to do exactly what you tell them.

T: What number is on the display? (0)
Cover the display. Press $\boxed{3}\ \boxed{\times}\ \boxed{4}\ \boxed{+}\ \boxed{5}\ \boxed{=}$.

When you are telling the students which buttons to press, you should pause after each numeral key to give the students an opportunity to do the calculation mentally. For example, say: "$\boxed{3}\ \boxed{\times}\ \boxed{4}$", then pause before continuing, "$\boxed{+}\ \boxed{5}\ \boxed{=}$".

T: Don't look. What number should be on the display?

S: 17.

---

[*] See the note on ROLE AND USE OF HAND-CALCULATORS in the General Introduction.

Allow several students to answer before asking the students with calculators to look at the display. If any of the calculators have a number other than 17 on the display, ask the students with those calculators to push $\boxed{C}$ and then to put 17 on the display.

T: Cover the display again. Now press $\boxed{+}\ \boxed{3}\ \boxed{=}\ \boxed{=}\ \boxed{=}$. Don't look. What number should be on the display? (26)

Allow several students to answer before asking the students with calculators to look at the display. Ask the students with calculators to correct the number on the display if necessary.

Continue this activity with the following sequences.

Starting with 26 on the display: $\boxed{-}\ \boxed{2}\ \boxed{=}\ \boxed{=}$ (22)

Starting with 22 on the display: $\boxed{\times}\ \boxed{2}\ \boxed{+}\ \boxed{6}\ \boxed{=}$ (50)

Starting with 50 on the display: $\boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ (62)

Starting with 62 on the display: $\boxed{+}\ \boxed{1}\ \boxed{0}\ \boxed{=}\ \boxed{=}$ [*] (82)

Ask the students with calculators to press $\boxed{C}$.

Exercise 2: Detective Story

T: I have a number friend named "Flip" and if you are good detectives, you can figure out who Flip is. This piece of paper hides Flip's identity. (Show the class the folded paper you prepared before this lesson began.)

First clue

T: Flip is one of the numbers we get if we start at 3 and count by fives.

---

[*] Read: plus ten equals equals.

Record "3" near the top of the board. Ask the students with calculators:

T: Press $\boxed{3}$ $\boxed{+}$ $\boxed{5}$ $\boxed{=}$. What number is on the display?

S: 8.

Record "8" below "3" on the board.

T: If we continue adding fives, what is the next number we will get? (13)
Press $\boxed{=}$. What number is on the display?

S: 13.

Ask a volunteer to predict what the next number will be before you let the students find it on the calculators. Continue this activity until you have eight numbers in the sequence recorded on the board.

3
8
13
18
23
28
33
38

T: Do you see any patterns in this list of numbers?

Your class should notice that the last digits of these numbers are alternating 3 and 8.

T: What number comes next? Why?

S: 43, because we are counting by fives. 38 + 5 = 43.

S: The next number ends in 3.

Record "43" in the list on the board.

T: If we continued pressing $\boxed{=}$, what are some other numbers we would get on the display?

Encourage the students to suggest many numbers which Flip could be. During this discussion you should ask the following and any other questions which you feel are appropriate.

T: What is a number more than 100 that Flip could be?

. . . more than 250?
. . . between 500 and 600?
. . . less than 1,000?
. . . the largest possible number less than 1,000? (998)

Take whispers for the last question before letting someone answer aloud.

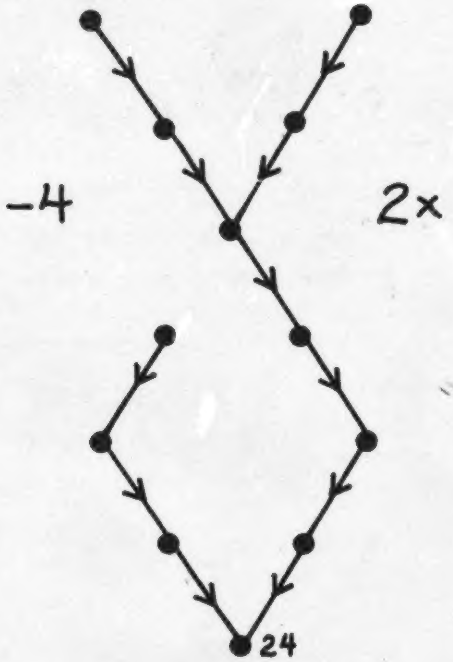
T: Do you know who Flip is?

S: All we know is that Flip ends in 3 or 8.

Second clue

T: My next clue is that Flip is in the arrow picture I am going to draw on the board.

Draw this arrow picture on the board.



T: Where should we start if we want to know which numbers are in this arrow picture?

S: The only dot that is labeled is the dot for 24.

T: We will have to start at 24 and follow the arrows to label the other dots.

S: All the arrows are going the wrong way.

S: We could draw the return arrows.

W 15

Trace the red arrow which ends at 24 as you say:

T: This red arrow is for -4. What is its return arrow for?

S: +4.

Trace the blue arrow which ends at 24.

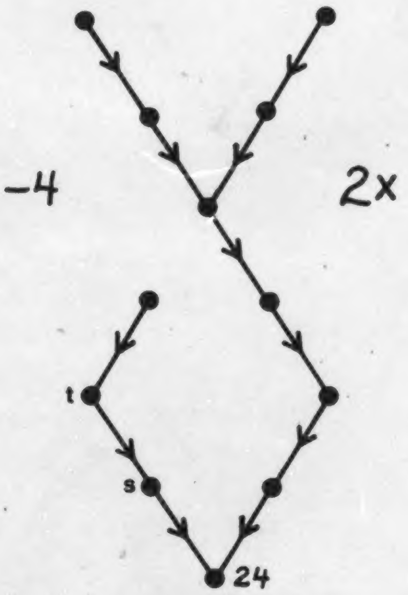
T: This blue arrow is for 2×. What is its return arrow for?

S: $\frac{1}{2}$×.

Distribute copies of Worksheet W15.

T: Label the dots in this arrow picture and be very careful about the directions of the arrows. Perhaps you can discover which number is Flip.

This worksheet is difficult and perhaps only a few students will be able to complete it correctly on their own. Walk around the room and observe the students' work. Whenever necessary, emphasize the directions of the arrows.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

After a few minutes call the class' attention back to the board.

T (pointing to s): How did you calculate this number?

S: $\frac{1}{2} \times 20 = 10$ and $\frac{1}{2} \times 4 = 2$, so $\frac{1}{2} \times 24 = 12$.

T: If this dot (s) is for 12, then we have the number story "$2 \times 12 = 24$". Is that true?

S: Yes.

Label s "12". Point to t.

T: What number is this? How do you know?

S: 6, because $2 \times 6 = 12$.

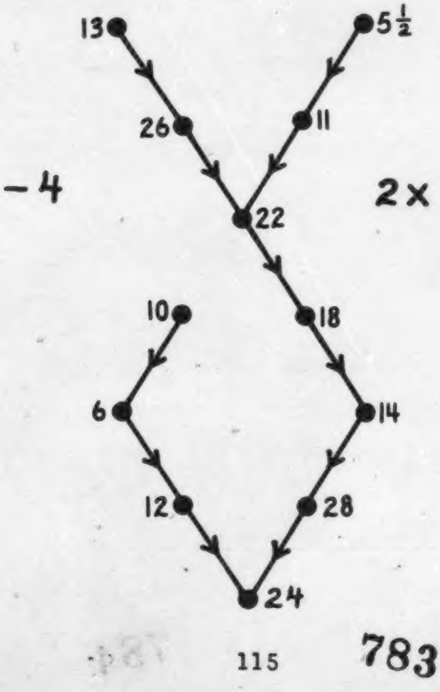
T: The blue arrows are for 2 ×, so their return arrows are for $\frac{1}{2}$ ×.

Trace the return arrow from s to t.

T: What number is $\frac{1}{2} \times 12$?

S: 6.

Label t "6". Continue this activity until all the dots are labeled.



W 15

T: Does this arrow picture tell us anything about Flip?

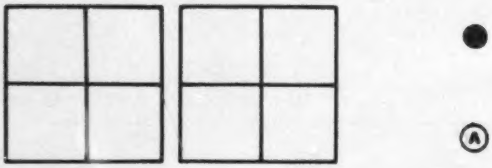
S: Flip could be 13, 18, or 28 because those are the only numbers in this arrow picture which end in 3 or 8.

Circle "13", "18", and "28" in the arrow picture or record these numbers in a prominent place on the board.

T: We know that Flip is either 13, 18, or 28.

Third clue

Display two Minicomputer boards. Draw a regular checker and a negative checker to the side of the boards.



T: Flip can be put on the Minicomputer using exactly one regular and one negative checker. Which numbers could Flip be?

If some students think they know which number is Flip, walk over to them and ask them to point to Flip in their arrow pictures.

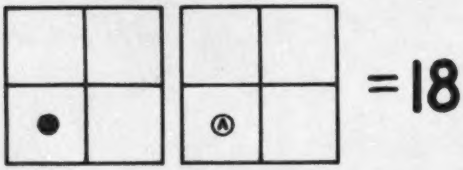
T: Could Flip be 13?

Encourage students to attempt by trial and error to find a way to put 13 on the Minicomputer using just one regular and one negative checker. Continue this activity until several students observe that 13 cannot be put on this Minicomputer using just those two checkers. Perhaps a student will explain that 13 = 20 + 7 and that it is impossible to put 7 on the Minicomputer using only one negative checker.

Cross out "13".

T: Flip is either 18 or 28. Could Flip be 18? (Yes)

Ask for a volunteer to put 18 on the Minicomputer using exactly one regular and one negative checker.



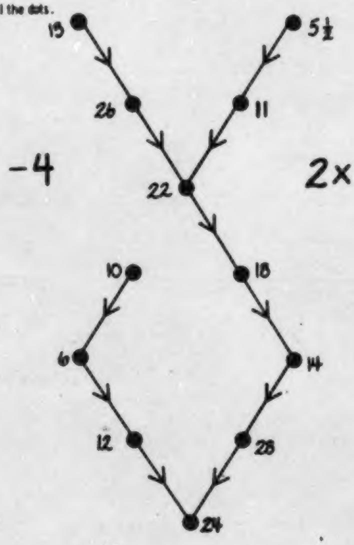
T: Flip could be 18. Could Flip be 28?

Encourage the students to attempt by trial and error to find a way to put 28 on the Minicomputer using just one regular and one negative checker. Continue this activity until several students observe that 28 cannot be put on this Minicomputer using just those two checkers. Perhaps a student will explain that $28 = 30 + \widehat{2}$ or $28 = 40 + \widehat{12}$, but it is impossible to put 30 on the Minicomputer using only one regular checker and it is impossible to put $\widehat{12}$ on the Minicomputer using only one negative checker.

Cross out "28" and conclude that Flip must be 18. Reveal that "18" is written on the folded piece of paper.

Name ______________________ **W 15**

Label the dots.


15
5½
26
11
-4
22
2x
10
18
6
14
12
28
24

W16 20? 100? (Lesson 1)

CAPSULE LESSON SUMMARY

Look at the number 20 in a variety of ways:

- as a multiple of certain whole numbers;
- on the Minicomputer using a specified number of checkers;
- in one of two +4 arrow roads;
- in a three-string picture.

MATERIALS

Teacher: Minicomputer set; colored chalk
Student: Workbook "20? 100?"; colored pencils

DESCRIPTION OF LESSON

T: Today we are going to become more acquainted with the number 20. What can you tell me about 20? Is 20 a multiple of any of the whole numbers?

Encourage the students to think of many numbers of which 20 is a multiple. A sample dialogue is given here.

S: 20 is a multiple of 5.

T: How do you know?

S: I counted by fives: 5, 10, 15, 20.

T: 20 is how many fives? (4)
4 x 5 = 20.

Record "4 x 5 = 20" on the board.

W 16

T: Is 20 a multiple of 3? (No)
How do you know?

S: If we count by threes: 3, 6, 9, 12, 15, 18, 21, we skip over 20.

T: 20 is not a multiple of 3.

Continue this activity until the class discovers that 20 is a multiple of 1, 2, 4, 5, 10, and 20. Then you should have the following multiplication number stories on the board.

$4 \times 5 = 20$

$5 \times 4 = 20$

$1 \times 20 = 20$

$20 \times 1 = 20$

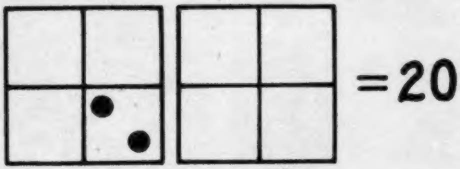
$2 \times 10 = 20$

$10 \times 2 = 20$

Display two Minicomputer boards.

T: Who can put 20 on this Minicomputer using exactly two checkers?

Perhaps a volunteer will suggest this configuration.



T: What calculation could we be doing on the Minicomputer?

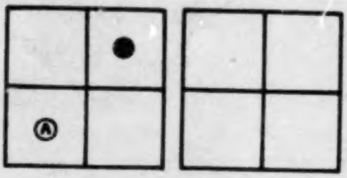
S: "10 + 10".

Record "10 + 10 = 20" on the board.

S: "2 × 10".

Point to the appropriate number story on the board.

Perhaps another student will suggest this configuration.

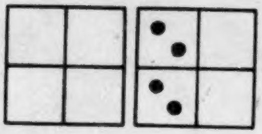


$= 40 + \widehat{20} = 20$

If necessary, suggest this configuration yourself. Record "$40 + \widehat{20} = 20$" on the board.

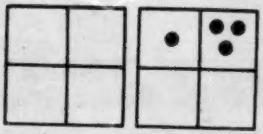
T: Who can put 20 on the Minicomputer using exactly four checkers?

Encourage the students to find several ways to solve this problem. As each configuration is put on the Minicomputer, record the number stories which the students suggest on the board or point to the appropriate number stories if they are already on the board. A few of the possible configurations are given here along with some of the number stories which students might suggest.



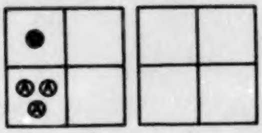
$= 20$

$(2 \times 8) + (2 \times 2) = 20$
$16 + 4 = 20$
$2 \times 10 = 20$



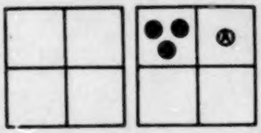
$= 20$

$8 + (3 \times 4) = 20$
$8 + 12 = 20$



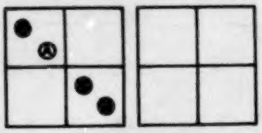
= 20

$$80+(3\times\widehat{20})=20$$
$$80+\widehat{60}=20$$



= 20

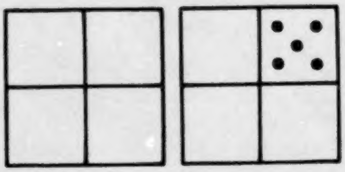
$$(3\times 8)+\widehat{4}=20$$
$$24+\widehat{4}=20$$



= 20

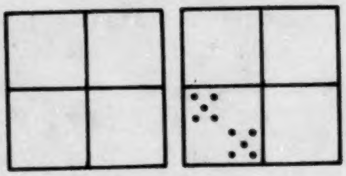
$$80+\widehat{80}+(2\times 10)=20$$
$$0+20=20$$

T: Who can put 20 on the Minicomputer using exactly five checkers on the same square?



$$=5\times 4=20$$

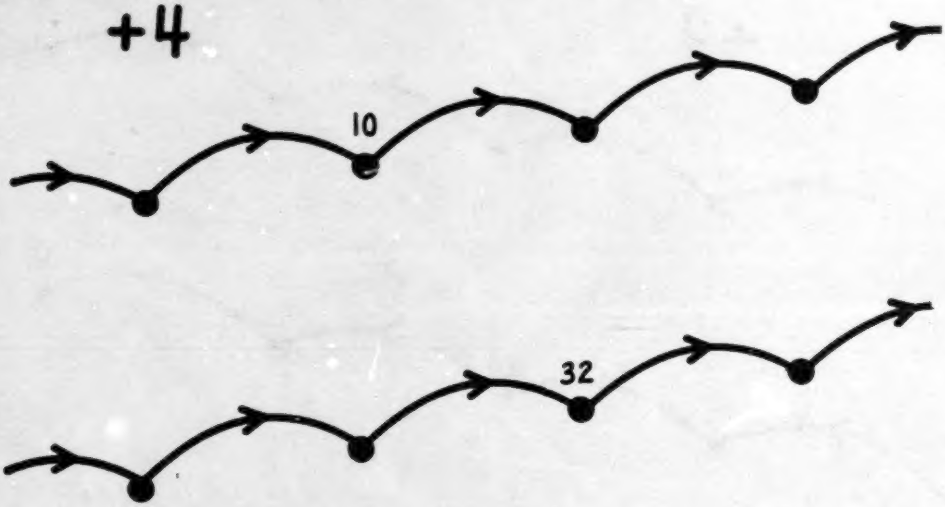
T: Who can put 20 on the Minicomputer using exactly ten checkers on the same square?



$$=10\times 2=20$$

T: We have found many ways to put 20 on the Minicomputer and have written many number stories about 20 on the board.

Erase the board and then draw this arrow picture.



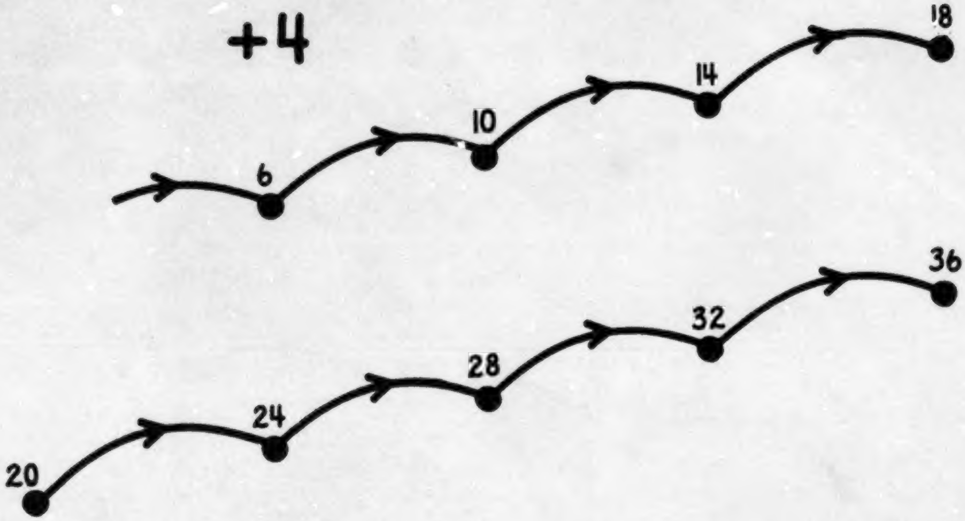
T: Can you find 20 in this arrow picture?

Encourage the students to comment. Very likely a student will claim that a particular dot in the arrow picture is for 20. In this case, label the suggested dot "20" and seek your class' disapproval. Continue this activity until the class concludes that none of these dots is for 20. If necessary, collectively label all the dots and show that none of them is for 20.

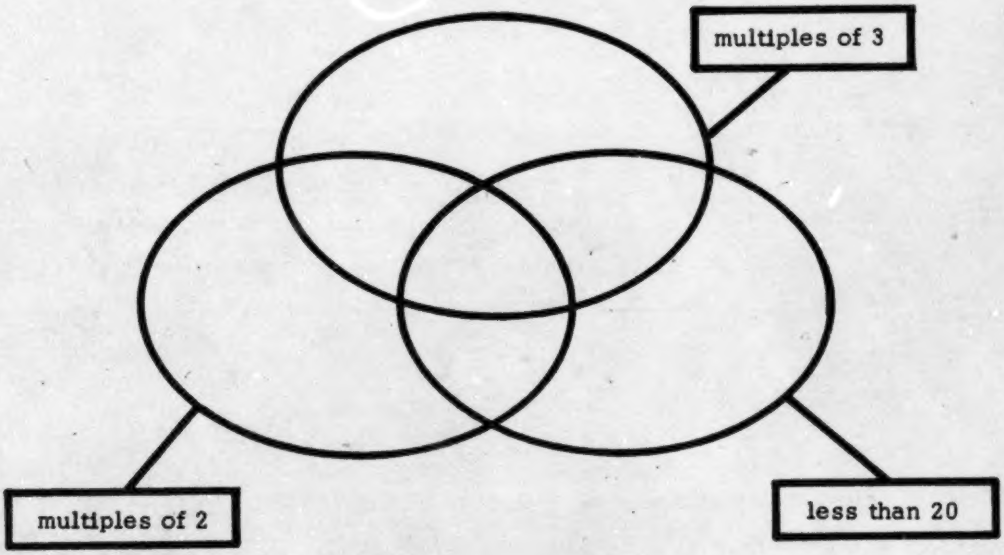
T: Could we extend this arrow picture to include 20?

Let one of the students extend the picture to include 20. If a student just draws a dot and labels it "20", ask if there are any new arrows that can be drawn. Of course, since 24 is already in the picture an arrow can be drawn from 20 to 24. (See the illustration on the next page.)

W 16


+4
6
10
14
18
20
24
28
32
36

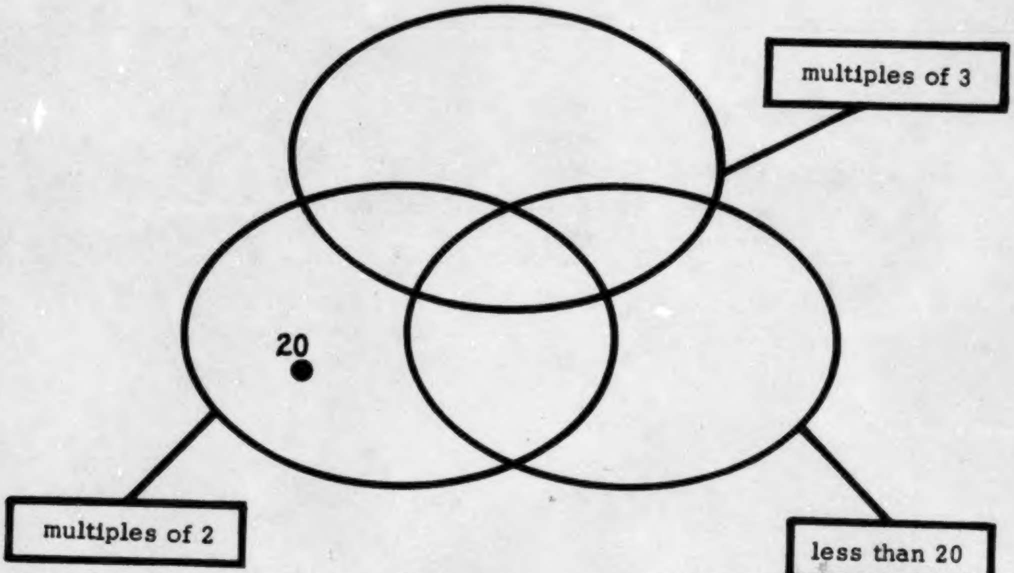
Erase the board and then draw this string picture.


multiples of 3
multiples of 2
less than 20

T: Where is 20 in this string picture?

Invite a student to draw and label a dot for 20. Ask the class if the student put 20 in the correct place. If necessary, erase an incorrect placement and ask for another volunteer to draw a dot for 20. Continue until 20 is located in the correct region.

T: Is 20 a multiple of 3? (No)
So 20 is outside of the red string.
Is 20 a multiple of 2? (Yes)
10 × 2 = 20, so 20 is inside the green string.
Is 20 less than 20? (No)
So 20 is outside the blue string.



Distribute the workbooks to the students. Stress that in the first section they are to find 20 in every picture or to extend the picture to include 20. The second section is similar but involves the number 100 instead of 20. Tell the students to begin working on page 2. After a few minutes, ask the class if they have found 20 on page 2. Be sure all the students understand that they need to extend the arrow picture on page 2 to include 20.

At the end of the lesson, collect the workbooks for your review and then store them for use in Lesson W17.

W 17

W17 20? 100? (Lesson 2)

CAPSULE LESSON SUMMARY

Look at the number 100 as a multiple of certain whole numbers and on the Minicomputer using a specified number of checkers.

MATERIALS

Teacher: Minicomputer set
Student: Workbook "20? 100?"; colored pencils

DESCRIPTION OF LESSON

T: Today we are going to become more acquainted with 100. What can you tell me about 100? Is 100 a multiple of any of the whole numbers?

Encourage the students to suggest many numbers of which 100 is a multiple. A sample dialogue is given here.

S: 100 is a multiple of 5.

T: How do you know?

S: 100 is one of the numbers we say when we count by fives starting at 0.

T: How many fives are there in 100?

If the students have difficulty with this question, guide them to the answer as follows.

T: How many dimes are there in a dollar? (10)
10 x 10 = 100.
How many nickels are there in a dollar? (20)
How do you know?

S: I know that two nickels is the same amount of money as one dime, so there are twice as many nickels in a dollar as there are dimes.

T: How many fives are there in 100? (20)
20 × 5 = 100.

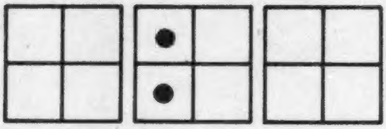
Record "20 × 5 = 100" on the board.

Continue this activity until the class discovers that 100 is a multiple of 1, 2, 4, 5, 10, 20, 25, 50, and 100.

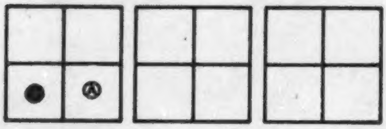
Display three Minicomputer boards.

T: Who can put 100 on this Minicomputer using exactly two checkers?

When a configuration has been put on the Minicomputer, ask the students what calculation is suggested by the checkers on the Minicomputer and record these number stories on the board. There are two ways to put 100 on the Minicomputer using exactly two checkers.



$= 80 + 20 = 100$

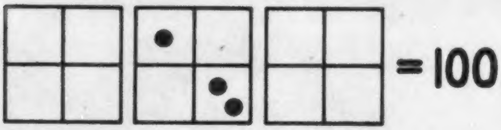


$= 200 + \widehat{100} = 100$

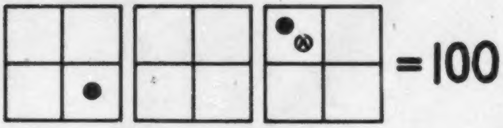
T: Who can put 100 on the Minicomputer using exactly three checkers?

W 17

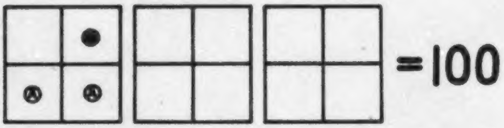
A few of the possible configurations are shown here.



$$80+(2\times 10)=100$$
$$80+20=100$$



$$100+(8+\widehat{8})=100$$
$$100+0=100$$



$$400+(\widehat{200}+\widehat{100})=100$$
$$400+\widehat{300}=100$$

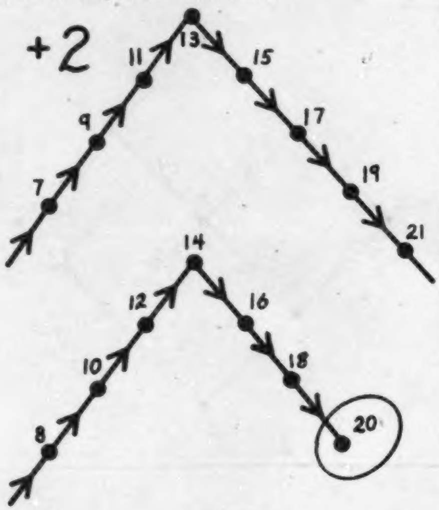
Distribute the students' copies of the Workbook "20? 100?". Stress that in the first section they are to find 20 in every picture or to extend the picture to include 20. The second section involves the number 100 rather than 20.

If some of the students finish the workbook early, encourage them to make up their own problems for a similar workbook about a number of their choice.

At the end of the lesson, collect the workbooks for your review.

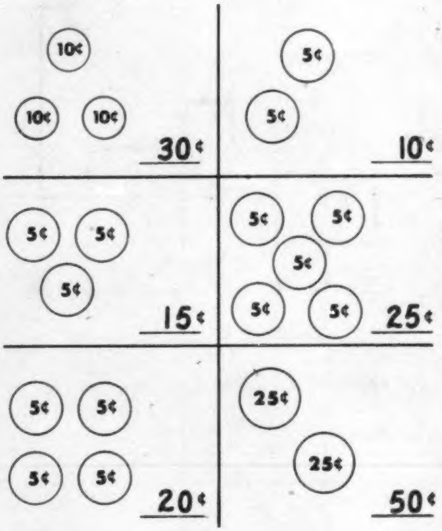
20? 100?

Label the dots.
If 20 is not in this picture, extend it to include 20.


+2
7
9
11
13
15
17
19
21
8
10
12
14
16
18
20

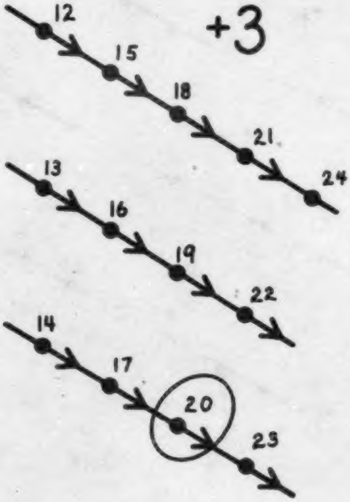
2

How much money?


10¢
10¢
10¢
30¢
5¢
5¢
10¢
5¢
5¢
5¢
15¢
5¢
5¢
5¢
5¢
5¢
25¢
5¢
5¢
5¢
5¢
20¢
25¢
25¢
50¢

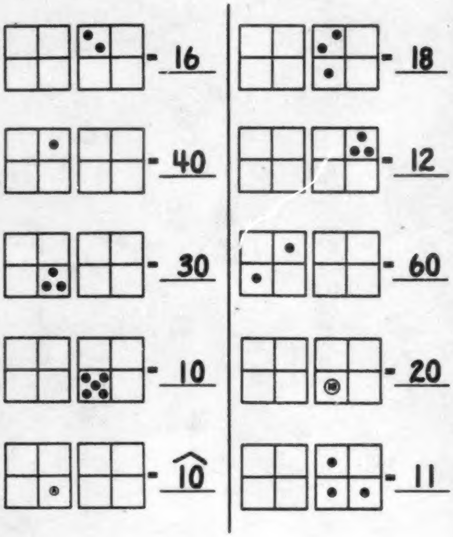
3

Label the dots.
If 20 is not in this picture, extend it to include 20.


12
+3
15
18
21
24
13
16
19
22
14
17
20
23

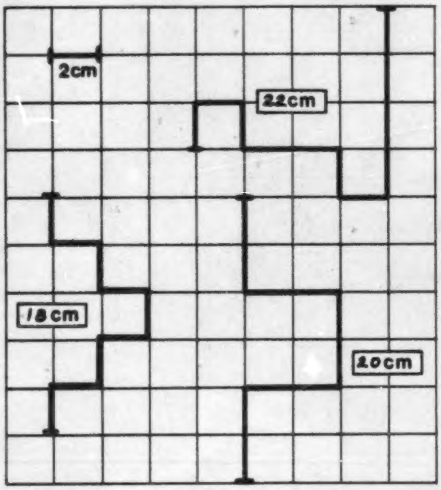
4

What number is on the Minicomputer?

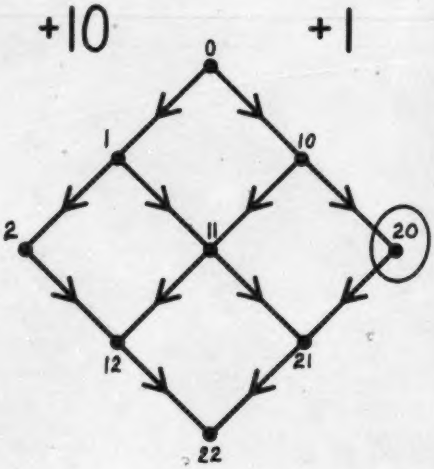

16
18
40
12
30
60
10
20
$\widehat{10}$
11

5

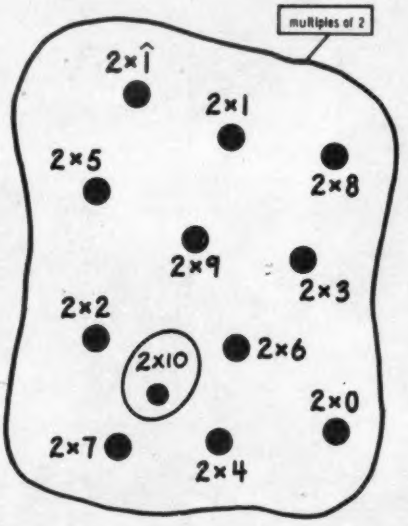
How long are these zigzag paths?
For each zigzag, put your answer in the [illegible] of the same color.


2cm
22cm
18cm
20cm

Label the dots. 20 is in this picture.

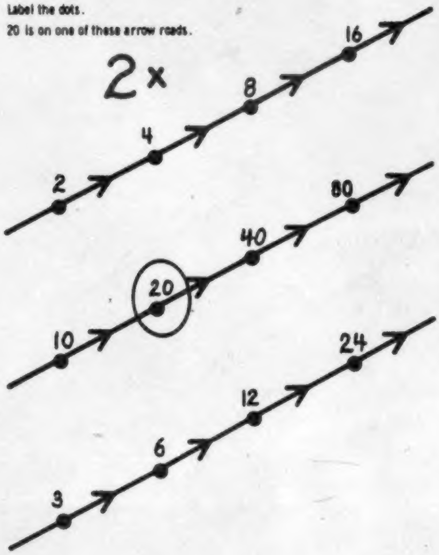

+10
+1
0
1
10
2
11
20
12
21
22

Find or put 20 in this picture.

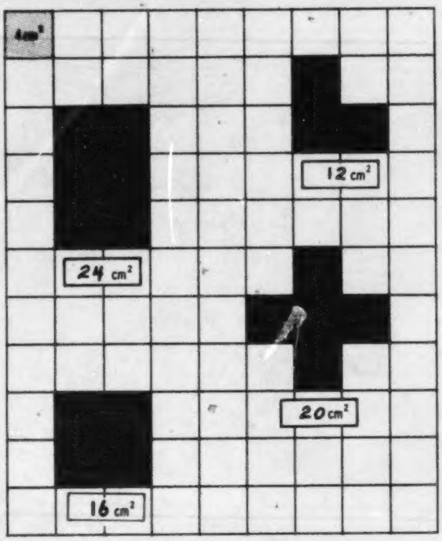

multiples of 2
2×1̂
2×1
2×5
2×8
2×9
2×3
2×2
2×10
2×6
2×0
2×7
2×4

Other names for 20 could be used to label its dot.

Label the dots.
20 is on one of these arrow roads.

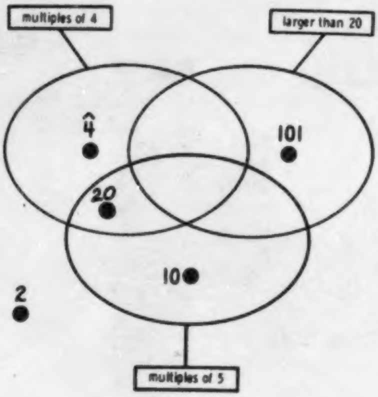

2×
2
4
8
16
10
20
40
80
3
6
12
24

What are the areas of these shapes?


4cm²
12 cm²
24 cm²
20cm²
16 cm²

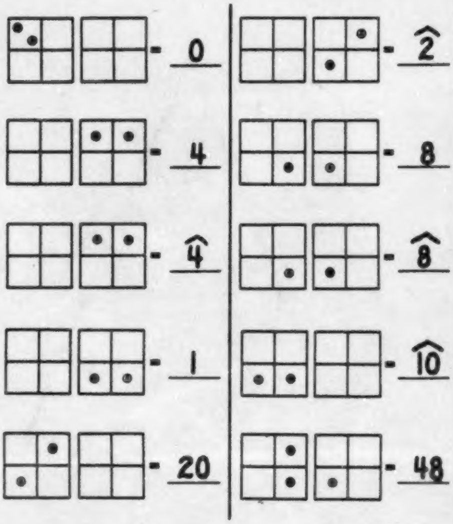
10

Draw and label a dot for 20.

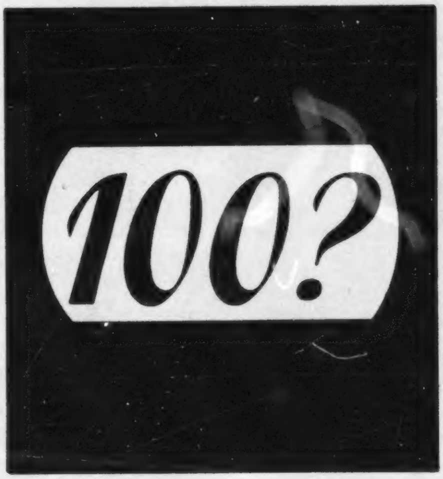

multiples of 4
larger than 20
$\hat{4}$
101
20
10
2
multiples of 5

11

What number is on the Minicomputer?

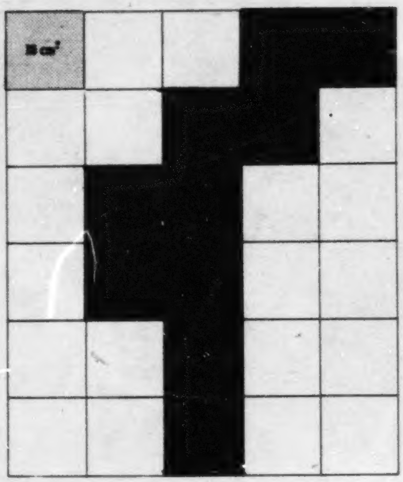

0
4
$\hat{4}$
1
20
$\hat{2}$
8
$\hat{8}$
$\widehat{10}$
48

12



13

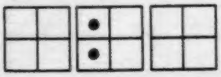
Color a shape with an area of 100 cm².



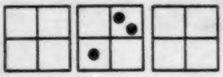
14

Many colorings are possible.

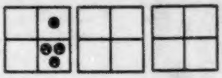
Put 100 on the Minicomputer using exactly two checkers.



Put 100 on the Minicomputer using exactly three checkers.



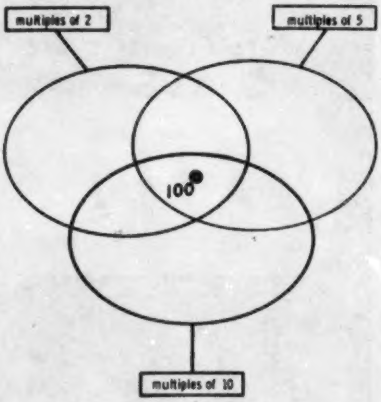
Put 100 on the Minicomputer using exactly four checkers.



15

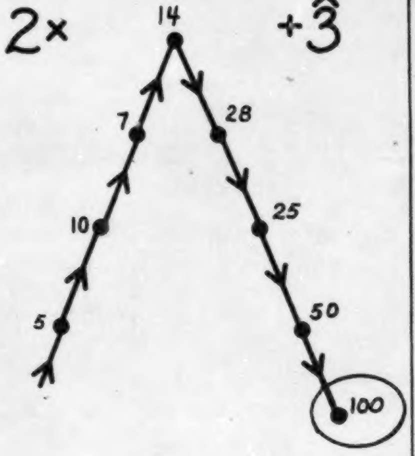
Many solutions are possible for each problem.

Draw and label a dot for 100 in this picture.


multiples of 2
multiples of 5
multiples of 10
100

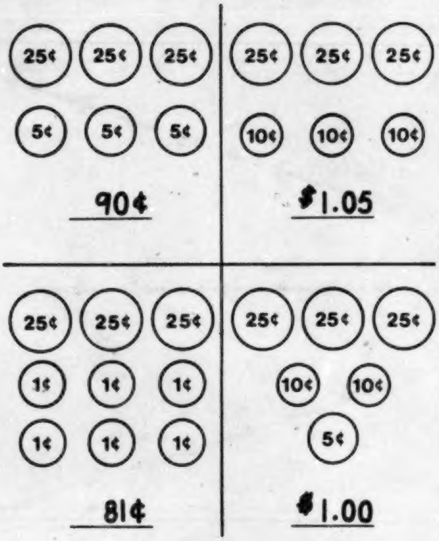
16

Label the dots.
If 100 is not in this picture, extend it to include 100.

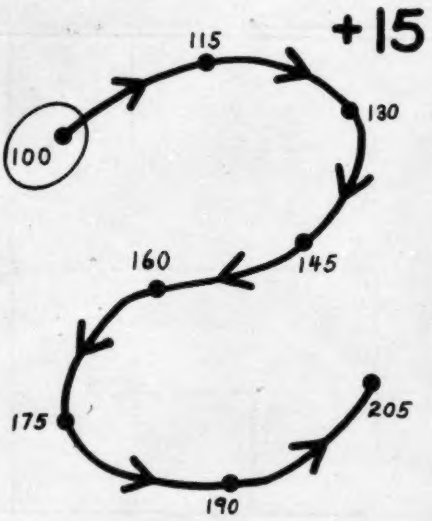

2×
+3̂
14
7
28
10
25
5
50
100

17

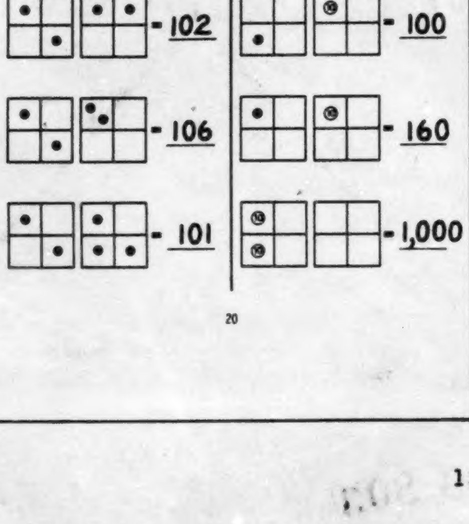
How much money?


25¢
25¢
25¢
25¢
25¢
25¢
5¢
5¢
5¢
10¢
10¢
10¢
90¢
$1.05
25¢
25¢
25¢
25¢
25¢
25¢
1¢
1¢
1¢
10¢
10¢
1¢
1¢
1¢
5¢
81¢
$1.00

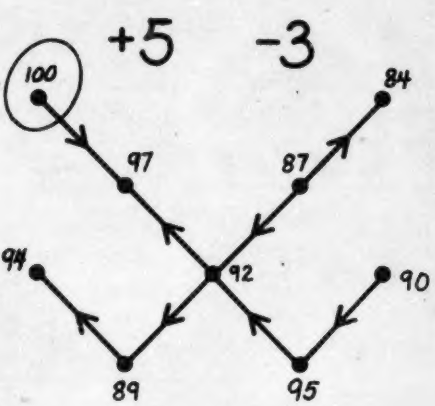
100 is the smallest number in this arrow picture.
Label the dots.


+15
115
130
100
145
160
175
205
190

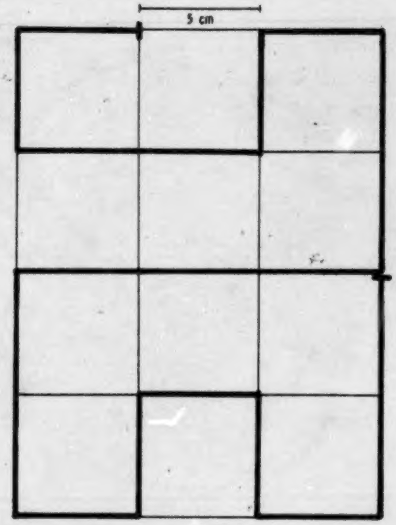
What number is on the Minicomputer?


99
10
98
110
102
100
106
160
101
1,000

Label the dots.
If 100 is not in this picture, extend it to include 100.

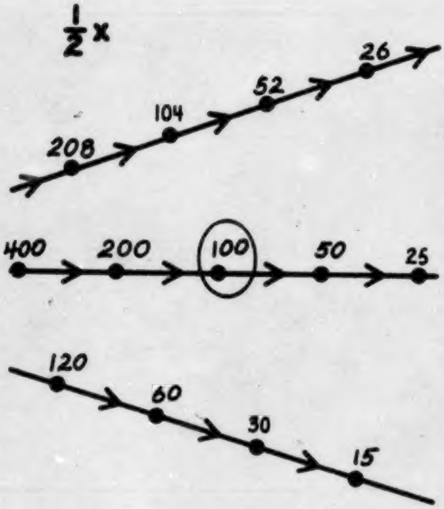

100
+5
-3
84
97
87
94
92
90
89
95

Draw a zigzag path which is 100 cm long.
Your path should follow the lines.

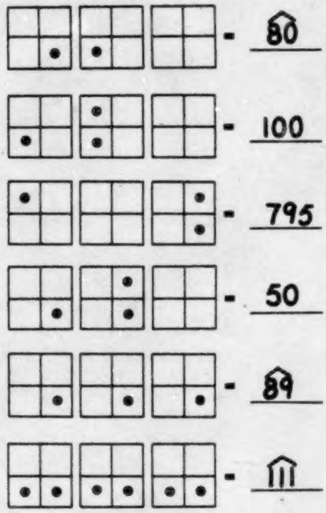


Many solutions are possible.

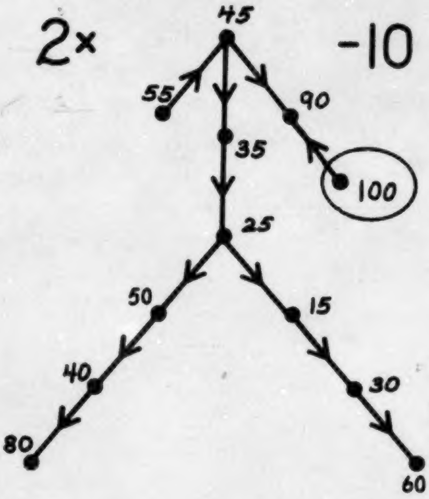
Label the dots.
If 100 is not in this picture, extend it to include 100.


$\frac{1}{2}$x
208
104
52
26
400
200
100
50
25
120
60
30
15

What number is on the Minicomputer?

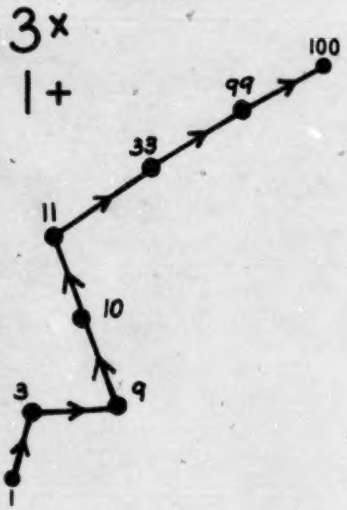

80
100
795
50
89
III

Label the dots.
If 100 is not in this picture, extend it to include 100.


2x
-10
45
55
35
90
100
25
50
15
40
30
80
60

There are other ways to extend the picture to include 100.

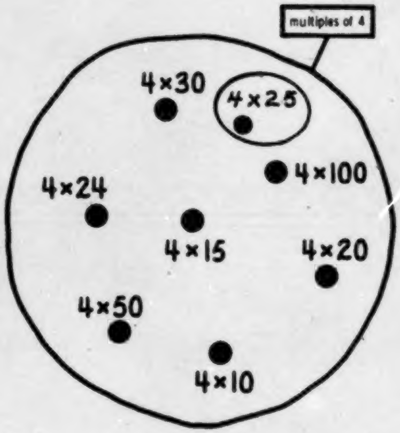
Build a red-blue road from 1 to 100.


3x
1+
100
99
33
11
10
3
9
1

26

There are many other solutions; however, this is a shortest arrow road.

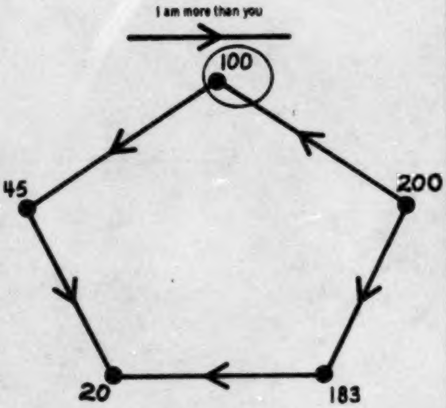
Find or put 100 in this picture.


multiples of 4
4×30
4×25
4×100
4×24
4×15
4×20
4×50
4×10

27

Other names for 100 could be used to label its dot.

100 is in this arrow picture. Label all the dots. Many answers are possible.

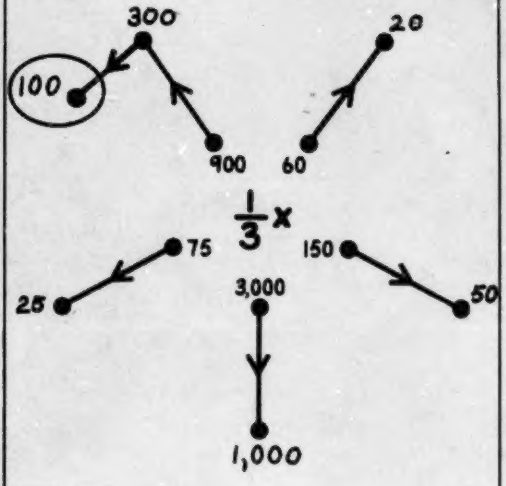

I am more than you
100
45
200
20
183

28

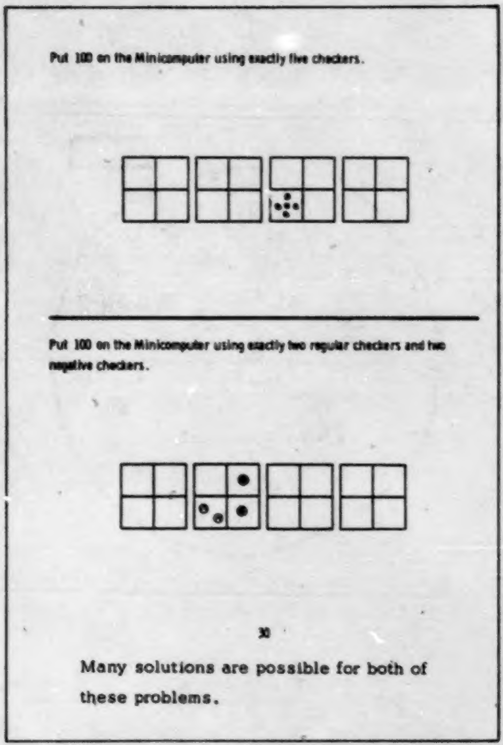
Many solutions are possible; however, 100 must be at the top dot.

Label the dots.

If 100 is not in this picture, extend it to include 100.


300
100
20
900
60
$\frac{1}{3}$x
75
150
25
3,000
50
1,000

29

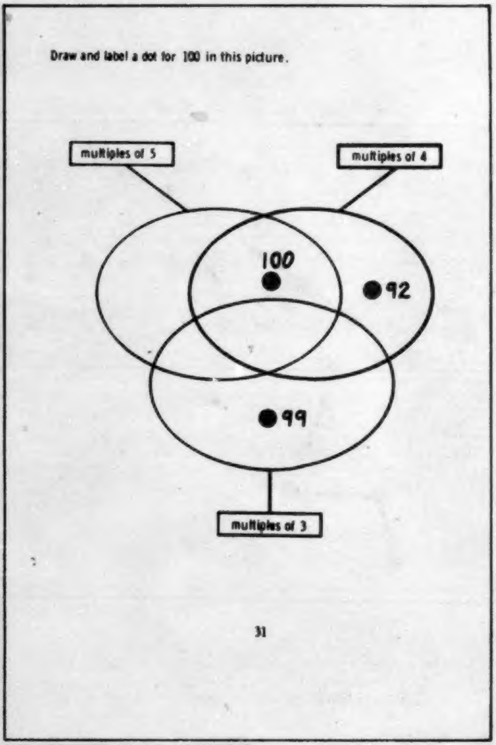


Put 100 on the Minicomputer using exactly five checkers.

Put 100 on the Minicomputer using exactly two regular checkers and two negative checkers.

30

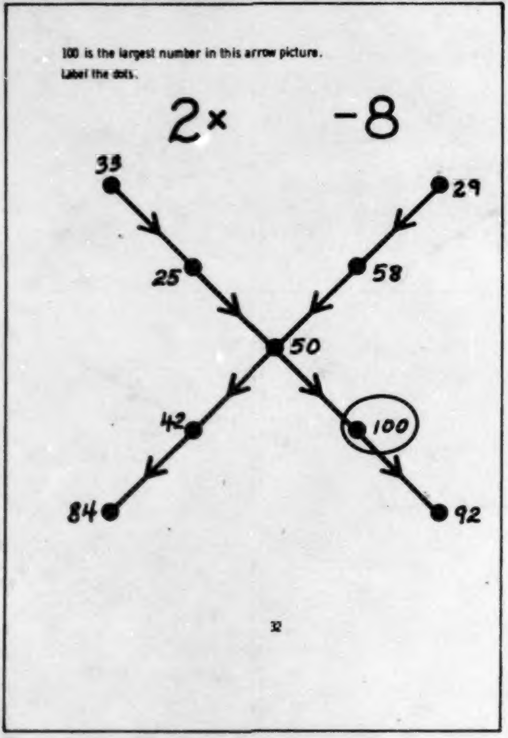
Many solutions are possible for both of these problems.



Draw and label a dot for 100 in this picture.

multiples of 5
multiples of 4
100
92
99
multiples of 3

31



100 is the largest number in this arrow picture.
Label the dots.

2×
-8
33
29
25
58
50
42
100
84
92

32

W18 Who are Tic and Tac?

CAPSULE LESSON SUMMARY

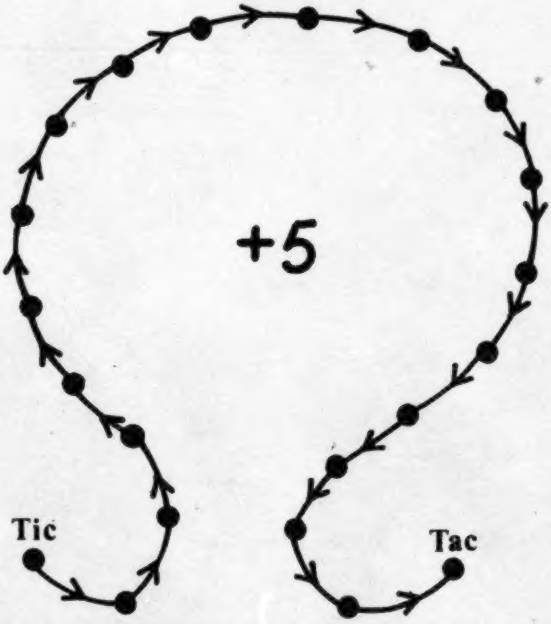
Identify two number friends using clues which involve arrow pictures.

MATERIALS

Teacher: Colored chalk
Student: Worksheets W18*, **, ***; pencil

DESCRIPTION OF LESSON

Draw this arrow picture on the board and distribute copies of Worksheet W18.



T: I have two number friends, Tic and Tac. Tic and Tac are whole numbers. Can you see them in this arrow picture?

Invite a student to point to Tic and Tac.

W 18

T: Look carefully at this arrow picture.
What are the red arrows for?

S: +5.

T: How many red arrows are there between Tic and Tac?

S: Twenty.

T: Which is the largest number in this arrow picture? (Tac)
Which is the smallest number? (Tic)

If Tic were 10, what would Tac be? Write your answer on the back of your worksheet.

Look at many of the students' answers. Indicate to students individually if their answers are correct or incorrect. Ask a student with the correct answer to answer aloud.

S: If Tic were 10, Tac would be 110.

T: How did you determine that Tac would be 110?

S: I started at 10 and counted by fives.

S: 20 x 5 = 100, so I added "10 + 100".

S: I figured out what number would be at each dot by adding 5 each time.

T: If Tic were 100, what would Tac be? Write your answer on the back of your worksheet.

Look at many of the students' answers before asking a student to answer aloud.

S: If Tic were 100, Tac would be 200.

T: How do you know?

Encourage several students to explain how they calculated the answer.

Draw this table on the board near the arrow picture.

Tic	Tac
10	
100	

T: If Tic were 10, what would Tac be?

S: 110.

Write "110" in the column for Tac.

T: If Tic were 100, what would Tac be?

S: 200.

Write "200" in the column for Tac; then write "20" in the column for Tic in the next row.

Tic	Tac
10	110
100	200
20	

W 18

T: If Tic were 20, what would Tac be? Write your answer on the back of your worksheet.

Look at many of the students' papers before you ask someone to answer aloud. Conclude that if Tic were 20, then Tac would be 120 and record "120" in your table. Continue this activity for Tic being 50, 27, and 184. Enter the appropriate numbers in the table.

Tic	Tac
10	110
100	200
20	120
50	150
27	127
184	284

T: Listen carefully. If Tac were 361, what would Tic be? Write your answer on the back of your worksheet.

Write "361" in the column for Tac. Emphasize that Tac is given as 361 and that they are to find Tic. Look at many of the students' papers; then ask someone to answer aloud.

S: If Tac were 361, Tic would be 261.

Enter "261" in the column for Tic.

Tic	Tac
10	110
100	200
20	120
50	150
27	127
184	284
261	361

T: Do you notice any patterns in this table?

S: Tac is always 100 more than Tic.

Draw a blue arrow from Tic to Tac in the arrow picture.

T: What could this blue arrow be for?

S: +100.

T: How do you know?

S: 20 x 5 = 100.

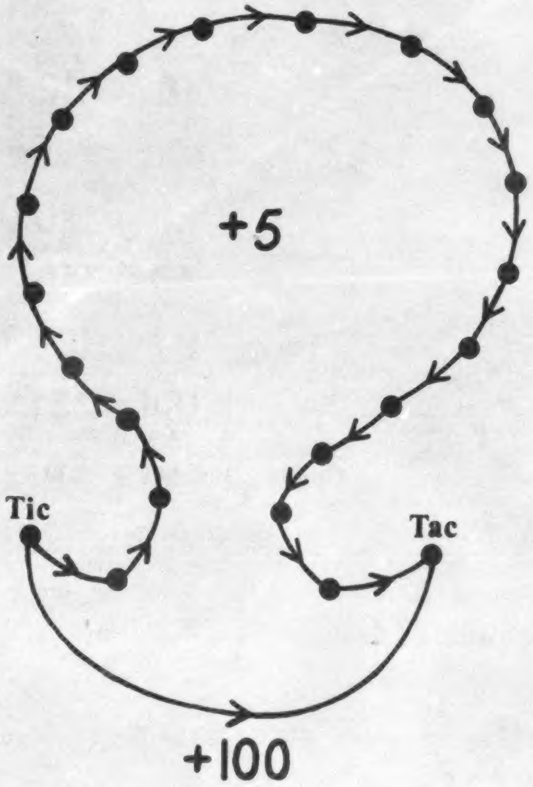
S: Tac is always 100 more than Tic.

S: I counted by fives.

W 18

Write "+100" in blue near the blue arrow and at the top of the table.

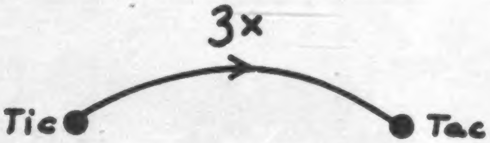
+100	
Tic	Tac
10	110
100	200
20	120
50	150
27	127
184	284
261	361



T: Do we know which numbers Tic and Tac are?

S: No, all we know is that Tac is 100 more than Tic.

On another part of the board draw this arrow picture.



T: What does this arrow picture tell you about Tic and Tac?

S: Tac is three times as large as Tic.

T: For the moment let's concentrate on this arrow picture only. If Tic were 5, what would Tac be? How do you know?

S: 15, because 3 × 5 = 15.

T: If Tic were 100, what would Tac be? Write your answer on the back of your worksheet.

Look at many of the students' papers; then ask a student to answer aloud.

S: If Tic were 100, then Tac would be 300 because 100 + 100 + 100 = 300.

Draw this table on the board near the 3 × arrow picture.

3×	
Tic	Tac
5	
100	

T: Tic × 3 = Tac.
If Tic were 5, what would Tac be?

S: 15.

T: 3 × 5 = 15.

W 18

Write "15" in the column for Tac.

T: If Tic were 100, what could Tac be?

S: 300.

Write "300" in the column for Tac and then write "31" in the column for Tic.

3×	
Tic	Tac
5	15
100	300
31	

T: If Tic were 31, what would Tac be? Write your answer on the back of your worksheet.

Look at many of the students' papers before you ask a student to answer aloud. Conclude that Tac would be 93. Ask a few students to explain how they calculated the answer, and record "93" in the table. Continue this activity for Tic being 120 and 251. Enter the appropriate numbers in the table.

3×	
Tic	Tac
5	15
100	300
31	93
120	360
251	753

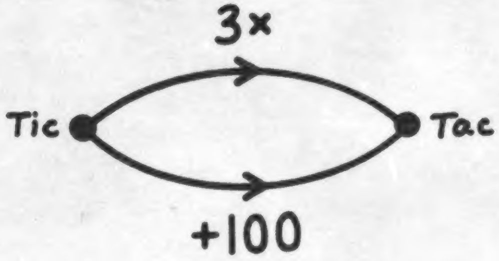
T: Listen carefully. If Tac were 126, what would Tic be? Write your answer on the back of your worksheet.

Enter "126" in the column for Tac. Emphasize that Tac is given as 126 and that they are to find Tic. Look at many of the students' papers before asking a student to answer aloud. Conclude that Tic would be 42 and enter "42" in the table.

NOTE: This is a difficult problem. If none of your students are able to find Tic, suggest that they determine what Tic would be if Tac were 120. When one of the students discovers that 3 × 40 = 120, you should return to the original problem. If necessary, suggest that since 3 × 40 = 120, Tic would be just a little bit more than 40 if Tac were 126.

T: We know that Tac is 3 × Tic. We also know that Tac is 100 more than Tic. Now I will put these two clues together in this arrow picture.

Draw a blue arrow from Tic to Tac in the second arrow picture. Write "+100" in blue near the blue arrow.



T: Which numbers are Tic and Tac?

Encourage the students to investigate this problem on their own; do not expect any of them to solve it quickly. After a few minutes discuss the students suggestions for Tic and Tac collectively. A sample dialogue is given here.

S: I think that Tic is 100 and Tac is 300.

Label the dot for Tic "100" and then trace the green arrow.

T: 3 × 100 = . . . ?

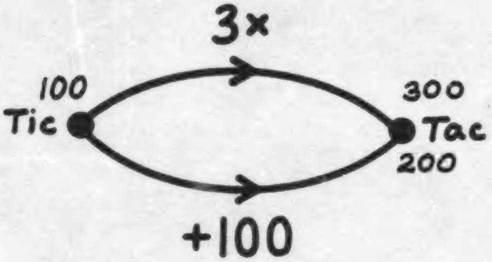
S: 300.

Write "300" near the dot for Tac. Trace the blue arrow.

T: 100 + 100 = . . . ?

S: 200.

Write "200" near the dot for Tac.



T: Tac cannot be both 300 and 200, so Tic cannot be 100.

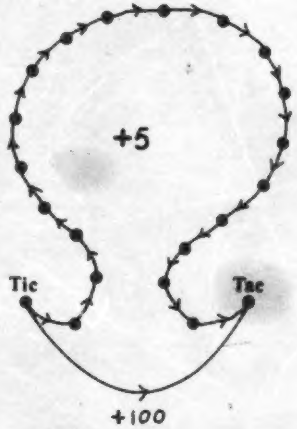
Erase "100", "300", and "200" in the arrow picture, and in both tables cross out the rows corresponding to "Tic = 100".

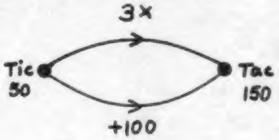
Encourage other students to suggest their ideas. Continue this activity until someone suggests that Tic is 50 and Tac is 150.

If no one suggests that Tic is 50, refer to the numbers in the +100 table for Tic and Tac and ask if any of these numbers would work in the second arrow picture.

Conclude that 3 × 50 = 150 and 50 + 100 = 150, so Tic is 50 and Tac is 150. When you finish, your board might look something like this.

+100	
Tic	Tac
10	110
[illegible]	[illegible]
[illegible]	[illegible]
50	150
27	127
184	284
261	361



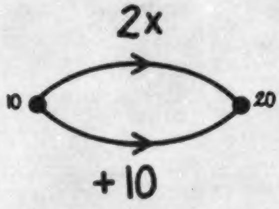


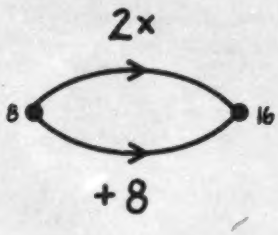
3 ×	
Tic	Tac
5	15
[illegible]	[illegible]
31	93
120	360
251	753
[illegible]	[illegible]

Worksheets W18** and *** are available for individual work.

Name ______________________ W 18 **

Label the dots.





Name ______________________ W 18 ***

Label the dots.